reinforced concrete construction

Concrete Construction
- cast-in-place
- tilt-up
- prestressing
- post-tensioning

Concrete Materials
- low strength to weight ratio
- relatively inexpensive
  - Portland cement
  - aggregate
  - water
Concrete Materials

- reinforcement
  - deformed bars
  - prestressing strand
  - stirrups
  - development length
  - anchorage
  - splices

Concrete Beams

- types
  - reinforced
  - precast
  - prestressed

- shapes
  - rectangular, I
  - T, double T’s, bulb T’s
  - box
  - spandrel

Concrete Materials

- fire resistance
  - most fire-resistive structural material
  - low rate of penetration
  - retains strength if exposure not too long
    - stable to 900 – 1200 °F internally
    - loses 50% after that
  - no toxic fumes
  - cover necessary to protect steel

Concrete Beams

- deformation
  - camber (elastic)
    - hogging
    - sagging
  - shrinkage strain
    - 200-400 x 10^-6
    - about 2-3 years
  - creep strain
    - 2~3 times elastic strain
    - about 2-3 years
Concrete Beams

- shear
  - vertical
  - horizontal
  - combination:
    - tensile stresses at 45°
- bearing
  - crushing

Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for failure
  - strength design (LRFD)
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity
  - concrete strength = f'c

Behavior of Composite Members

- plane sections remain plane
- stress distribution changes

Transformation of Material

- n is the ratio of E’s
  \[ n = \frac{E_2}{E_1} \]
- effectively widens a material to get same stress distribution
Stresses in Composite Section

- with a section transformed to one material, new
  \[ n = \frac{E_2}{E_1} = \frac{E_{\text{steel}}}{E_{\text{concrete}}} \]
  - stresses in that material are determined as usual
  - stresses in the other material need to be adjusted by \( n \)

\[ f_c = -\frac{M_y}{I_{\text{transformed}}} \]
\[ f_s = -\frac{M_{yn}}{I_{\text{transformed}}} \]

Reinforced Concrete Analysis

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent stress block
  - concrete takes no tension
  - steel takes tension
  - force ductile failure

Reinforced Concrete - stress/strain

Location of n.a.

- ignore concrete below n.a.
- transform steel
- same area moments, solve for \( x \)

\[ bx \cdot \frac{x}{2} - nA_s (d - x) = 0 \]
**T sections**

- n.a. equation is different if n.a. below flange

\[
bf \left(x - \frac{hf}{2}\right) + \left(x - hf\right)bw \left(x - hf\right) - nA_s (d - x) = 0
\]

**ACI Load Combinations**

- 1.4D
- 1.2D + 1.6L + 0.5(L_r or S or R)
- 1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)
- 1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)
- 1.2D + 1.0E + 1.0L + 0.2S
- 0.9D + 1.0W
- 0.9D + 1.0E

*can also use old ACI factors

**Reinforcement**

- deformed steel bars (rebar)
  - Grade 40, \(F_y = 40\) ksi
  - Grade 60, \(F_y = 60\) ksi - most common
  - Grade 75, \(F_y = 75\) ksi
  - US customary in # of 1/8” \(\phi\)
- longitudinally placed
  - bottom
  - top for compression reinforcement
  - spliced, hooked, terminated...

**Reinforced Concrete Design**

- stress distribution in bending

\[A_c = 0.85f'_c\]

Wang & Salmon, Chapter 3
Force Equations

- \( C = 0.85 \, f' \, c \, ba \)
- \( T = A_s \, f_y \)

where
- \( f' \, c \) = concrete compressive strength
- \( a \) = height of stress block
- \( \beta_1 \) = factor based on \( f' \, c \)
- \( c \) = location to the n.a.
- \( b \) = width of stress block
- \( f_y \) = steel yield strength
- \( A_s \) = area of steel reinforcement

Over and Under-reinforcement

- **over-reinforced**
  - steel won’t yield

- **under-reinforced**
  - steel will yield

reinforcement ratio
- \( \rho = \frac{A_s}{bd} \)
  - use as a design estimate to find \( A_s, b, d \)
  - max \( \rho \) is found with \( \varepsilon_{steel} \geq 0.004 \) (not \( \rho_{bal} \))
  - *with \( \varepsilon_{steel} \geq 0.005, \phi = 0.9 \)

Equilibrium

- \( T = C \)
- \( M_n = T(d - a/2) \)
  - \( d \) = depth to the steel n.a.

with \( A_s \)
- \( a = A_s \, f_y \)
- \( \phi = 0.65 + (\varepsilon_y - \varepsilon_s) \frac{0.25}{(0.005 - \varepsilon_y)} \geq 0.65 \)

- \( M_u \leq \phi M_n \)
  - \( \phi = 0.9 \) for flexure*
  - \( \phi M_n = \phi T(d - a/2) = A_s \, f_y \, (d - a/2) \)

\( A_s \) for a given Section

- several methods
  - guess \( a \) and iterate
    1. guess \( a \) (less than n.a.)
    2. \( A_s = \frac{0.85 \, f' \, c \, ba}{f_y} \)
    3. solve for \( a \) from \( M_u = \phi A_s \, f_y \, \left(d - \frac{a}{2}\right)\)
      \[
      a = 2 \left( d - \frac{M_u}{\phi A_s \, f_y} \right)
      \]
    4. repeat from 2. until \( a \) from 3. matches \( a \) in 2.
A_s For Given Section (cont)

- chart method
  - Wang & Salmon
    - Fig. 3.8.1 $R_n$ vs. $\rho$
  1. calculate $R_n = \frac{M_n}{bd^2}$
  2. find curve for $f'_c$ and $f_y$ to get $\rho$
  3. calculate $A_s$ and $a$
- simplify by setting $h = 1.1d$

Shear in Concrete Beams

- flexure combines with shear to form diagonal cracks

Shear in Concrete Beams

- horizontal reinforcement doesn’t help
- stirrups = vertical reinforcement

ACI Shear Values

- $V_u$ is at distance $d$ from face of support
- shear capacity: $V_c = \nu_c \times b_w d$
  - where $b_w$ means thickness of web at n.a.
- shear stress (beams)
  - $\nu_c = 2\lambda \sqrt{f'_c}$
  - $\phi = 0.75$ for shear
  - $f'_c$ is in psi
  - $\phi V_c = \phi 2\lambda \sqrt{f'_c} b_w d$
  - $\lambda$ for lightweight materials
- shear strength: $V_u \leq \phi V_c + \phi V_s$
  - $V_s$ is strength from stirrup reinforcement

Stirrup Reinforcement

- shear capacity:
  - $V_s = \frac{A_v f_y d}{s} \leq 8\sqrt{f'_c b_w d}$
    - $A_v$ = area in all legs of stirrups
    - $s$ = spacing of stirrup
- may need stirrups when concrete has enough strength!
Required Stirrup Reinforcement

- spacing limits

<table>
<thead>
<tr>
<th>Stirrup spacing, a</th>
<th>Required area of stirrups, $A_{st}$</th>
<th>Recommended Minimum $l$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\psi_V &gt; V_u$; $\psi_V = 0.75 \psi_{V, max}$</td>
<td>--</td>
</tr>
<tr>
<td></td>
<td>$\psi_V = \psi_V$</td>
<td>--</td>
</tr>
</tbody>
</table>

Concrete Deflections

- elastic range
  - I transformed
  - $E_c$ (with $f_c'$ in psi)
    - normal weight concrete (~ 145 lb/ft³)
      \[
      E_c = 57,000 \sqrt{f_c'}
      \]
    - concrete between 90 and 155 lb/ft³
      \[
      E_c = \frac{w_{c, f}}{33.5} \sqrt{f_c'}
      \]
- cracked
  - I cracked
  - $E$ adjusted

Deflection Limits

- relate to whether or not beam supports or is attached to a damageable non-structural element
- need to check service live load and long term deflection against these

<table>
<thead>
<tr>
<th>Deflection Limit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/180</td>
<td>roof systems (typical) – live</td>
</tr>
<tr>
<td>L/240</td>
<td>floor systems (typical) – live + long term</td>
</tr>
<tr>
<td>L/360</td>
<td>supporting plaster – live</td>
</tr>
<tr>
<td>L/480</td>
<td>supporting masonry – live + long term</td>
</tr>
</tbody>
</table>

Prestressed Concrete

- impose a longitudinal force on a member in order to withstand more loading until the member reaches a tensile limit
Prestressed Concrete

- pretensioned
  - reinforcement bonded
- post-tensioned
  - bonded or unbonded
  - end bearing
- precast
  - concrete premade in a position other than its final position in the structure

Prestressed Concrete

- high strength tendons
  - grade 250
  - grade 270

\[
\begin{align*}
\text{Axial prestress (e=0)}: \\
& f' = \frac{P}{A} - \frac{M_c}{I_g} \\
& f_b = \frac{P}{A} + \frac{M_c}{I_g} \\
& c - \text{distance to fiber} \\
& I_g - \text{gross cross section inertia}
\end{align*}
\]

\[
\begin{align*}
\text{Axial prestress (e\neq 0)}: \\
& f' = \frac{P}{A} - \frac{P \cdot f_c}{A} - \frac{M_c}{I_g} \\
& f_b = \frac{P}{A} + \frac{P \cdot f_c}{A} + \frac{M_c}{I_g} \\
& (\text{remember} \quad f = \frac{f_b}{I_g})
\end{align*}
\]
**Prestressed Concrete**

![Diagram of Prestressed Concrete](image)

**Composite Beams**
- concrete
  - in compression
- steel
  - in tension
- shear studs

**Continuous Beams**
- reduced size
- reduced moments
- moments can reverse with loading patterns
- need top & bottom reinforcement
- sensitive to settlement
Concrete Columns

- columns require
  - ties or spiral reinforcement to “confine” concrete (#3 bars minimum)
  - minimum amount of longitudinal steel (4 bars minimum)

Concrete Columns

- effective length in monolithic casts must be found with respect to stiffness of joint
- not slender when
  \[
  \frac{kL_i}{r} < 22 
  \]
  \( r \) not braced
Concrete Columns

- \( P_o \) – no bending
  \[
  P_o = 0.85 f'_c (A_g - A_{st}) + f_y A_{st}
  \]
- \( \phi_c = 0.65 \) for ties with \( P_n = 0.8P_o \)
- \( \phi_c = 0.70 \) for spirals with \( P_n = 0.85P_o \)
- \( P_u \leq \phi_c P_n \)
- nominal axial capacity:
  -presumes steel yields
  -concrete at ultimate stress

Columns with Bending

- eccentric loads can cause moments
- moments can change shape and induce more deflection \((P-\Delta)\)

Columns with Bending

- for ultimate strength behavior, ultimate strains can’t be exceeded
  -concrete 0.003
  -steel \( \frac{f_y}{E_s} \)
- \( P \) reduces with \( M \)

Columns with Bending

- need to consider combined stresses
  \[
  \frac{P_n}{P_o} + \frac{M_n}{M_o} \leq 1
  \]
- plot interaction diagram
Concrete Floor Systems

- types & spanning direction

Concrete Floor Systems

- one-way and two-way moments
- flexure design as T-beams (+/- M)
- increase of 10% $V_c$ permitted
- slabs need steel
- effective width is
  - $L/4$
  - $b_w + 16t$
  - center-to-center of beams

One-way Joists

- standard stems
- 2.5” to 4.5” slab
- ~30” widths
- reusable forms
One-way Joists
- wide pans
- 5', 6' up
- light loads & long spans
- one-leg stirrups

Two-way Joists
- domed pans
- 3', 4' & 5'

Construction Supervision
- proper placement of all reinforcement
  - welding
  - splices
- mix design
  - slump
- in-situ strength
  - cast cylinders
  - cylinder cores – if needed