



VAULTING OF NARBONNE CATHEDRAL

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Abstract

The design of Narbonne Cathedral constructed in the 13th century followed the rayonnant gothic style of cathedrals in the north of France, rather than the regional style of south-central France. The structure has been carefully measured and recorded in order to identify the design issues and decisions made by the builders. The ribs of the choir in the eastern straight bay form half a sexpartite vaulted unit with the hemicycle vaults, possibly eccentrically loading the flying buttresses. This investigation models the structural behavior of the vault construction and its effect on the buttressing by addressing the stiffness of the transverse arch, rubble fill placement, vault thickness, bay width, and wind and roof loading. The results of the finite element modeling of the vaulting are compared to recorded measurements and crack formation, and the rationale of the construction is presented.

Keywords: historical masonry, structural modeling, finite element analysis, vaulting

Introduction

According to historical documents, the Cathedral of Narbonne in south central France was built, “in the manner of the noble cathedrals of the Kingdom of France”. For those who began the construction of the cathedral in 1272, this implied something quite different from regional preferences for low, wide, aisleless structures with wall buttresses. It meant a building that would convey an impression of power and authority suitable to the Archbishopric of Narbonne. It meant a building of imposing, monumental scale. It meant a building with a

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three-aisled plan, ambulatory, radiating chapels and transept. It meant a tall, three-storied elevation with triforium and clerestory. It meant a particular collection of formal details. And it meant a rib-vaulted structure with flying buttresses. But these were at best vague, generic concepts. It was the various masters of the works who translated these notions into physical form (See Figure 1).



Figure 1. Narbonne Cathedral Exterior with Buttressing

The building was never completed (See Figure 2). By about 1320 construction had reached the city wall, and order to finish construction it would have been necessary to breach the wall. That necessity was successfully challenged between 1349 and 1354 by the town consuls in a legal battle with the cathedral chapter before the king's representative. The consuls were particularly concerned with what appeared to be fortifications: the crenellated turret towers atop the outer piers of the flying buttresses. The masters of the works, called to give "expert witness" downplayed the idea of fortifications. Although there is no record of their exact words, they apparently stated that what the consuls called towers were really piers, and they and the arches in the upper parts of the building were there to buttress (push against) the vaults and roof. The crenellation was there as decoration.

It is true that the solution adopted for the buttresses at Narbonne is unique, not only in the crenellated turret towers and their connecting bridges, but also in the fact that the lower flyer of the outer rank along the straight bays of the choir assumes the form of a flat "strut" carried by a round arch rather than the angled configuration used in many Gothic buildings and in the outer flyers of the radiating chapels at Narbonne itself (Figure 3). The form seems vaguely to suggest comparisons with timber architecture, but the question here is not to determine the model upon which the master of this portion of Narbonne Cathedral drew, but whether or not

the adopted solution is more structurally efficient or effective than the more common solution in which the outer flyers approximates the angle of the inner flyers.

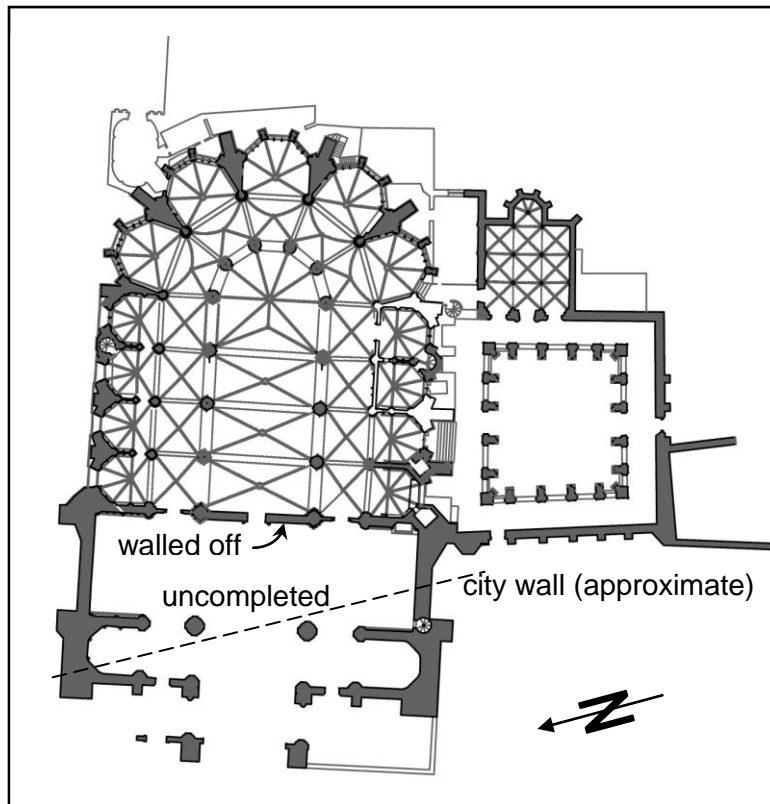


Figure 2. Narbonne Cathedral Plan with Vaulting Pattern

During a ten year period the cathedral of Narbonne was measured and documented by a team from Texas A&M University. The drawings and measurements resulting from that process have been used to produce a model that could be subjected to various structural analyses in an attempt to test the efficacy of solutions such as the unusual buttressing pattern.

Background

The measurements collected and the historical records suggest that the large crossing piers of 6.4 ft (1.96 m) in diameter (shown on Figure 2 embedded in the wall constructed to close off the unfinished choir) were placed at the highest elevation in the city, presumably directly over bedrock. The topography slopes steeply away from that point, and the massive size of the foundations is indicative of the loads anticipated from the building weight and height of the structure. The presence of an underground Roman

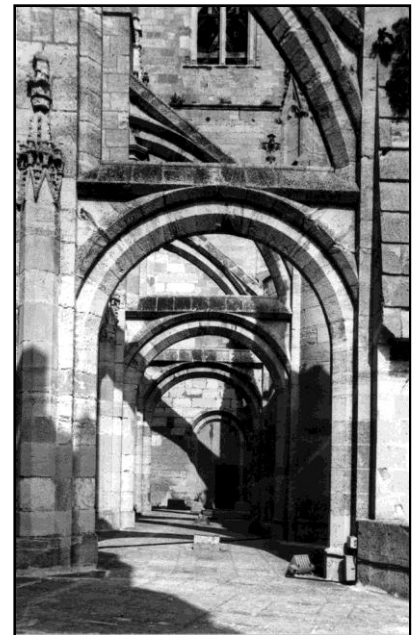


Figure 3. Flat Arch or "Strut" of Buttressing

horreum (storehouse) to the north of the eastern portions of the choir, may have given rise to additional settlement concerns.

The building is tall, being the fourth tallest cathedral in France, measuring over 133 feet (40.6 m) from floor to the interior of the main vault. Given its location approximately 8 miles (13 km) from the Mediterranean coast, the effect of wind gusts were most likely a concern, particularly given the building's height and location at the highest point in the city.

Construction

The main arcade piers of the choir are not quite aligned, with the south side of the building being slightly longer than the north. In addition, the buttresses on the north side do not project to the same extent as those on the south side of the building (See Figure 4) due to the presence of a street. The two eastern chapels on the south side of the building are walled off from the aisles to form two-storied chambers. The lower level serves as a sacristy and the upper as the treasury (Figure 4).

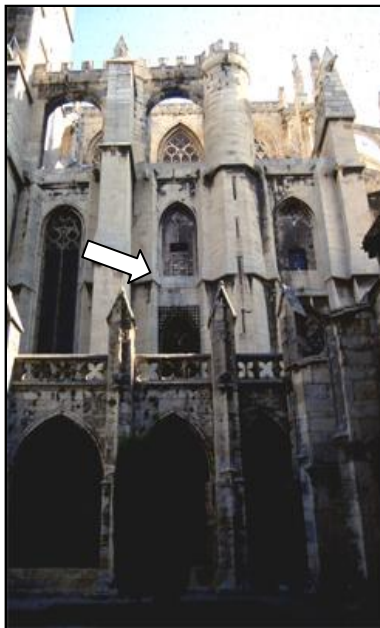


Figure 4. South Elevation Indicating Location of Treasury



Figure 5. Interior North Elevation Toward East

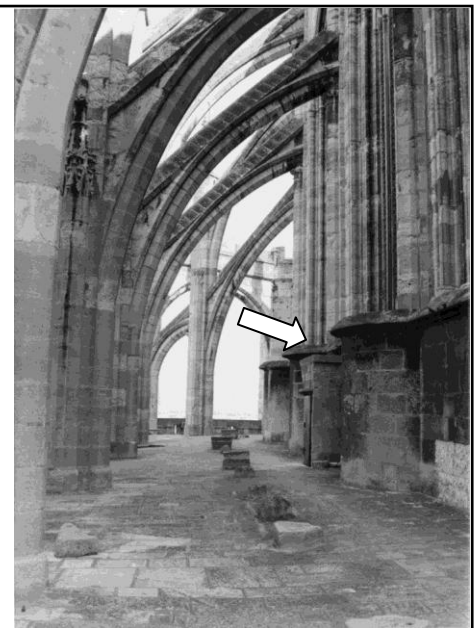


Figure 6. Triforium Passage Wall and Roof, North Side Toward East

The cathedral is constructed of large, hard fossiliferous limestone blocks to the level of the triforium roof. The triforium passage (Figure 5) is unusual in that it turns outward at the point of intersection with the main aisle piers. The mass of the main arcade piers thus continues without interruption to the upper reaches of the building (Figure 6). Above the level of the triforium, limestone from a second quarry was used. This limestone has quartz veins and has not been as resistant to weathering as the stone in the lower portions of the building.

As the building progress halted at the city wall, the arcade and aisle openings in the west wall of the choir were walled in, leaving unfilled the clerestory window opening into the transept (Figure 2). The roof terminates at this wall as well. Two massive towers are constructed over the chapel abutting the transept on both the southeast and the northeast (See Figure 1).

The choir vaults have a clear span of nearly 50 feet (14.3 m) at the level of the clerestory. The ridge of the window penetration curves gradually upward toward the center of the bay, making the vaults slightly domical in shape. The hemicycle is vaulted with the easternmost bay of the choir, resulting in half a sexpartite unit while the remaining bays are vaulted with quadripartite vaults (Figure 7). Although this arrangement is not uncommon, it is likely to produce eccentric loading at the point where the two vaulted units meet.

A stepped wall is constructed over the transverse arches. There is no reason to suppose that such a wall was not part of the original construction, as it is also found in other Gothic buildings. A wall of this type is likely to have been intended to increase the stiffness of the transverse arch so that during the construction of the vaulting and addition of rubble fill, the arches would not be compressed inward and rise at the peak. The rubble fill is placed in the void formed by the intersection of the transverse arches with the diagonal rib arches at the clerestory wall and is level at approximately the height of the vaulting at the window penetrations.

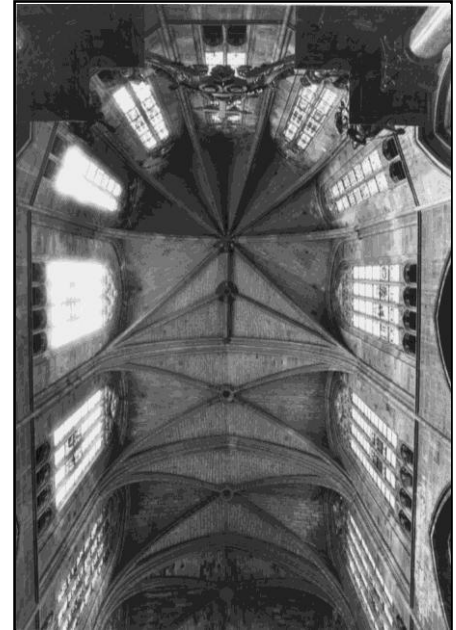


Figure 7. Choir Vaulting

The upper flyers of the buttressing meet the clerestory walls at roughly the height of the top of the clerestory windows, transmitting the roof loads through the buttressing, while the lower flyers meet the clerestory walls just above the convergence of the diagonal rib and transverse arches (Figure 8). The lower flyers are intended to transmit lateral forces from the vaulting. The method used to locate the upper position of the lower flyers is possibly a function of the clear span, but this is not certain.

There are two ranks of flyers, with a slender inner pier and pilasters imbedded in large turret towers over 11 feet (3.47 m) in diameter with the exception of the flyers at the junction with the hemicycle (Figure 1). (The westernmost flyers abut the towers flanking the transept.) The upper and lower flyers of the inner rank have the same radius for the arch and a slope of 45 degrees, while the upper flyers of the outer rank have a slope of 38 degrees. The lower flyers of the outer rank along the straight bays of the choir are not angled, but instead are nearly horizontal, carried by a semicircular arch that gives the appearance of a flat "strut" (Figure 3). The lower flyers in the outer rank of the hemicycle buttresses are at the same angle as the lower flyers in the inner rank. The span of the flyers in the outer rank of the

hemicycle buttresses is less than the span of the outer flyers along the length of the choir. All adjoining turrets have narrow bridges with crenellation.

The existing roof has a relatively shallow pitch (roughly 1:0.6) for the gable. This roof is not medieval, although it is likely that the original pitch was not dissimilar to that of the present structure.

Structural Investigations

With the unusual structural features of the triforium passage, the turret towers on the outer buttress piers, and the horizontal flyer or "strut", the question arises as to why the builders would vary from the more typical arrangement in which both ranks of flyers slope. Given the massive size of the main piers and the height of the structure, were the builders being conservative or daring in this deviation?

With these questions in mind, using the hand-recorded measurements and the scaled drawings made on site, much of which had been transferred to an electronic format, an initial finite element model was constructed in the mid-80's to evaluate the structural behavior of the buttressing system. While Robert Mark had used photoelastic modeling based on interference patterns to provide reasonable assessment of buttressing stresses under wind and gravity loading, finite element analysis was also being used to investigate buttressing behavior, although at a computational cost [Mark 1985].

Early Modeling

Robert Warden, who had assisted in the original recording and documentation project, modeled a planar section that included the aisles (at the lower level adjacent to the arcade), chapel walls, and openings (near the exterior) [Warden 1988, ANSYS 1980]. The triforium passage can be seen in his model adjacent to the main piers at the terrace level, and the horizontal flyers are buttressed by turret towers at the exterior, shown as solid piers (Figure 9-Figure 10). The vaults were modeled with a portion of the rubble infill.

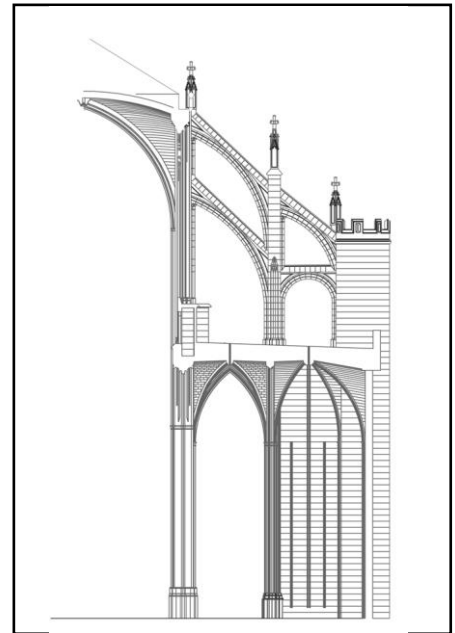


Figure 8. Buttress Section

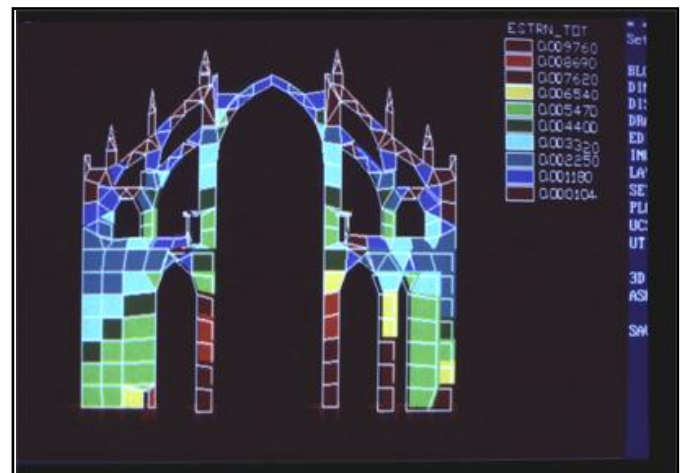


Figure 9. Strain Analysis

The results indicate that under gravity and wind loading there is a tendency for the walls to bow outward, flattening the transverse arch and separating the rubble fill from the clerestory walls. It also implies large displacements at the base of each pier at the upper flyers and at the base of the piers and solid turret towers, with the inner rank of lower flyers tending to separate at the clerestory walls.

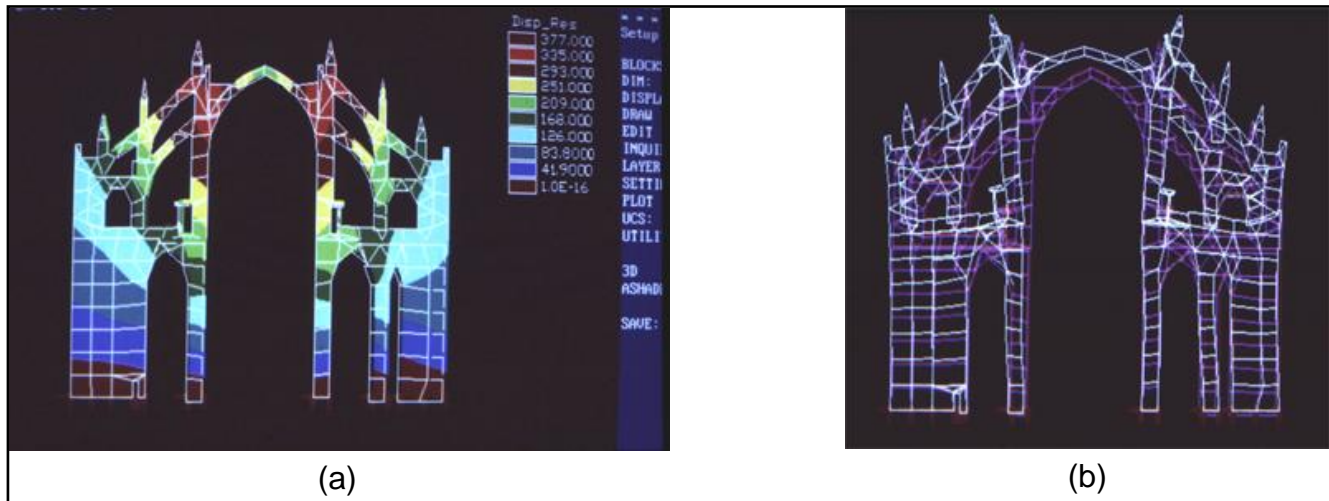


Figure 10. Stress Analysis (a) and Displacement (b)

While this investigation provided insight to potential deformation at the intersection of the lower diagonal flyer with the inner pier and outward tilting of the wall, it did little to explain the reasoning behind the use of horizontal flyers or "struts" and how the turret towers with imbedded pilasters respond to the flyers.

Student Modeling

The questions surrounding the buttressing design, construction, and performance were posed to students in a graduate seminar course on architectural history of Gothic structure and design as potential topics for a research paper. A team of two master's students, one from architecture and one from civil engineering, took up the question "Is the unusual arrangement of the outer rank of flyers...in the form of a straight piece carried by a round arch...efficacious?" Darren Truelock and James Haliburton modeled a three-dimensional section of the main pier, flyers and buttress piers as originally constructed, then modeled a second version with a diagonal flyer in place of the horizontal "strut" [Truelock 2006, ABAQUS 2003]. The piers were modeled as cylinders, while the turret tower was modeled as a solid prism, although the tower is not solid but constructed of perimeter walls. The students chose to model the roof and vaulting loads as lateral pressures with the pressure to the lower flyers 10 times the magnitude of the pressure to the upper flyers and over a wider surface area representing the depth of the rubble fill. A linear elastic material with modulus of elasticity of 7,250 ksi (50 GPa) and Poisson's ratio of 0.25 was specified with von Mises yield criteria.

The analysis results were largely dependent upon the loading, and showed similar strain and stress characteristics with both models (Figure 11-Figure 12). The large stiffness of the "solid" turret towers limited the displacement of the lower flyers similarly, but showed that with a diagonal flyer, the compressive stress against the tower was distributed over a larger area. The diagonal flyer, which is much closer to an axial member showed less distortion than the flat arch which has curvature susceptible to bending.

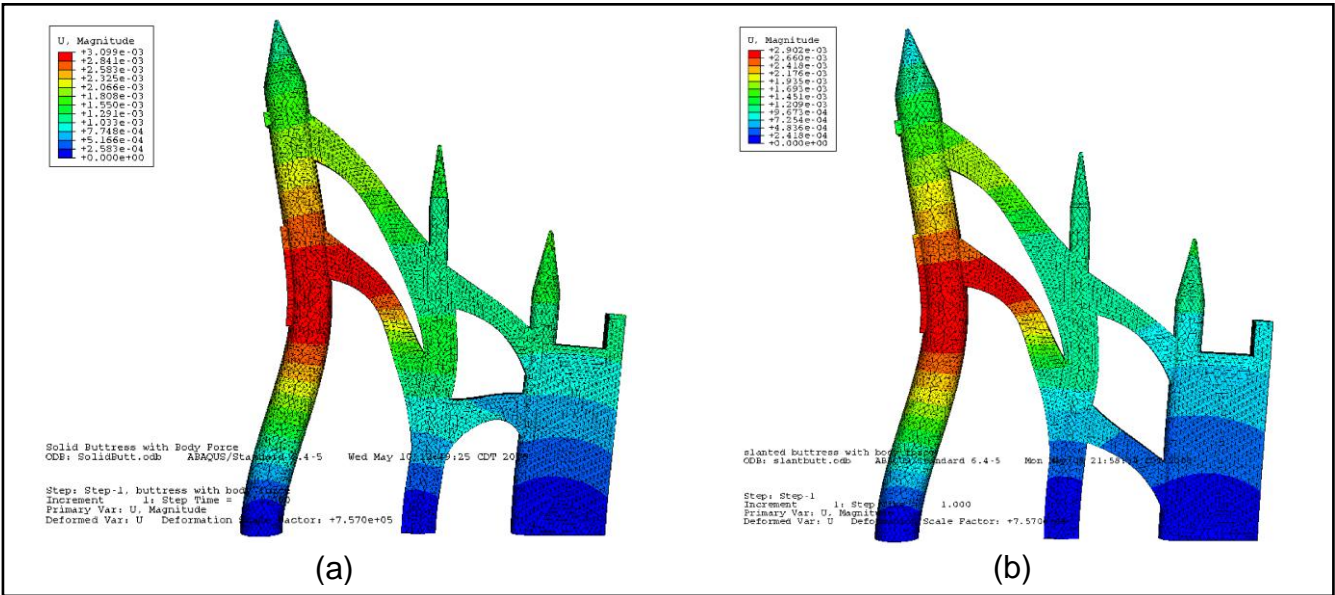


Figure 11. Displacement of (a) Horizontal Flyer and (b) Modified Flyer

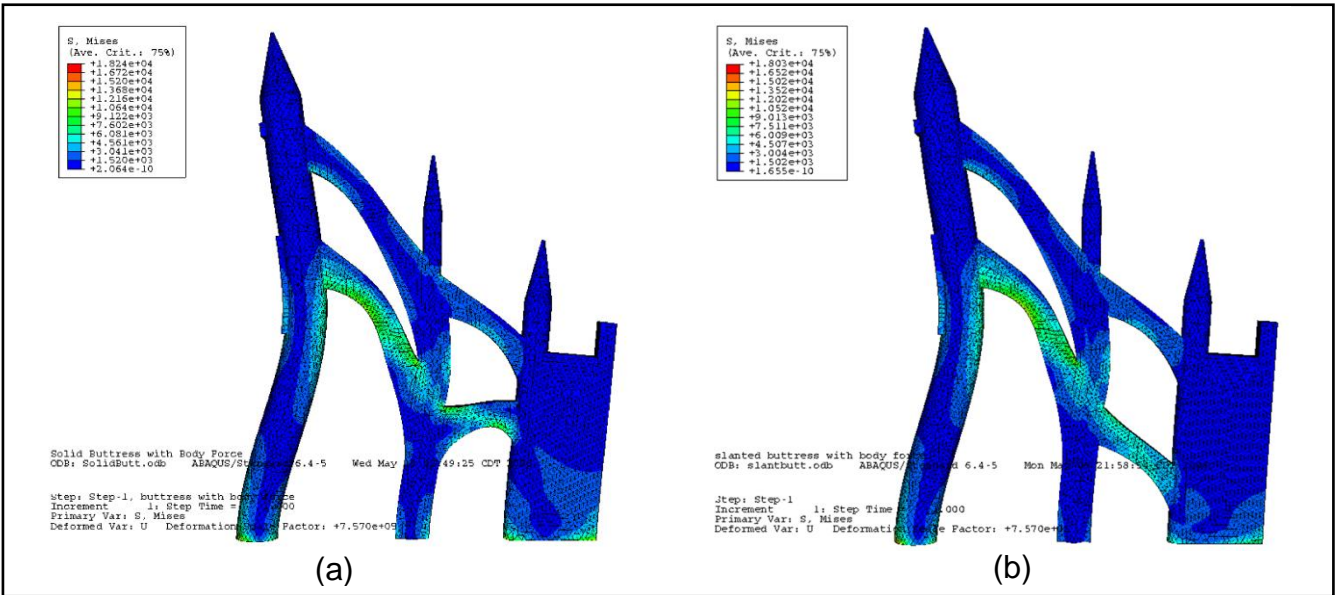


Figure 12. Stress Analysis of (a) Horizontal Flyer and (b) Modified Flyer

Modeling Development

A recent investigation relied on the detailed cross section geometry of the main arch to construct a three-dimensional model of the buttressing with the turret towers for one bay of quadripartite vaulting through the choir (Figure 13) [Nichols 2010].

The configuration of the radiating chapels, the triforium passage, and the vaulting was simplified. The vaulting itself was not modeled, but the rib arches were included and the profile of the vaulting (at mid bay) to the window penetrations was used as a stabilizing element over each main arch, much like the wall would provide. The connecting bridges between turret towers were not included in the model.

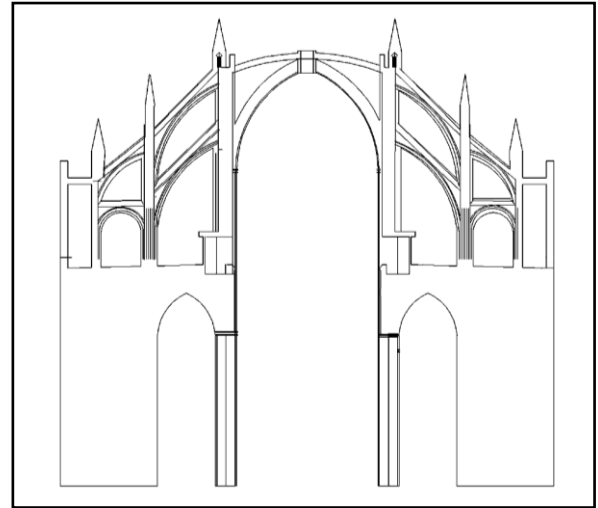


Figure 13. Main Arch Section

The model was defined using a linear elastic material with modulus of elasticity of 7,250 ksi (50 GPa), Poisson's ratio of 0.2, and density of density of 150 lb/ft³ (2.4×10^{-6} kg/mm³) with von Mises yield criterion. Wind loading was applied to the walls, terrace (aisle roof), clerestory walls (externally and internally), and gabled roof using a design wind speed of 89 mi/hr (39.6 m/sec). In addition, the foundation block was modeled to evaluate the dynamic response with a modal analysis.

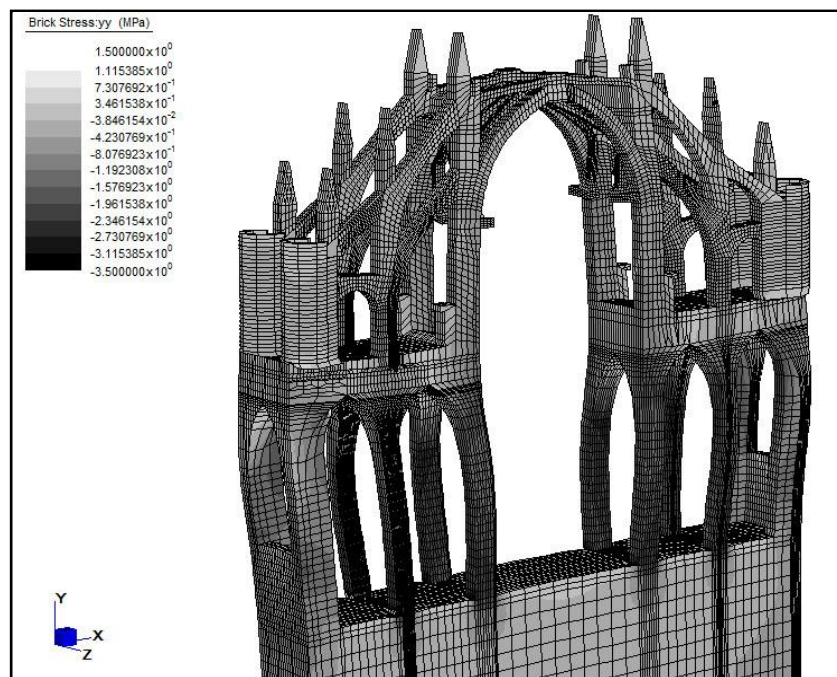


Figure 14. Stress Analysis of Three Dimensional Model

The analysis results for wind and gravity loading indicated that the gravity loads were significant with respect to the lateral wind loading to the response of the buttressing, and that the out-of-plane displacement by the lower story piers without the restraint provided by the rest of the choir contributed to the outward tilting of the buttresses and flattening of the arches of the vaulting (Figure 14). Peak tensile stresses resulted in the main arches, and the lower flyers and horizontal flyer or "strut" showed high stress at the clerestory wall and at the turret towers. The analysis indicated that the lateral thrust to the turret towers was adequately resisted.

Current Modeling

Given the inaccuracies of the earlier finite element model and comparison with a recently completed three-dimensional model constructed from detailed measurements, the finite element model geometry was reconstructed in order to be able to investigate the sexpartite vaulting arrangement, the placement of the upper and lower flyers at the clerestory wall, the geometry of the horizontal flyers or "struts", and the configuration of the outer piers. The buttressing from the level of the terrace roof, walkways between turret towers, the sexpartite and quadripartite vaulting, and the wall over the main arches were included in the model (Figure 15).

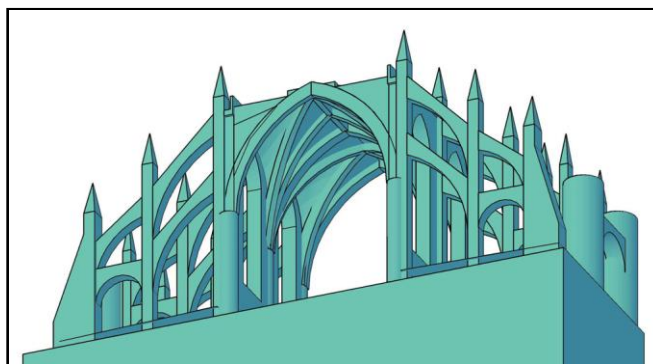


Figure 15. New Model Geometry

Analysis and Discussion

The model geometry was imported into a finite element analysis package, and an elastic-plastic linear analysis was performed with Mohr-Coulomb yield criterion (as suggested for frictional materials) using wind and gravity load [Strand7 2010]. The static wind pressure used for the analysis was based on the coastal region, alignment of the major axis of the structure, and impermeable windward clerestory wall as determined using the Australian/New Zealand Standard which has a comprehensive selection of wind loading diagrams and detailed guidance on appropriate loading coefficients [AS/NZS 1170.2:2002]. For category 3 terrain (no other large structures, but urban), wind region A2, and an average recurrence interval of 20 years, the design wind speed was 89 mi/hr (39.6 m/sec). The pressures were applied as pushing on the windward side of the roof and pulling on the leeward side, pushing and pulling on the windward and leeward towers and clerestory walls, and pulling on the interior of the windward clerestory wall (impermeable).

The wind and gravity analysis indicates that the gravity loads are predominate with respect to the wind loading, and that the buttressing experiences similar stress for both sides (Figure 16). The turret towers, which are modeled as tubular members and are connected with the

arched walkways, have larger localized stress at the junction with the horizontal flyers on the windward side. When the vaulting, the wall over the main arches, and rubble fill are included there will be additional dead load transferred through the lower flyers to the outer piers. The modeling shows that the transverse and rib arches at the sexpartite vaults (forefront in Figure 15) experience larger stresses than the transverse arches at the quadripartite vaults. The large turret towers appear to adequately resist the lateral thrust.

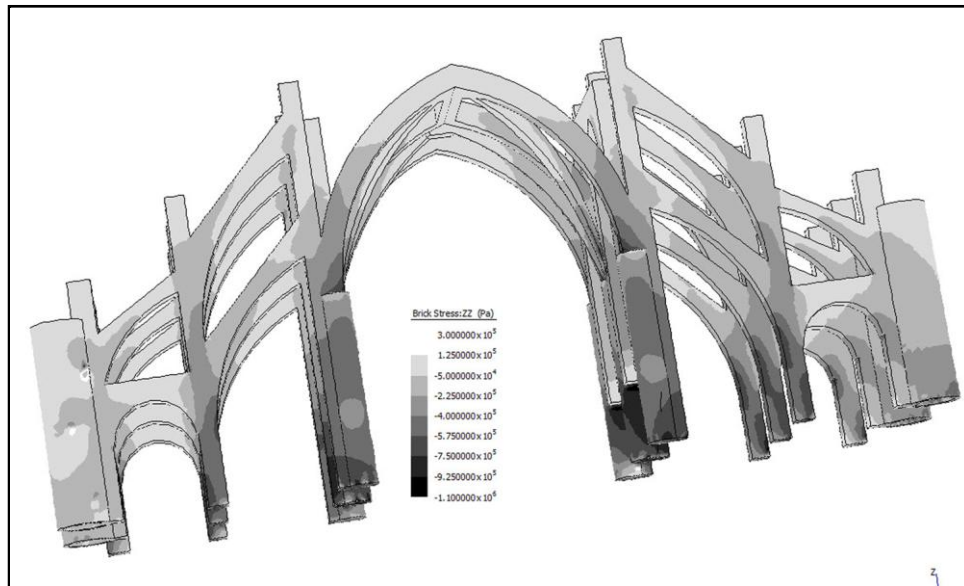


Figure 16. Stress Analysis of Vaulted Model

Conclusions

The several models of Narbonne cathedral that have been generated from measurements and data collected at the site have provided considerable insight into both the problems of modeling a building as complex as Narbonne and the complexities of the forces active within the physical structure itself. That the unusual form of the second rank of flyers and the crenellated turret towers do not seem to have been particularly advantageous structurally, has prompted the investigators to consider that the arrangement could represent a change in design or an attempt to manipulate architectural form for the sake of creating a particular “image”, one that suggested power and authority or a possible reference to the presence of a royal burial since the flesh of Philip IV was entombed on his death in 1285 first in the old Carolingian cathedral and later in the new cathedral. Continuing to increase the levels of geometric detail in the structural models may provide further clarity to the intent of the designer by the unusual buttressing.

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