lecture twenty one

concrete construction:
shear & deflection
Shear in Concrete Beams

- *flexure combines with shear to form diagonal cracks*

- *horizontal reinforcement doesn’t help*

- *stirrups = vertical reinforcement*
ACI Shear Values

- $V_u$ is at distance $d$ from face of support
- shear capacity: $V_c = v_c \times b_w d$

- where $b_w$ means thickness of web at n.a.
ACI Shear Values

- **shear stress (beams)**
  
  \[ v_c = 2 \sqrt{f'_c} \]

  \[ \phi V_c = \phi 2 \sqrt{f'_c} b_w d \]

  \( \phi \) = 0.75 for shear

  \( f'_c \) is in psi

- **shear strength:**

  \[ V_u \leq \phi V_c + \phi V_s \]

  - \( V_s \) is strength from stirrup reinforcement

![Figure 13.17](image_url) Consideration for spacing of a single stirrup.
Stirrup Reinforcement

• shear capacity:

\[ V_s = \frac{A_v f_y d}{s} \]

– \( A_v = \text{area in all legs of stirrups} \)
– \( s = \text{spacing of stirrup} \)

• may need stirrups when concrete has enough strength!
Required Stirrup Reinforcement

- spacing limits

Table 3-8 ACI Provisions for Shear Design*

<table>
<thead>
<tr>
<th></th>
<th>$V_u \leq \frac{\phi V_c}{2}$</th>
<th>$\phi V_c \geq V_u &gt; \frac{\phi V_c}{2}$</th>
<th>$V_u &gt; \phi V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required area of stirrups, $A_v$ **</td>
<td>none</td>
<td>$\frac{50b_ws}{f_y}$</td>
<td>$\frac{(V_u - \phi V_c)s}{\phi f_y d}$</td>
</tr>
<tr>
<td>Stirrup spacing, $s$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Required</td>
<td>$-\leq\frac{A_v f_y}{50b_w}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recommended</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum †</td>
<td></td>
<td></td>
<td>4 in.</td>
</tr>
<tr>
<td>Maximum ††</td>
<td></td>
<td>$\frac{d}{2}$ or 24 in.</td>
<td>$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \leq \phi 4\sqrt{f'_c} b wd$</td>
</tr>
<tr>
<td>(ACI 11.5.4)</td>
<td></td>
<td></td>
<td>$\frac{d}{2}$ or 12 in. for $(V_u - \phi V_c) &gt; \phi 4\sqrt{f'_c} b wd$</td>
</tr>
</tbody>
</table>

*Members subjected to shear and flexure only; $\phi V_c = \phi 2 \sqrt{f'_c} b wd$, $\phi = 0.75$ (ACI 11.3.1.1)

**$A_v = 2 \times A_b$ for U stirrups; $f_y \leq 60$ ksi (ACI 11.5.2)

†A practical limit for minimum spacing is $d/4$

††Maximum spacing based on minimum shear reinforcement ($= A_v f_y/50b_w$) must also be considered (ACI 11.5.5.3).
Torsional Stress & Strain

• can see torsional stresses & twisting of axi-symmetrical cross sections
  – torque
  – remain plane
  – undistorted
  – rotates

• not true for square sections....
Shear Stress Distribution

- depend on the deformation
- $\phi = \text{angle of twist}$
  - measure
- can prove planar section doesn’t distort
Shearing Strain

- related to $\phi$
  \[ \gamma = \frac{\rho \phi}{L} \]

- $\rho$ is the radial distance from the centroid to the point under strain

- shear strain varies linearly along the radius: $\gamma_{max}$ is at outer diameter
Torsional Stress - Strain

- know \( f_v = \tau = G \cdot \gamma \) and \( \gamma = \frac{\rho \phi}{L} \)
- so \( \tau = G \cdot \frac{\rho \phi}{L} \)
- where G is the Shear Modulus
Torsional Stress - Strain

- from

\[ T = \Sigma \tau(\rho) \Delta A \]

- can derive

\[ T = \frac{\tau J}{\rho} \]

- where \( J \) is the polar moment of inertia

- elastic range

\[ \tau = \frac{T \rho}{J} \]
Shear Stress

- $\tau_{\text{max}}$ happens at outer diameter

- combined shear and axial stresses
  - maximum shear stress at 45° “twisted” plane
Shear Strain

- knowing \( \tau = G \cdot \frac{\rho \phi}{L} \) and \( \tau = \frac{T\rho}{J} \)

- solve: \( \phi = \frac{TL}{JG} \)

- composite shafts: \( \phi = \sum_i \frac{T_iL_i}{J_iG_i} \)
Noncircular Shapes

- torsion depends on $J$
- plane sections don’t remain plane
- $\tau_{\text{max}}$ is still at outer diameter

$$\tau_{\text{max}} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}$$

- where $a$ is longer side ($> b$)

<table>
<thead>
<tr>
<th>$a/b$</th>
<th>$c_1$</th>
<th>$c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.208</td>
<td>0.1406</td>
</tr>
<tr>
<td>1.2</td>
<td>0.219</td>
<td>0.1661</td>
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<tr>
<td>1.5</td>
<td>0.231</td>
<td>0.1958</td>
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<tr>
<td>2.0</td>
<td>0.246</td>
<td>0.229</td>
</tr>
<tr>
<td>2.5</td>
<td>0.258</td>
<td>0.249</td>
</tr>
<tr>
<td>3.0</td>
<td>0.267</td>
<td>0.263</td>
</tr>
<tr>
<td>4.0</td>
<td>0.282</td>
<td>0.281</td>
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<tr>
<td>5.0</td>
<td>0.291</td>
<td>0.291</td>
</tr>
<tr>
<td>10.0</td>
<td>0.312</td>
<td>0.312</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.333</td>
<td>0.333</td>
</tr>
</tbody>
</table>
Open Thin-Walled Sections

- with very large $a/b$ ratios:

\[ \tau_{\text{max}} = \frac{T}{\frac{1}{3}ab^2} \]
\[ \phi = \frac{TL}{\frac{1}{3}ab^3G} \]
Shear Flow in Closed Sections

- $q$ is the internal shear force/unit length

$$
\tau = \frac{T}{2t\alpha}
$$

$$
\phi = \frac{TL}{4t\alpha^2} \sum_i \frac{s_i}{t_i}
$$

- $\alpha$ is the area bounded by the centerline
- $s_i$ is the length segment, $t_i$ is the thickness
Shear Flow in Open Sections

- each segment has proportion of $T$ with respect to torsional rigidity,

\[ \tau_{\text{max}} = \frac{T t_{\text{max}}}{\frac{1}{3} \Sigma b_i t_i^3} \]

- total angle of twist:

\[ \phi = \frac{TL}{\frac{1}{3} G \Sigma b_i t_i^3} \]

- I beams - web is thicker, so $\tau_{\text{max}}$ is in web
Torsional Shear Stress

- **twisting moment**
- **and beam shear**

![Diagram of architectural structures showing torsional and shear stresses in hollow and solid sections.](image)

Fig. R11.6.3.1—Addition of torsional and shear stresses
Torsional Shear Reinforcement

- closed stirrups
- more longitudinal reinforcement
- area enclosed by shear flow

Fig. R11.6.3.6(a)—Space truss analogy

Fig. R11.6.3.6(b)—Definition of $A_{oh}$
Development Lengths

• required to allow steel to yield ($f_y$)
• standard hooks
  – moment at beam end

• splices
  – lapped
  – mechanical connectors
Development Lengths

- $l_d$, embedment required both sides

- proper cover, spacing:
  - No. 6 or smaller
    $$l_d = \frac{d_b F_y}{25 \sqrt{f'_c}} \text{ or 12 in. minimum}$$
  - No. 7 or larger
    $$l_d = \frac{d_b F_y}{20 \sqrt{f'_c}} \text{ or 12 in. minimum}$$
Development Lengths

- **hooks**
  - bend and extension

\[ l_{dh} = \frac{1200d_b}{\sqrt{f'_c}} \]

**Figure 9-17**: Minimum requirements for 90° bar hooks.

**Figure 9-18**: Minimum requirements for 180° bar hooks.
Development Lengths

- **bars in compression**
  \[ l_d = \frac{0.02d_b F_y}{\sqrt{f'_c}} \leq 0.0003d_b F_y \]

- **splices**
  - tension minimum is function of \( l_d \) and splice classification
  - compression minimum
  - is function of \( d_b \) and \( F_y \)
Concrete Deflections

- elastic range
  - I transformed
  - $E_c$ (with $f'_c$ in psi)
    - normal weight concrete ($\sim 145$ lb/ft$^3$)
      $$E_c = 57,000 \sqrt{f'_c}$$
    - concrete between 90 and 160 lb/ft$^3$
      $$E_c = w_c^{1.5} 33 \sqrt{f'_c}$$
  - cracked
    - I cracked
    - $E$ adjusted

\[ nA_s \]
Deflection Limits

• relate to whether or not beam supports or is attached to a damageable non-structural element

• need to check service live load and long term deflection against these

<table>
<thead>
<tr>
<th>Limit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/180</td>
<td>roof systems (typical) – live</td>
</tr>
<tr>
<td>L/240</td>
<td>floor systems (typical) – live + long term</td>
</tr>
<tr>
<td>L/360</td>
<td>supporting plaster – live</td>
</tr>
<tr>
<td>L/480</td>
<td>supporting masonry – live + long term</td>
</tr>
</tbody>
</table>