beams –
internal forces & diagrams
Beams

- span horizontally
  - floors
  - bridges
  - roofs

- loaded transversely by gravity loads
- may have internal axial force
- will have internal shear force
- will have internal moment (bending)
Beams

• *transverse loading*

• **sees:**
  – bending
  – shear
  – deflection
  – torsion
  – bearing

• **behavior depends on cross section shape**
Beams

- **bending**
  - bowing of beam with loads
  - one edge surface stretches
  - other edge surface squishes
Beam Stresses

- stress = relative force over an area
  - tensile
  - compressive
  - bending

- tension and compression + ...
Beam Stresses

Unreinforced concrete beam fails in tension (cracks on bottom)

Steel reinforcing in bottom of beam resists tension
Beam Stresses

- tension and compression
  - causes moments

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Beam Stresses

- **prestress or post-tensioning**
  - put stresses in tension area to "pre-compress"
Beam Stresses

- shear – horizontal & vertical

[Diagram showing beam section with labels for horizontal and vertical slices, and steps for replacing and holding with tape.]
Beam Stresses

- shear – horizontal & vertical
Beam Stresses

• shear – horizontal
Beam Deflections

- depends on
  - load
  - section
  - material
Beam Deflections

• “moment of inertia”
Beam Styles

- vierendeel
- open web joists
- manufactured

http://nisee.berkeley.edu/godden
Internal Forces

- **trusses**
  - axial only, (compression & tension)

- **in general**
  - axial force
  - shear force, $V$
  - bending moment, $M$
Beam Loading

- concentrated force
- concentrated moment
  - spandrel beams
Beam Loading

- uniformly distributed load (line load)
- non-uniformly distributed load
  - hydrostatic pressure = $\gamma h$
  - wind loads
Beam Supports

- **statically determinate**

- **statically indeterminate**
Beam Supports

- *in the real world, modeled type*

(a) Beam supported by a neoprene pad.

(c) Timber beam–column connection with T-plate.
Internal Forces in Beams

- like method of sections / joints
  - no axial forces
- section must be in equilibrium
- want to know where biggest internal forces and moments are for designing
V & M Diagrams

- tool to locate $V_{\text{max}}$ and $M_{\text{max}}$ (at $V = 0$)
- necessary for designing
- have a different sign convention than external forces, moments, and reactions
Sign Convention

• shear force, V:
  – cut section to LEFT
  – if $\Sigma F_y$ is positive by statics, V acts down and is POSITIVE
  – beam has to resist shearing apart by V
Shear Sign Convention

(+) Shear.

(-) Shear.

(+) Shear.

(-) Shear.
Sign Convention

- **bending moment, \( M \):**
  - cut section to LEFT
  - if \( \sum M_{\text{cut}} \) is clockwise, \( M \) acts ccw and is POSITIVE – flexes into a “smiley” beam has to resist bending apart by \( M \)
Bending Moment Sign Convention

(+) Moment.

(−) Moment.

Holds Water

Sheds Water

(+) Moment.

(−) Moment.
Deflected Shape

- **positive bending moment**
  - tension in bottom, compression in top
- **negative bending moment**
  - tension in top, compression in bottom
- **zero bending moment**
  - inflection point
Constructing V & M Diagrams

- along the beam length, plot V, plot M

---

**load diagram**
Mathematical Method

- cut sections with $x$ as width
- write functions of $V(x)$ and $M(x)$
Method 1: Equilibrium

- cut sections at important places
- plot V & M
Method 1: Equilibrium

• **important places**
  - supports
  - concentrated loads
  - start and end of distributed loads
  - concentrated moments

• **free ends**
  - zero forces
Method 2: Semigraphical

- by knowing
  - area under loading curve = change in \( V \)
  - area under shear curve = change in \( M \)
  - concentrated forces cause “jump” in \( V \)
  - concentrated moments cause “jump” in \( M \)

\[
V_D - V_C = - \int_{x_c}^{x_D}wdx \quad M_D - M_C = \int_{x_c}^{x_D}Vdx
\]
Method 2

- relationships
Method 2: Semigraphical

- $M_{\text{max}}$ occurs where $V = 0$ (calculus)
Curve Relationships

- integration of functions
- line with 0 slope, integrates to sloped

- ex: load to shear, shear to moment
Curve Relationships

- line with slope, integrates to parabola

- ex: load to shear, shear to moment
Curve Relationships

- parabola, integrates to $3^{rd}$ order curve

- ex: load to shear, shear to moment
Basic Procedure with Sections

1. Find reaction forces & moments
   Plot axes, underneath beam load diagram

V:

2. Starting at left

3. Shear is 0 at free ends

4. Shear has 2 values at point loads

5. Sum vertical forces at each section
Basic Procedure with Sections

M:

6. Starting at left
7. Moment is 0 at free ends
8. Moment has 2 values at moments
9. Sum moments at each section
10. Maximum moment is where shear = 0! (locate where V = 0)
Basic Procedure by Curves

1. Find reaction forces & moments
   Plot axes, underneath beam load diagram

V:

2. Starting at left

3. Shear is 0 at free ends

4. Shear jumps with concentrated load

5. Shear changes with area under load
Basic Procedure by Curves

**M:**

6. **Starting at left**

7. **Moment is 0 at free ends**

8. **Moment jumps with moment**

9. **Moment changes with area under V**

10. **Maximum moment is where shear = 0!**  
    (locate where V = 0)
Shear Through Zero

• slope of V is \( w \) \((-w:1)\)

\[
\begin{align*}
\text{height} &= V_A \\
\text{width} &= x \\
x \cdot w &= V_A \\
\Rightarrow x &= \frac{V_A}{w}
\end{align*}
\]
Parabolic Shapes

- cases

up fast, then slow
up slow, then fast
down fast, then slow
down slow, then fast
Deflected Shape & $M(x)$

- $-M(x)$ gives shape indication
- boundary conditions must be met
Boundary Conditions

- at pins, rollers, fixed supports: $y = 0$
- at fixed supports: $\theta = 0$
- at inflection points from symmetry: $\theta = 0$
- $y_{\text{max}}$ at $\frac{dy}{dx} = 0$
Tabulated Beam Formulas

- how to read charts

1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD

- Total Equiv. Uniform Load \( wl \)
- Reaction \( R = V = \frac{wl}{2} \)
- Shear \( V_x = w \left( \frac{l}{2} - x \right) \)
- Moment \( M_{\text{max. (at center)}} = \frac{wl^2}{8} \)
- Moment \( M_x = \frac{wx}{2} (l - x) \)
- Deflection \( \Delta_{\text{max. (at center)}} = \frac{5wl^4}{384EI} \)
- Deflection \( \Delta_x = \frac{wx}{24EI} \left( l^3 - 2lx^2 + x^3 \right) \)
Tools

- software & spreadsheets help
Tools – Multiframe

- in computer lab
Tools – Multiframe

- frame window
  - define beam members
  - select points, assign supports
  - select members, assign section

- load window
  - select point or member, add point or distributed loads
Tools – Multiframe

- to run analysis choose
  - Analyze menu
    - Linear

- plot
  - choose options
  - double click (all)

- results
  - choose options