steel construction: columns & tension members

Design Methods (revisited)

• know
  – loads or lengths
• select
  – section or load
  – adequate for strength and no buckling

Structural Steel

• standard rolled shapes (W, C, L, T)
• tubing
• pipe
• built-up

Allowable Stress Design (ASD)

• AICS 9th ed

\[
F_a = \frac{f_{\text{critical}}}{F.S.} = \frac{12\pi^2 E}{23(Kl/r)^2}
\]

• slenderness ratio \( \frac{Kl}{r} \)
  – for \( kl/r \geq C_c \) = 126.1 with \( F_y = 36 \text{ ksi} \)
  = 107.0 with \( F_y = 50 \text{ ksi} \)
\[ C_c \quad \text{and Euler's Formula} \]

- \( KL/r < C_c \)
  - short and stubby
  - parabolic transition

- \( KL/r > C_c \)
  - Euler's relationship
  - < 200 preferred

\[ C_c = \sqrt{\frac{2\pi^2 E}{F_y}} \]

\[ \text{Short / Intermediate} \]

- \( L_e/r < C_c \)
  \[ F_a = 1 - \left( \frac{KL/r}{2C_c^2} \right) \frac{F_y}{F.S.} \]
  - where

\[ F.S. = \frac{5}{3} + \frac{3(KL/r)}{8C_c} - \frac{(KL/r)^3}{8C_c^3} \]

\[ \text{Unified Design} \]

- limit states for failure
  \[ P_a \leq \frac{P_n}{\Omega} \]

\[ \phi_c = 0.90 \quad P_n = F_{cr} A_g \quad P_u \leq \phi_c P_n \]

1. yielding \( \frac{KL}{r} \leq 4.71 \frac{E}{F_y} \) or \( F_e \geq 0.44F_y \)

2. buckling \( \frac{KL}{r} > 4.71 \frac{E}{F_y} \) or \( F_e < 0.44F_y \)

\( F_e \) – elastic buckling stress (Euler)
Unified Design

- $P_n = F_{cr}A_g$
  - for $\frac{KL}{r} \leq 4.71 \sqrt{\frac{E}{F_y}} F_{cr} = \left[0.658 \frac{F_y}{F_e}\right] F_y$
  - for $\frac{KL}{r} > 4.71 \sqrt{\frac{E}{F_y}} F_{cr} = 0.877 F$
  - where $F_e = \frac{\pi^2 E}{(KL/r)^2}$

Procedure for Analysis

1. calculate $KL/r$
   - biggest of $KL/r$ with respect to $x$ axes and $y$ axis
2. find $F_a$ or $F_{cr}$ from appropriate equation
   - tables are available
3. compute $P_{allowable} = F_a A$ or $P_n = F_{cr}A_g$
   - or find $f_{actual} = P/A$
4. is $P \leq P_{allowable}$ (or $P_n \leq \phi P_n$)?
   - yes: ok
   - no: insufficient capacity and no good

Procedure for Design

1. guess a size (pick a section)
2. calculate $KL/r$
   - biggest of $KL/r$ with respect to $x$ axes and $y$ axis
3. find $F_a$ or $F_{cr}$ from appropriate equations
   - or find a chart
4. compute $P_{allowable} = F_a A$ (or $P_n/\Omega = F_{cr}A$)
   - or $P_n = F_{cr}A_g$
   - or find $f_{actual} = P/A$
5. is $P \leq P_{allowable}$? or is $P_u \leq \phi P_n$?
   - yes: ok
   - no: pick a bigger section and go back to step 2.
6. check design efficiency
   - percentage of stress $= \frac{P}{P_c} \cdot 100\%$
   - if between 90-100%: good
   - if < 90%: pick a smaller section and go back to step 2.
Column Charts, $F_a$ (pg. 361-364)

Table 10.1  Allowable stress for compression members ( $F_y = 36$ ksi and $F_a = 250$ MPa).

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<th>KL</th>
<th>$F_a$ (ksi)</th>
<th>$F_a$ (MPa)</th>
<th>KL</th>
<th>$F_a$ (ksi)</th>
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Factors of safety:
- $F_y = 50$ ksi
- $W_{12}$

Beam-Column Design

- moment magnification ($P-\Delta$)

$$M_u = B_1M_{max \text{ factored}} \quad B_1 = \frac{C_m}{1 - (P_u / P_{el})}$$

- $C_m$ = modification factor for end conditions
- $P_{el}$ = Euler buckling strength

$$P_{el} = \frac{\pi^2 EA}{(KL/r)^2}$$

Available Critical Stress, $\phi F_{cr}$, for Compression Members, kls ($F_y = 50$ ksi and $\phi = 0.90$)

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Steel Columns & Tension 11
Foundations Structures
ARCH 331
Lecture 20

Steel Columns & Tension 14
Foundations Structures
ARCH 331
Lecture 17
Beam-Column Design

• LRFD (Unified) Steel
  – for
  $$\frac{P_r}{P_c} \geq 0.2 : \quad \frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{\phi_u M_{ux}}{\phi_b M_{nx}} + \frac{\phi_u M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0$$
  – for
  $$\frac{P_r}{P_c} < 0.2 : \quad \frac{P_u}{2\phi_c P_n} + \left( \frac{\phi_u M_{ux}}{\phi_b M_{nx}} + \frac{\phi_u M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0$$

Pr is required, Pc is capacity

$$\phi_c$$ - resistance factor for compression = 0.9

$$\phi_b$$ - resistance factor for bending = 0.9

Design Steps Knowing Loads (revisited)

1. assume limiting stress
   • buckling, axial stress, combined stress
2. solve for r, A or S
3. pick trial section
4. analyze stresses
5. section ok?
6. stop when section is ok

Rigid Frame Design (revisited)

• columns in frames
  – ends can be “flexible”
  – stiffness affected by beams and column = EI/L
  \[
  G = \Psi = \frac{\sum EI}{l_c}
  \]
  – for the joint
    • \(l_c\) is the column length of each column
    • \(l_b\) is the beam length of each beam
    • measured center to center

Rigid Frame Design (revisited)

• column effective length, \(k\)

**Tension Members**

- Steel members can have holes
- Reduced area
  \[ A_n = A_g - A_{\text{of all holes}} + t\Sigma \frac{s}{4g} \]  
  (AISC - Steel Structures of the Everyday)
- Increased stress

**Effective Net Area**

- Likely path to “rip” across
- Bolts divide transferred force too
- Shear lag  
  \[ A_e \leq A_n U \]

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**Tension Members**

- Limit states for failure  
  \[ P_d \leq \frac{P_n}{\Omega} \quad P_u \leq \phi_t P_n \]

1. Yielding  
   \[ \phi_t = 0.90 \quad P_n = F_y A_g \]

2. Rupture*  
   \[ \phi_t = 0.75 \quad P_n = F_u A_e \]

- \( A_g \) - Gross area
- \( A_e \) - Effective net area
- (holes 1/8” + d)
- \( F_u \) = The tensile strength of the steel (ultimate)