Center of Gravity

- location of equivalent weight
- determined with calculus

![Diagram of Center of Gravity](image)

\[ W = \int dW \]

Centroid

- “average” x & y of an area
- for a volume of constant thickness
  - \( \Delta W = \gamma \Delta A \) where \( \gamma \) is weight/volume
  - center of gravity = centroid of area

\[
\bar{x} = \frac{\sum (x \Delta A)}{A} \\
\bar{y} = \frac{\sum (y \Delta A)}{A}
\]
Centroid
• for a line, sum up length
\[
\bar{x} = \frac{\sum(x\Delta L)}{L} \\
\bar{y} = \frac{\sum(y\Delta L)}{L}
\]
**Basic Procedure**

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate $\bar{x}$ and $\bar{y}$

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**Area Centroids**

- Table 7.1 – pg. 242

<table>
<thead>
<tr>
<th>Shape</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular area</td>
<td>$\frac{b}{3}$</td>
<td>$\frac{A}{3}$</td>
</tr>
<tr>
<td>Quarter-circular area</td>
<td>$\frac{4\pi}{3}$</td>
<td>$\frac{4\pi}{3\pi}$</td>
</tr>
<tr>
<td>Semicircular area</td>
<td>0</td>
<td>$\frac{4\pi}{3\pi}$</td>
</tr>
<tr>
<td>Semiparabolic area</td>
<td>$\frac{3h}{8}$</td>
<td>$\frac{3h}{5}$</td>
</tr>
<tr>
<td>Parabolic area</td>
<td>0</td>
<td>$\frac{3h}{5}$</td>
</tr>
</tbody>
</table>

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**Moments of Inertia**

- $2^{nd}$ moment area
  - math concept
  - area $x$ (distance)$^2$
- need for behavior of
  - beams
  - columns

- **Moment of Inertia**
  - about any reference axis
  - can be negative

\[
I_y = \int x^2 \, dA \\
I_x = \int y^2 \, dA
\]

- resistance to bending and buckling
Moment of Inertia

- same area moved away a distance
  - larger $I$

Polar Moment of Inertia

- for roundish shapes
- uses polar coordinates ($r$ and $\theta$)
- resistance to twisting

\[ J_o = \int r^2 dA \]

Radius of Gyration

- measure of inertia with respect to area

\[ r_x = \sqrt{\frac{I_x}{A}} \]

Parallel Axis Theorem

- can find composite $I$ once composite centroid is known (basic shapes)

\[ I_x = I_{cx} + Ad_y^2 \]
\[ = \bar{I}_x + Ad_y^2 \]
\[ I = \sum \bar{I} + \sum Ad^2 \]
\[ \bar{I} = I - Ad^2 \]
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with $A$, $\bar{x}$, $\bar{y}$, $\bar{I}$'s, $d$'s, and $Ad^2$'s
5. Fill in table and get $\hat{x}$ and $\hat{y}$ for composite
6. Sum necessary columns
7. Sum $I$'s and $Ad^2$'s

Area Moments of Inertia

- Table 7.2 – pg. 252: (bars refer to centroid)
  - $x$, $y$
  - $x'$, $y'$
  - $C$

\[ \begin{array}{c|c}
\text{Rectangle} & d_x = \hat{x} - \bar{x} \\
\text{Triangle} & d_x = \hat{x} - \bar{x} \\
\text{Circle} & d_y = \hat{y} - \bar{y} \\
\end{array} \]