Beams: bending and shear stress

**Beam Bending**

- **Galileo**
  - relationship between stress and depth\(^2\)
- can see
  - top squishing
  - bottom stretching
- what are the stress across the section?

**Pure Bending**

- bending only
- no shear
- axial normal stresses from bending can be found in
  - homogeneous materials
  - plane of symmetry
  - follow Hooke’s law

**Bending Moments**

- sign convention:
- size of maximum internal moment will govern our design of the section
Normal Stresses

• geometric fit
  – plane sections remain plane
  – stress varies linearly

Neutral Axis

• stresses vary linearly
  • zero stress occurs at the centroid
  • neutral axis is line of centroids (n.a.)

Derivation of Stress from Strain

• pure bending = arc shape

\[ L = R\theta \]

\[ L_{outside} = (R + y)\theta \]

\[ \varepsilon = \frac{\delta}{L} = \frac{L_{outside} - L}{L} = \frac{(R + y)\theta - R\theta}{R\theta} = \frac{y}{R} \]

Derivation of Stress

• zero stress at n.a.

\[ f = E\varepsilon = \frac{Ey}{R} \]

\[ f_{max} = \frac{Ec}{R} \]

\[ f = \frac{y}{c} f_{max} \]
Bending Moment

- resultant moment from stresses = bending moment!

\[ M = \sum f_y \Delta A \]

\[ = \sum \frac{y f_{max}}{c} y \Delta A = \frac{f_{max}}{c} \sum y^2 \Delta A = \frac{f_{max}}{c} I = f_{max} S \]

Bending Stress Relations

\[ \frac{1}{R} = \frac{M}{EI} \]

\[ f_b = \frac{My}{I} \]

\[ S = \frac{I}{c} \]

curvature  general bending stress  section modulus

maximum bending stress  required section modulus for design

Transverse Loading and Shear

- perpendicular loading
- internal shear
- along with bending moment

Bending vs. Shear in Design

- bending stresses dominate
- shear stresses exist horizontally with shear
- no shear stresses with pure bending
Shear Stresses

- horizontal & vertical

Beam Stresses

- horizontal with bending

Equilibrium

- horizontal force V needed

\[ V_{longitudinal} = \frac{V_T Q}{I} \Delta x \]

- Q is a moment area
Moment of Area

• Q is a moment area with respect to the n.a. of area above or below the horizontal

- \( Q_{\text{max}} \) at \( y=0 \) (neutral axis)

- \( q \) is shear flow:
  \[
  q = \frac{V_{\text{longitudinal}}}{\Delta x} = \frac{V I}{I}
  \]

Shearing Stresses

- \( f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x} \)
- \( f_{v-\text{ave}} = \frac{VQ}{Ib} \)

- \( f_{v-\text{ave}} = 0 \) on the top/bottom
- \( b \) min may not be with \( Q \) max
- with \( h/4 \geq b, f_{v-\text{max}} \leq 1.008 f_{v-\text{ave}} \)

Rectangular Sections

\[
I = \frac{bh^3}{12} \quad Q = A\bar{y} = bh^2/8
\]

- \( f_v \) = \( \frac{VQ}{Ib} = \frac{3V}{2A} \)

- \( f_{v-\text{max}} \) occurs at n.a.

Steel Beam Webs

- \( W \) and \( S \) sections
  - \( b \) varies

  - stress in flange negligible
  - presume constant stress in web
  \[
  f_{v-\text{max}} = \frac{3V}{2A} \approx \frac{V}{A_{\text{web}}}
  \]
Shear Flow
• loads applied in plane of symmetry
• cut made perpendicular
\[ q = \frac{VQ}{I} \]

Shear Flow Quantity
• sketch from Q
\[ q = \frac{VQ}{I} \]

Connectors Resisting Shear
• plates with
  – nails
  – rivets
  – bolts
• splices

Vertical Connectors
• isolate an area with vertical interfaces
\[ nF_{\text{connector}} \geq \frac{VQ_{\text{connected area}}}{I} \cdot p \]
Unsymmetrical Shear or Section

- member can bend and twist
  - not symmetric
  - shear not in that plane
- shear center
  - moments balance