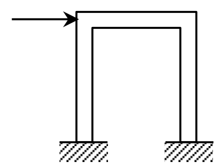
Pinned Frames and Arches

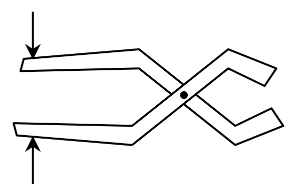
Notation:

F	= name for force vectors	R	= name for reaction force vector
F_x	= force component in the x direction	w	= name for distributed load
F_y	= force component in the y direction	W	= name for total force due to distributed load
FBD	= free body diagram		
M	= name for reaction moment, as is M_R	Σ	= summation symbol

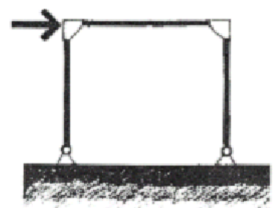
- A FRAME is made up of members where at least <u>one</u> member has more than 3 forces on it
  - Usually stationary and fully constrained

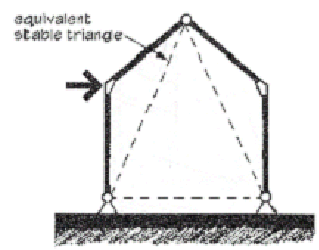


- A PINNED FRAME has member connected by pins
  - Considered *non-rigid* if it would collapse when the supports are removed
  - Considered *rigid* if it retains it's original shape when the supports are removed

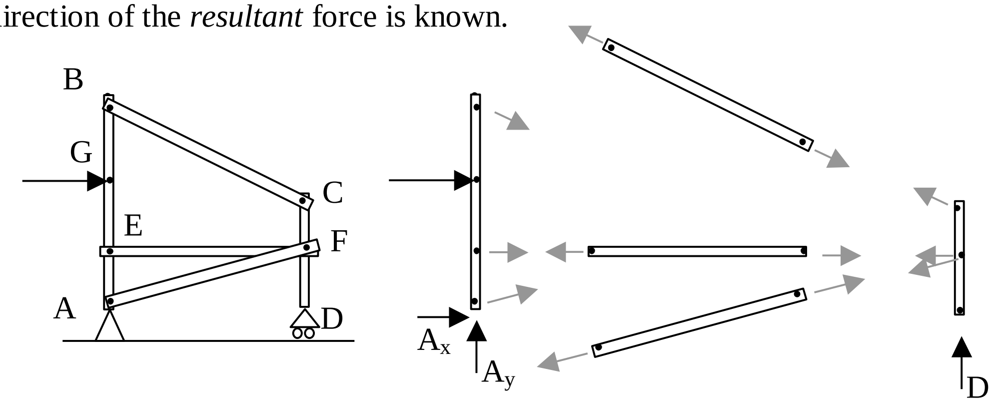


- A RIGID FRAME is all one member with no internal pins
  - Typically *statically indeterminate*
  - **Portal** frames look like door frames
  - Gable frames have a peak.

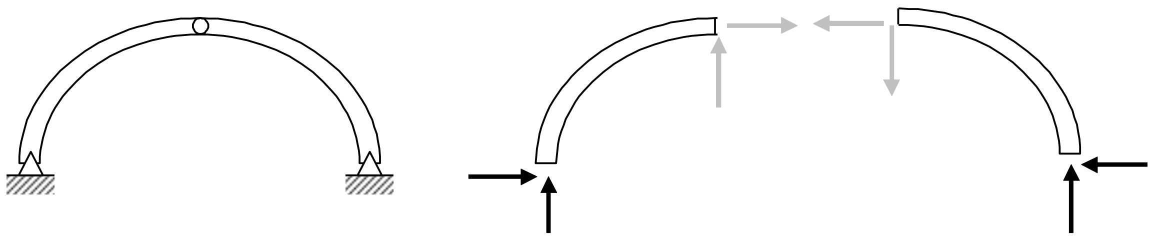




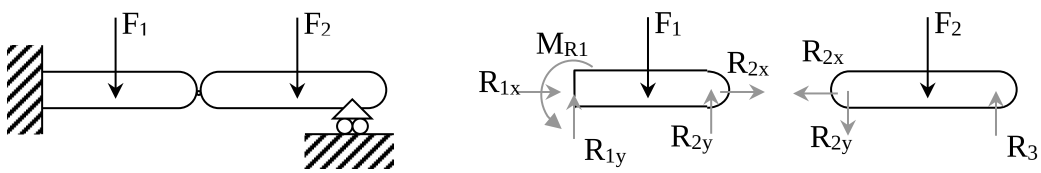
- INTERNAL PIN CONNECTIONS:
  - Pin connection forces are **equal** and **opposite** between the bodies they connect.
  - There are 2 unknown forces at a pin, but if we know a body is a **two-force** body, the direction of the *resultant* force is known.



- AN ARCH is a structural shape that can span large distances and sees compression along its slope. It may have no hinges (or pins), two hinges at the supports, or two hinges at the supports with a hinge at the apex. The three-hinged arch types are statically determinate with 2 bodies and **6** unknown forces.



- CONTINUOUS BEAMS WITH PINS:
  - If pins within the span of a beam over multiple supports result in static determinacy (the right number of unknowns for the number of equilibrium equations), the internal forces at the pins are applied as reactions to the adjacent span.



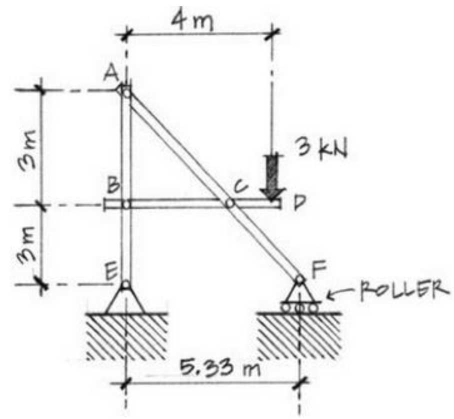
  - The location of the internal pins can be chosen to increase or decrease the moments in order to make the section economical for both positive bending and negative bending (similar values for the moments).

<u>Solution Procedure</u>

1. Solve for the support forces on the entire frame (FBD) if possible.
2. Draw a FBD of each member:
   - Consider all two-force bodies first.
   - Pins are integral with members
   - Pins with applied forces should belong to members with greater than two forces [Same if pins connect 3 or more members]
   - Draw forces on either side of a pin <u>equal</u> and <u>opposite</u> with arbitrary direction chosen for the first side
   - Consider all multi-force bodies
   - Represent connection forces <u>not known</u> by x & y components
   - There are still three equilibrium equations available, but the moment equations may be more helpful when the number of unknowns is greater than two.

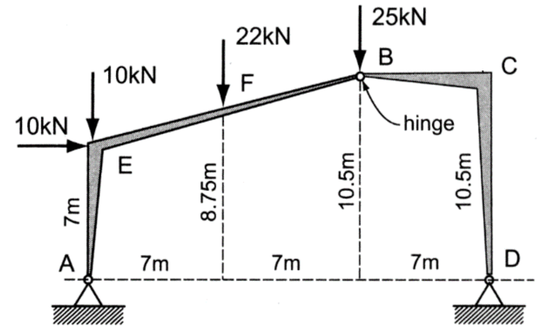
Example 1 (pg 162)

3.16 For the pinned frame shown in Figure 3.66, determine the support reactions at *E* and *F* and the pin reactions at *A*, *B*, and *C*.



Example 2

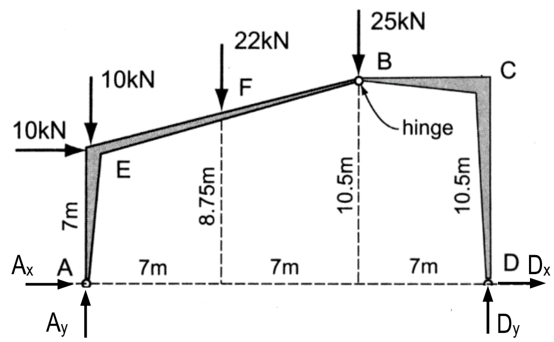
Example (Three-Hinged Arch)



An industrial building is framed using tapered steel sections (haunches) and connected with three hinges (Figure 4.70). Assuming that the loads shown are from gravity loads and wind, determine the support reactions at *A* and *D* and the pin reactions at *B*.

Solution:

Draw the system FBD and write three equilibrium equations:



$$\sum M_A = -10^{kN}\cdot 7m - 22^{kN}\cdot 7m - 25^{kN}\cdot 14m + D_y\cdot 21m = 0$$

D_y = 27.33 kN

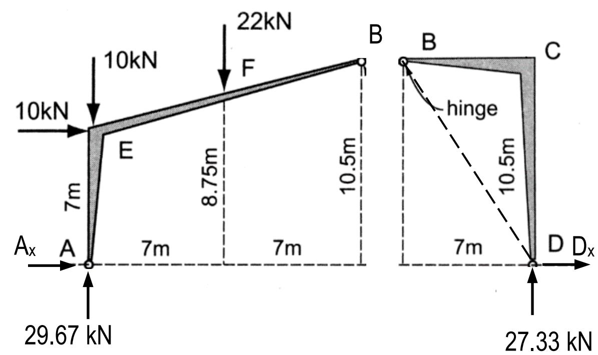
$$\sum F_y = -10^{kN} - 22^{kN} - 25^{kN} + A_y + D_y = 0$$

A_y = 29.67 kN

$$\sum F_x = 10^{kN} + A_x + D_x = 0$$

Draw the FBD's for the parts, and wait to put on the 25 kN load at B because it can only go on one FBD. We'll also wait to draw the connection force components at B:

Notice that body BCD is a 2-force body if we choose to put the 25 kN at B on body AEFB! That means that the force at B and the force at D must be equal, opposite and in the direction of the line from B to D (shown as a long-dashed line)

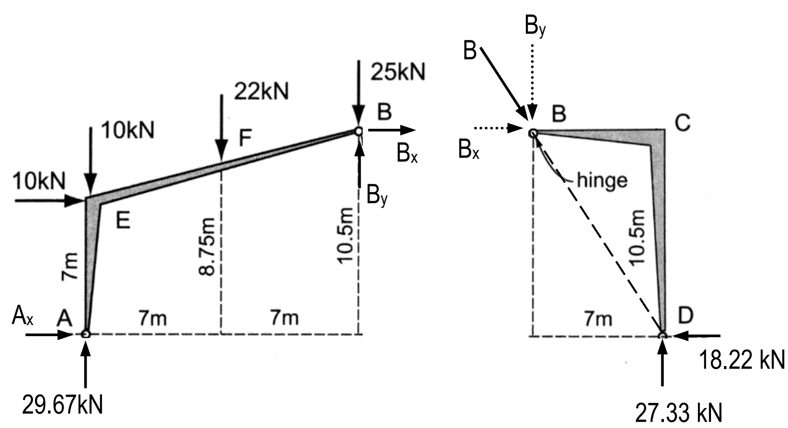


We can find D_x (which must be pointing into D like D_y is) with similar triangles:

$$\frac{D_x}{D_y} = \frac{7m}{10.5m} = \frac{D_x}{27.33kN}$$ so D_x = 18.22 kN (to the left)

and B = $\sqrt{(27.33m)^2 + (18.22m)^2}$ = 32.85 kN

Even if we hadn't figured out the connection force at B by knowing it was on a 2-force body, we could still use the FBD for AEFB because there are only 3 unknowns. Drawing B_x and B_y in the positive direction on body AEFB yields:



$$\sum M_B = A_x\cdot 10.5m - 29.67^{kN}\cdot 14m +$$
$$10^{kN}\cdot 3.5m + 10^{kN}\cdot 14m + 22^{kN}\cdot 7m = 0$$

A_x = 8.23 kN

$$\sum F_y = -10^{kN} - 22^{kN} - 25^{kN} + 29.33^{kN} + B_y = 0$$

B_y = 27.33 kN (up *wrt* AEFB)*

$$\sum F_x = 10^{kN} + A_x + B_x = 0$$

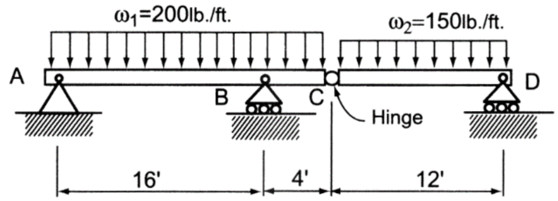
B_x = -18.23 kN (to the left *wrt* AEFB*)

So the horizontal reaction force at B pushes to the left on the left FBD and to the right (opposite) on the right FBD, while the vertical reaction force at B pushes up on the left FBD and down (opposite) on the right FBD.

**wrt - with respect to*

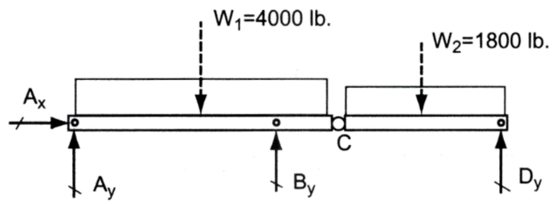
Example 3

A compound beam has three supports at ***A***, ***B*** and ***D*** and an internal hinge at ***C***. Two uniformly distributed loads cover the entire length of the beams. Draw the appropriate FBDs and determine the reactions at the supports and the internal pin forces at ***C***, and construct the shear and bending moment diagrams.



Solution:

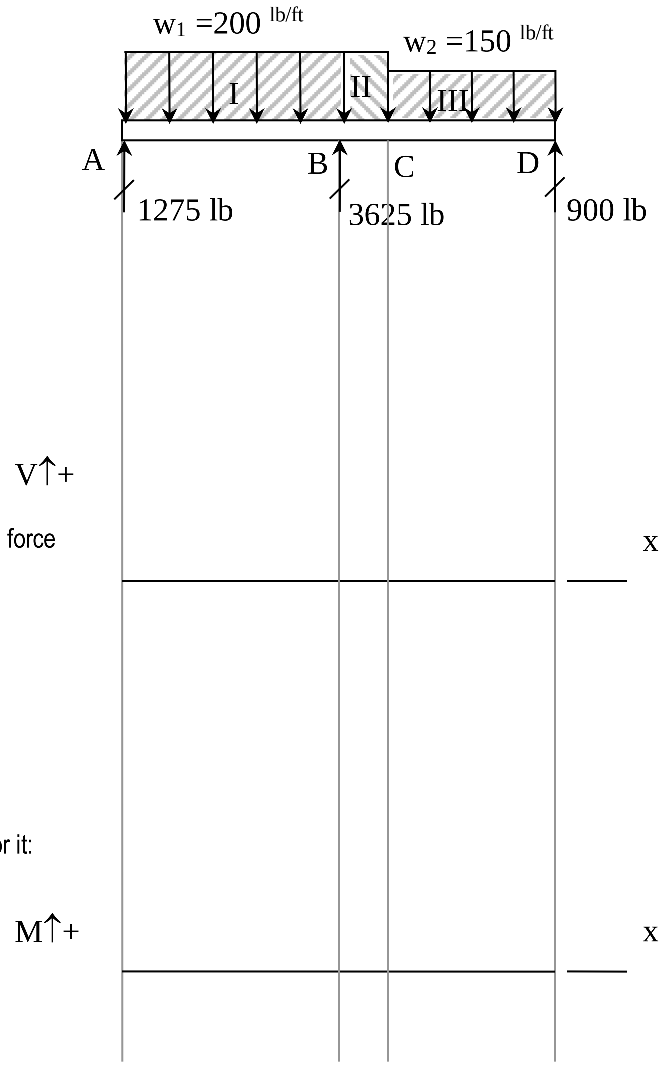
Draw the system FBD and write three equilibrium equations:



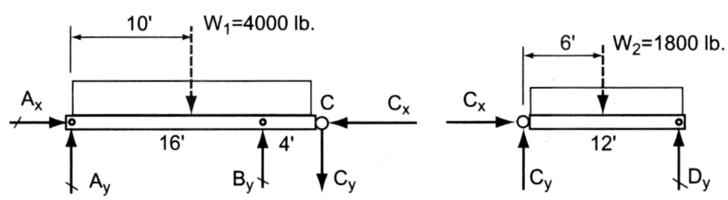
$$\sum M_A = -4^k \cdot 10^{ft} - 1.8^k \cdot 26^{ft} + B_y \cdot 16^{ft} + D_y \cdot 32^{ft} = 0$$

$$\sum F_y = -4^k - 1.8^k + A_y + B_y + D_y = 0$$

$$\sum F_x = A_x = 0$$



Draw the FBD's for the parts. We'll also wait to draw the connection force components at C:



The right body has 3 unknowns, and we can write three equations for it:

$$\sum M_C = -1.8^k \cdot 6^{ft} + D_y \cdot 12^{ft} = 0$$

D_y = 0.9 k = 900 lb

$$\sum F_y = C_y - 1800^{lb} + 900^{lb} = 0$$

C_y = 900 lb

$$\sum F_x = C_x = 0$$

The left body now has 3 unknowns because we've found C_x and C_y, and we can write three equations for it:

$$\sum M_A = -4^k \cdot 8^{ft} + B_y \cdot 16^{ft} - 0.9^k \cdot 20^{ft} = 0$$

B_y = 3.625 k = 3625 lb

$$\sum F_y = A_y - 4000^{lb} + 3625 - 900^{lb} = 0$$

A_y = 1275 lb

Load diagram:

Area I = -(200 lb/ft)(16 ft) = 3200 lb
Area II = -(200 lb/ft)(4 ft) = 800 lb
Area III = -(150 lb/ft)(12 ft) = 1800 lb