ARCH 614: Practice Quiz 8

Note: No aids are allowed for part 1. One side of a letter sized paper with notes is allowed during part 2, along with a silent, non-programmable calculator. There are reference charts on pages 2-6 for part 2.

Clearly show your work and answer.

Part 1) Worth 5 points (conceptual questions)

Part 2) Worth 45 points

(NOTE: The loading type [ex, live, dead, wind...] and sizes can and will be changed for the quiz with respect to the beam diagrams and formula provided.)

A wide flange beam of A992 steel (F_y = 50 ksi, E = 29 x 10^3 ksi) is needed to span 32 ft and support uniformly distributed loads of 850 lb/ft of dead load (from materials), the self weight, and 1150 lb/ft of linearly distributed live load. The beam is simply supported with a maximum unbraced length of 11 ft.

a) Select the most economical beam based on flexural strength using the provided chart (including self weight). Assume that the dead load will determine the location of the maximum bending moment and superimpose the live load moment at that location.

b) If a W21 x 44 (A = 13.0 in.\(^2\), d = 20.66 in., t_w = 0.35 in., b_f = 6.50 in., t_f = 0.45 in., I_x = 843 in.\(^4\)) is chosen, is it adequate for shear?

c) Determine the moment of inertia required such that the total [or live load or dead load...] deflection, ignoring self weight, does not exceed 1.25 inches. Assume that the distributed load determines the location of the maximum deflection.

Answers – Not provided on actual quiz!

a) M_u = 248.3 k-ft, use W14x48 (M_u* > 250.5 k-ft)

b) V_u* = 36.8 k, φV_u = 216.9 k, OK

c) I_{req’d} = 927.4 in.\(^4\) [I_{req’d-dead} = 553.2 in.\(^4\), I_{req’d-live} = 374.8 in.\(^4\)]
1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load \( = \frac{w}{2} \)

\[ R = V = \frac{w}{2} \]

\[ V_x = w \left( \frac{l}{2} - x \right) \]

\[ M_{\text{max. (at center)}} = \frac{wx}{8} \]

\[ \Delta x = \frac{wx}{24EI} (l^3 - 2lx^2 + x^3) \]

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END

Total Equiv. Uniform Load \( = \frac{16W}{9} = 1.0264W \)

\[ R_1 = V_1 = \frac{W}{3} \]

\[ R_2 = V_2 \text{ max.} = \frac{2W}{3} \]

\[ V_x = \frac{W}{2} - \frac{Wx^2}{L^2} \]

\[ M_{\text{max. (at}} x = \frac{l}{3} = 0.5774l \]

\[ W = \frac{Wx}{3} (\frac{l}{2} - x) \]

\[ \Delta x = \frac{wx}{180EI} (3x^4 - 10lx^3 + 7l^2) \]

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER

Total Equiv. Uniform Load \( = \frac{4W}{3} \)

\[ R = V = \frac{W}{2} \]

\[ V_x \text{ (when } x < \frac{l}{2} \text{)} = \frac{W}{2} \left( \frac{l}{2} - 4x^2 \right) \]

\[ M_{\text{max. (at center)}} = \frac{Wl}{6} \]

\[ M_x \text{ (when } x < \frac{l}{2} \text{)} = \frac{WX}{24} \left( \frac{1}{2} - \frac{2x^2}{l^2} \right) \]

\[ \Delta x \text{ (when } x < \frac{l}{2} \text{)} = \frac{WX}{480EI} (\frac{9l^4}{8} - 6x^4 + x^2) \]

4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED

\[ R_1 = V_1 \text{ (max. when } a < c \text{)} = \frac{w}{2} \left( 2c + b \right) \]

\[ R_2 = V_2 \text{ (max. when } a > c \text{)} = \frac{w}{2} \left( 2a + b \right) \]

\[ V_x \text{ (when } x > a \text{ and } (a + b) \text{)} = R_1 - w(x - a) \]

\[ M_{\text{max. (at } x = a + \frac{R_1}{w}) = R_1 \left( a + \frac{R_1}{2w} \right) \]

\[ M_x \text{ (when } x < a \text{)} = R_1x - \frac{w^2}{2} (x - a)^2 \]

\[ M_x \text{ (when } x > a \text{ and } (a + b) \text{)} = R_2 \left( l - x \right) \]

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END

\[ R_1 = V_1 \text{ max.} = \frac{la}{2} \]

\[ R_2 = V_2 \]

\[ V_x \text{ (when } x < a \text{)} = R_3 - wx \]

\[ M_{\text{max. (at } x = \frac{R_3}{w}) = R_3 \left( l - x \right) \]

\[ M_x \text{ (when } x > a \text{)} = R_3 \left( l - x \right) - \frac{wx^2}{2} \]

\[ \Delta x \text{ (when } x < a \text{)} = \frac{wx}{24EI} \left( \frac{4x^3}{2} - 2ax^2 + x^2 \right) \]

\[ \Delta x \text{ (when } x > a \text{)} \]
REFERENCE CHARTS FOR QUIZ 8

7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load = 2P
R = V = P/2
M max. (at point of load) = Px/4
Mx (when x < 1/2) = Px/2

Δmax. (at point of load) = Px/24EI
Δx (when x < 1/2) = Px/48EI (3/4 - 4x^2)

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT

R1 = V1 (max. when a < b) = Pab/l
R2 = V2 (max. when a > b) = Pb/l
M max. (at point of load) = Pbx/l
Mx (when x < a) = Pab(l+2b)/3

Δmax. (at point of load) = Pab(l+2b)/3EI
Δa (when x < a) = Pab(l+2b)/3EI
Δx (when x < a) = Pab(l+2b)/6EI (3 - b^2 - x^2)

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED

Total Equiv. Uniform Load = 8Pa/l
R = V = Pa
M max. (between loads) = Pa
Mx (when x < a) = Px

Δmax. (at center) = Pa/24EI (3/4 - 4a^2)
Δx (when x > a and < (l - a)) = Pa/6EI (3/4 - 3a^2 - x^2)

10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED

R1 = V1 (max. when a < b) = P(l - a + b)
R2 = V2 (max. when a > b) = P(l - b + a)
Vx (when x > a and < (l - b)) = P(l - b - a)

M1 (max. when a > b) = R1a
M2 (max. when a < b) = R2b
Mx (when x < a) = R1x

Mx (when x > a and < (l - b)) = R1x - P(x - a)

11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED

R1 = V1 = Pab/l
R2 = V2 = Pab(l - b)/l
Vx (when x > a and < (l - b)) = R1 - P1

M1 (max. when R1 > P1) = R1a
M2 (max. when R2 < P2) = R2b
Mx (when x < a) = R1x

Mx (when x > a and < (l - b)) = R1x - P1(x - a)

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load = w/l
R1 = V1 = 2w/l
R2 = V2 = 5w/l
Vx = R1 - wx

M max. = w/L^2
M1 (at x = L/2) = 3w/L^2

M2 (at point of load) = 8w/L^2
Mx = R2 - wx^2/2

Δmax. (at x = L/16) = 1/8w/L^2
Δx (when x > a and < (l - a)) = 415w/48E1 (1 - 3x^2 + 2x^2)
13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load \( = \frac{3P}{2} \)

\( R_1 = V_1 \) \( = \frac{5P}{16} \)

\( R_2 = V_2 \) \( = \frac{11P}{16} \)

\( M_{\text{max}} \) \( \text{at fixed end} \) \( = \frac{3P}{16} \)

\( M_1 \) \( \text{at point of load} \) \( = \frac{5P}{16} \)

\( M_x \) \( \text{when } x < \frac{1}{2} \) \( = \frac{P}{2} \left( \frac{1}{2} - \frac{11x}{16} \right) \)

\( \Delta_{\text{max}} \) \( \text{at } x = \frac{1}{2} \) \( = \frac{P^2}{48EI} \left( \frac{1}{5} \right) - \frac{0.009317 P^2}{3EI} \)

\( \Delta_x \) \( \text{at point of load} \) \( = \frac{7P}{24EI} \)

\( \Delta_x \) \( \text{when } x < \frac{1}{2} \) \( = \frac{P}{3} \left( \frac{1}{2} - a \right) \)

\( \Delta_x \) \( \text{when } x > \frac{1}{2} \) \( = \frac{P}{3} \left( \frac{1}{2} - a \right) \)


14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT ANY POINT

\( R_1 = V_1 \) \( = \frac{Pb^3}{27a^2} (a + 2l) \)

\( R_2 = V_2 \) \( = \frac{Pb^3}{27a^2} (3l - 2a) \)

\( M_1 \) \( \text{at point of load} \) \( = R_1a \)

\( M_2 \) \( \text{at fixed end} \) \( = \frac{Pab}{2I} \)

\( M_x \) \( \text{when } x < a \) \( = R_1x \)

\( M_x \) \( \text{when } x > a \) \( = R_1x - \frac{P}{3} (x - a) \)

\( \Delta_{\text{max}} \) \( \text{when } a < 0.414l \text{ at } x = \frac{1}{2} \left( \frac{a + 2a}{2} \right) \)

\( = \frac{Pb^4}{3EI} \left( \frac{1}{2} - a \right)^2 \)

\( \Delta_{\text{max}} \) \( \text{when } a > 0.414l \text{ at } x = \frac{1}{2} \left( \frac{a + 2a}{2} \right) \)

\( = \frac{Pb^4}{6EI} \left( \frac{a}{2} + a \right) \)

\( \Delta_x \) \( \text{at point of load} \) \( = \frac{Pab^3}{12EI} \left( l + a \right) \)

\( \Delta_x \) \( \text{when } x < a \) \( = \frac{Pb^4}{12EI} \left( \frac{3a^2l - 2x^2 - ax^2}{x} \right) \)

\( \Delta_x \) \( \text{when } x > a \) \( = \frac{Pb^4}{12EI} \left( \frac{1}{2} - a \right) \left( 3lx - 3ax - 2ax \right) \)

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS

Total Equiv. Uniform Load \( = \frac{2wI}{3} \)

\( R = V \) \( = \frac{w}{2} \)

\( V_x \) \( = \frac{w}{12} \)

\( M_{\text{max}} \) \( \text{at ends} \) \( = \frac{wI^2}{24} \)

\( M_1 \) \( \text{at center} \) \( = \frac{wI^2}{24} \)

\( M_x \) \( = \frac{w}{6} \left( \frac{1}{2} - 6x^2 \right) \)

\( \Delta_{\text{max}} \) \( \text{at center} \) \( = \frac{384EI}{w} \)

\( \Delta_x \) \( \text{at point of load} \) \( = \frac{384EI}{w} \left( 1 - x \right)^2 \)

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load \( = P \)

\( R = V \) \( = \frac{P}{2} \)

\( M_{\text{max}} \) \( \text{at center and ends} \) \( = \frac{P}{8} \)

\( M_x \) \( \text{when } x < \frac{1}{2} \) \( = \frac{P}{3} (4x - l) \)

\( \Delta_{\text{max}} \) \( \text{at center} \) \( = \frac{Pb^2}{192EI} \)

\( \Delta_x \) \( \text{when } x < \frac{1}{2} \) \( = \frac{Pb^2}{48EI} \left( 3l - 4x \right) \)

17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT

\( R_1 = V_1 \) \( \text{max. when } a < b \) \( = \frac{Pb^2}{3I} (3a + b) \)

\( R_2 = V_2 \) \( \text{max. when } a > b \) \( = \frac{Pb^2}{I} (a + 3b) \)

\( M_1 \) \( \text{max. when } a < b \) \( = \frac{Pab^2}{I} \)

\( M_2 \) \( \text{max. when } a > b \) \( = \frac{Pab^2}{I} \)

\( M_a \) \( \text{at point of load} \) \( = \frac{2Pab^2}{I} \)

\( M_x \) \( \text{when } x < a \) \( = \frac{R_1x - \frac{Pab}{2}}{I} \)

\( \Delta_{\text{max}} \) \( \text{when } a > b \text{ at } x = \frac{1}{2} \left( \frac{3a + b}{2} \right) \)

\( = \frac{Pab^2}{3EI} \left( \frac{3a + b}{2} \right)^2 \)

\( \Delta_x \) \( \text{at point of load} \) \( = \frac{Pab^2}{6EI} \left( 3a + 3ax - bx \right) \)

\( \Delta_x \) \( \text{when } x < a \) \( = \frac{Pab^2}{6EI} \left( 3a - 3ax - bx \right) \)
REFERENCE CHARTS FOR QUIZ 8

18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END

- Total Equiv. Uniform Load = \( \frac{8}{3} W \)
- \( R = V \) = \( W \)
- \( V_x \) = \( W \frac{x^2}{l^2} \)
- \( M_{\text{max.}} \text{ (at fixed end)} \) = \( \frac{Wl}{3} \)
- \( \Delta_{\text{max.}} \text{ (at fixed end)} \) = \( \frac{Wl^2}{12EI} \)
- \( A_{\text{free}} \) = \( \frac{W}{6EI} \frac{(x^2 - 5/4x + 4/8)}{l^2} \)

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD

- Total Equiv. Uniform Load = \( 4wl \)
- \( R = V \) = \( wl \)
- \( V_x \) = \( wx \)
- \( M_{\text{max.}} \text{ (at fixed end)} \) = \( \frac{wl^2}{2} \)
- \( M_x \) = \( \frac{wx^2}{2} \)
- \( \Delta_{\text{max.}} \text{ (at fixed end)} \) = \( \frac{w}{8EI} \)
- \( A_{\text{free}} \) = \( \frac{w}{24EI} \frac{(x^2 - 4/3x + 3/4)}{l^2} \)

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD

- Total Equiv. Uniform Load = \( \frac{8}{3} \frac{wl}{l} \)
- \( R = V \) = \( \frac{wl}{l} \)
- \( V_x \) = \( wx \)
- \( M_{\text{max.}} \text{ (at fixed end)} \) = \( \frac{wl^2}{3} \)
- \( M_x \) = \( \frac{wx^2}{2} \)
- \( \Delta_{\text{max.}} \text{ (at deflected end)} \) = \( \frac{w}{24EI} \)
- \( A_{\text{deflected end}} \) = \( \frac{w}{24EI} \frac{(x^2 - 4/3x + 3/4)}{l^2} \)

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT

- Total Equiv. Uniform Load = \( \frac{8Pb}{l} \)
- \( R = V \) = \( P \)
- \( M_{\text{max.}} \text{ (at fixed end)} \) = \( Pb \)
- \( M_x \) (when \( x > a \)) = \( P(b - a) \)
- \( \Delta_{\text{max.}} \text{ (at fixed end)} \) = \( \frac{Pb^2}{6EI} \)
- \( \Delta_a \) (at point of load) = \( \frac{Pb^2}{3EI} \)
- \( \Delta_x \) (when \( x < a \)) = \( \frac{Pb^2}{6EI} \frac{3I - 3x - b}{6I} \)
- \( \Delta_x \) (when \( x > a \)) = \( \frac{Pb^2}{6EI} \frac{(I - x)^2}{6I} \frac{3b - l + x}{6I} \)

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END

- Total Equiv. Uniform Load = \( 8P \)
- \( R = V \) = \( P \)
- \( M_{\text{max.}} \text{ (at fixed end)} \) = \( Pl \)
- \( M_x \) = \( Px \)
- \( \Delta_{\text{max.}} \text{ (at fixed end)} \) = \( \frac{Pb^3}{3EI} \)
- \( \Delta_x \) = \( \frac{P}{6EI} \frac{(2/3 - 3/8x + x^3)}{2} \)

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END

- Total Equiv. Uniform Load = \( 4P \)
- \( R = V \) = \( P \)
- \( M_{\text{max.}} \text{ (at both ends)} \) = \( \frac{Pl}{2} \)
- \( M_x \) = \( P \left( \frac{l}{2} - x \right) \)
- \( \Delta_{\text{max.}} \text{ (at deflected end)} \) = \( \frac{Pb^3}{12EI} \)
- \( \Delta_x \) = \( \frac{P}{12EI} \frac{(l - x)^2}{12EI} \frac{1 - 2x}{12EI} \)