Forces and Vectors

Notation:

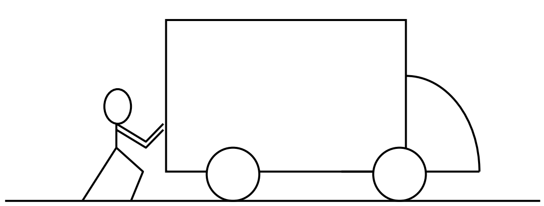
F = name for force vectors, as is *A, B, C, T and P*
F_x = force component in the x direction
F_y = force component in the y direction
h = cable sag height
L = span length
R = name for resultant vectors
R_x = resultant component in the x direction
R_y = resultant component in the y direction
tail = start of a vector (without arrowhead)
tip = direction end of a vector (with arrowhead)
T = name for a tension force
x = x axis direction
y = y axis direction
W = name for force due to weight
ϕ = angle
θ = angle, in a trig equation, ex. $\sin\theta$, that is measured between the x axis and *tail* of a vector

Force Characteristics

- Forces have *a point of application* -
  size – units of lb, K, N, kN
  direction – to a reference system,
  sense - indicated by an arrow, or by sign convention (+/-)
- Classifications include: *Static & Dynamic*
- Structural types separated primarily into *Dead Load* and *Live Load* with further identification as wind, earthquake (seismic), impact, etc.

Rigid Body

- *Ideal* material that doesn't deform
- Forces on rigid bodies can be *internal* - within or at connections
  or *external* - applied
- Rigid bodies can *translate* (move in a straight line)
  or *rotate* (change angle)

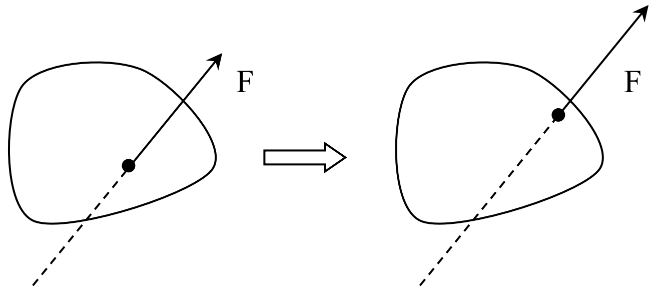


- Weight of truck is external (gravity)
- Push by driver is external
- Reaction of the ground on wheels is external

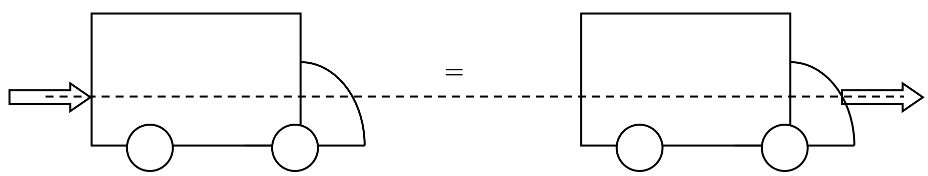
If the truck moves forward: *it translates*

If the truck gets put up on a jack: *it rotates*

- *Transmissibility:* We can replace a force at a point on a body by that force on another point on the body along the line of action of the force.



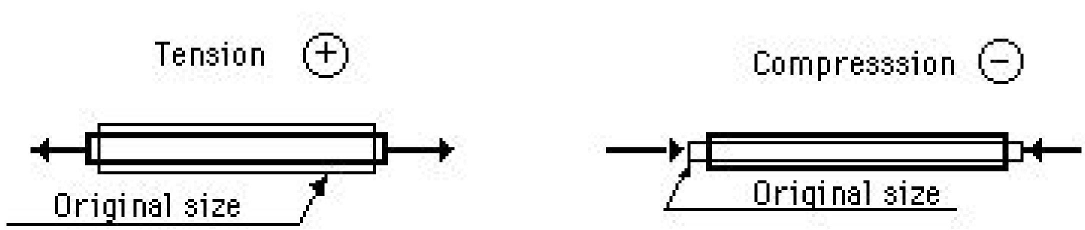
For the truck:



- The same external forces will result in the same conditions for motion
- Transmissibility applies to EXTERNAL forces. INTERNAL forces respond differently when an external force is moved.
- DEFINITION: *2D Structure* - A structure that is flat and may contain a plane of symmetry. All forces on this structure are in the same plane as the structure.

Internal and External Forces

- *Internal forces* occur within a member or between bodies within a system
- *External forces* represent the action of other bodies or gravity on the rigid body



Force System Types

- *Collinear* – all forces along the same **line**
- *Coplanar* – all forces in the same **plane**

- *Space* – not all concurrent or coplanar (all out there in 3 dimensions)

Further classification as

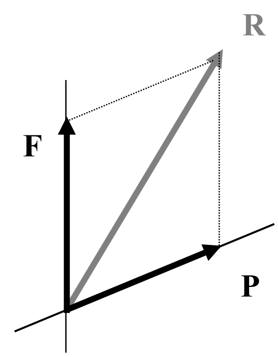
- *Concurrent* – all forces go through the same **point**
- *Parallel* – all forces are **parallel**

Static Equilibrium

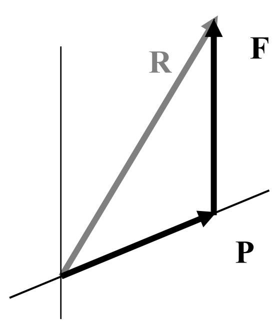
Equilibrium exists when the force system on a body or object produces no rotation or translation.

Graphical Addition of Forces and Resultants

- *Parallelogram law:* when adding two vectors acting at a point, the result is the **diagonal** of the parallelogram



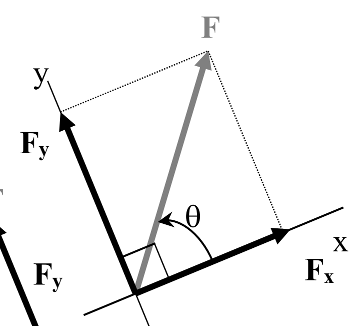
- The *tip-to-tail* method is another graphical way to add vectors.

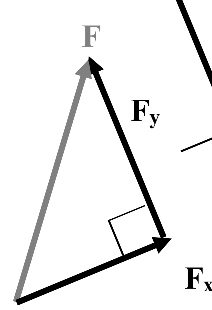


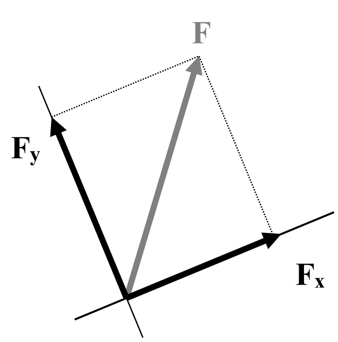
- With **3 (three)** or more vectors, successive application of the parallelogram law will find the resultant *OR* drawing all the vectors **tip-to-tail** in any order will find the resultant.

Rectangular Force Components and Addition

- It is convenient to resolve forces into perpendicular components (at 90°).
- Parallelogram law results in a rectangle.
- Triangle rule results in a right triangle.







θ is: *between x & F*

$F_x = F \cdot \cos\theta$

$F_y = F \cdot \sin\theta$

} magnitudes are *scalar* and can be negative; F_x & F_y are *vectors* in x and y direction

$F = \sqrt{F_x^2 + F_y^2}$

$\tan\theta = \dfrac{F_y}{F_x}$

When $90° < \theta < 270°$, F_x is *negative*

When $180° < \theta < 360°$, F_y is *negative*

When $0° < \theta < 90°$ and $180° < \theta < 270°$, $\tan\theta$ is *positive*

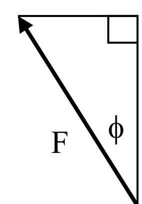
When $90° < \theta < 180°$ and $270° < \theta < 360°$, $\tan\theta$ is *negative*

- Addition (analytically) can be done by adding all the *x* components for a **resultant** *x* component and adding all the **y** components for a resultant *y* component.

$$R_x = \sum F_x \,,\ \ R_y = \sum F_y \quad \text{and } R = \sqrt{R_x^2 + R_y^2} \qquad \tan\theta = \frac{R_y}{R_x}$$

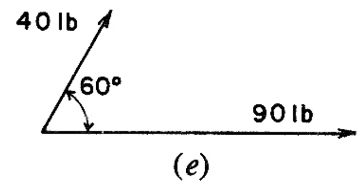
CAUTION: An interior angle, ϕ, between a vector and *either* coordinate axis can be used in the trig functions. BUT _No sign_ will be provided by the trig function, which means **you** must give a sign and determine if the component is in the x or y direction.

For example, $F\sin\phi =$ *opposite side, which whould be negative in x!*



Example 1 (page 10)

Example 2. The angle between two forces of 40 and 90 lb, as shown in Figure 1.4*e*, is 60°. Determine the resultant.



Steps:

1. GIVEN: Write down what's given (drawing and numbers).
2. FIND: Write down what you need to find. (resultant graphically)
3. SOLUTION:
4. Draw the 40 lb and 90 lb forces to scale with tails at O. (If the scale isn't given, you must choose one that fits on your paper; ie. 1 inch = 30 lb.)
5. Draw parallel reference lines at the ends of the vectors.
6. Draw a line from O to the intersection of the reference lines
7. Measure the length of the line
8. Convert the line length by the scale into pounds (by multiplying by the force measure and dividing by the scale value, i.e. X inches * 30 lb / 1 inch).
9. Measure the angle with respect to the positive x axis.

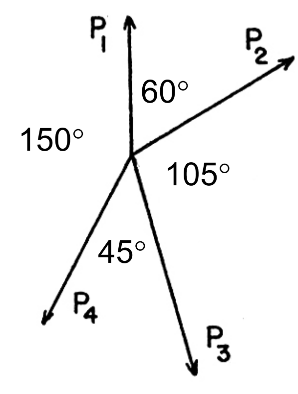
Alternate solution:

4. Draw one vector
5. Draw the other vector at the TIP of the first one (away from the tip).
6. Draw a line from 0 to the tip of the final vector and continue at step 7

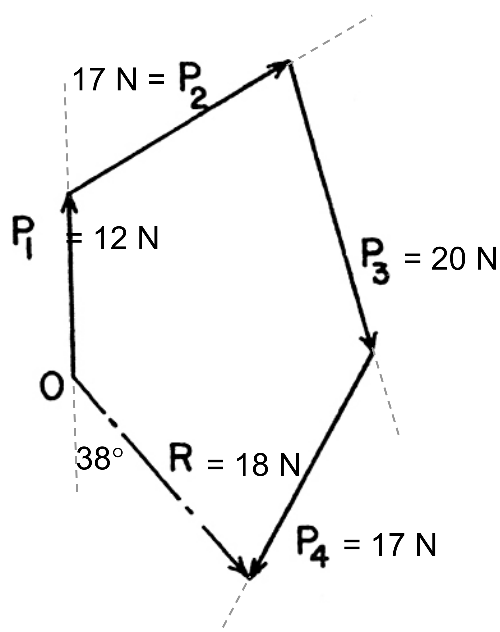
Equilibrant
The force equal and opposite to a resultant, that allows a system to be in equilibrium, is called an *equilibrant.*

Example 2 (pg 14)

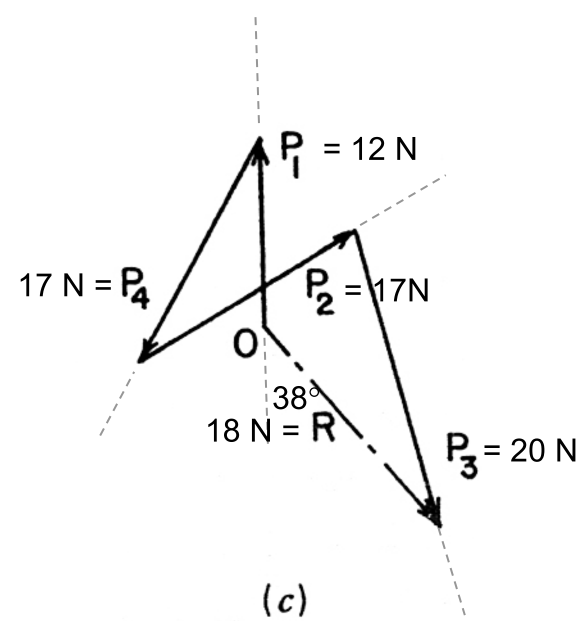
Example 4. Let it be required to find the resultant of the four concurrent forces P_1, P_2, P_3, and P_4 shown in Figure 1.9*a*. P_1 = 12 N, P_2 = 17 N, P_3 = 20 N, P_4 = 17 N, 10 mm = 5 N.



(*a*)

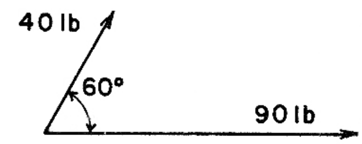


(*b*)



(*c*)

Example 3 (pg 10) Determine the resultant vector analytically with the component method.



(*e*)

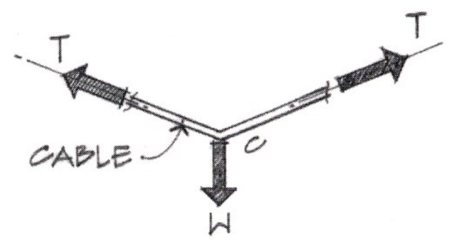
Cable Structures

Cables have the same tension all along the length if they are not cut. The force *magnitude* is the same everywhere in the cable *even if it changes angles.* Cables CANNOT be in **compression**. (They flex instead.)

High-strength steel is the most common material used for cable structures because it has a high strength to weight ratio.

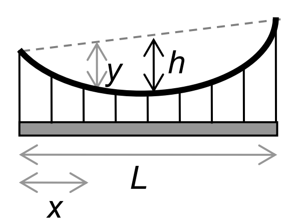
Cables must be supported by vertical supports or towers and must be anchored at the ends. Flexing or unwanted movement should be resisted. (Remember the Tacoma Narrows Bridge?)

Cables with a single load have a **concurrent** force system. It will only be in equilibrium if the cable is **symmetric** when the load can slide and come to rest**.**



The forces anywhere in a *straight segment* can be resolved into x and y components of $T_x = T\cos\theta$ and $T_y = T\sin\theta$.

Because the shape of the cable is derived from the applied loads, it is called a *funicular* structure.



The shape of a cable having a *uniform distributed load* is almost parabolic, which means the geometry and cable length can be found with:

$$y = 4h(Lx - x^2)/L^2$$

where y is the vertical distance from the straight line from cable start to end
h is the vertical sag (maximum y)
x is the distance from one end to the location of y
L is the horizontal span.

$$L_{total} = L(1 + \tfrac{8}{3}\tfrac{h^2}{L^2} - \tfrac{32}{5}\tfrac{h^4}{L^4})$$

where L_{total} is the total length of parabolic cable
h and L are defined above.

Example 4 (pg 16) Using force polygons and component relationships, determine the magnitudes in cables BC and CA.