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Course title and number        ARCH 614 – Elements of Architectural Structures (section 600)
Term                          Spring 2016
Meeting times and location    Lecture: 8-9:15 am T,R; Lab: 9:25-10:50 am in 403 Langford A (1:40 total)

Course Description and Prerequisites
Elements of Architectural Structures. (2-2). Credit 3. Investigation of the structural factors that influence the development of architectural space and form; introduction of the physical principles that govern statics and strength of materials through design of timber and steel components of architectural structures. Prerequisite: ARCH 612 or approval of instructor.

Learning Outcomes

- The student will be able to read a text or article about structural technology, identify the key concepts and related equations, and properly apply the concepts and equations to appropriate structural problems (relevance). The student will also be able to define the answers to key questions in the reading material. The student will be able to evaluate their own skills, or lack thereof, with respect to reading and comprehension of structural concepts, clarity of written communication, reasonable determination of precision in numerical data, and accuracy of computations.
- The student will be able to read a problem statement, interpret the structural wording in order to identify the concepts and select equations necessary to solve the problem presented (significance). The student will be able to identify common steps in solving structural problems regardless of the differences in the structural configuration and loads, and apply these steps in a clear and structured fashion (logic). The student will draw upon existing mathematical and geometrical knowledge to gather information, typically related to locations and dimensions, provided by representational drawings or models of structural configurations, and to present information, typically in the form of plots that graph variable values. The student will be able to draw representational structural models and diagrams, and express information provided by the figures in equation form. The student will compare the computational results in a design problem to the requirements and properly decide if the requirements have been met. The student will take the corrective action to meet the requirements.
- The student will create a structural model with a computer application based on the concepts of the behavior and loading of the structural member or assemblage. The student will be able to interpret the modeling results and relate the results to the solution obtained by manual calculations.
- The student will be able to articulate the physical phenomena, behavior and design criteria which influence structural space and form. (depth) The student will be able to identify the structural purpose, label, behavior, advantages and disadvantages, and interaction of various types of structural members and assemblies. (breadth) The student will be able to identify the configuration, label, behavior, advantages and disadvantages, and interaction of various types of structural members and assemblies with respect to materials (e.g. reinforced concrete beams or frames).
- The student will interact and participate in group settings to facilitate peer-learning and teaching. In addition, the student will be able to evaluate the comprehension of concepts, clarity of communication of these concepts or calculations, and the precision and accuracy of the data used in the computations in the work of their peers.

Instructor Information

Name                  Dr, Anne Nichols, Associate Professor of the Practice
Telephone number     (979) 845-6540
Email address        anichols@tamu.edu
Office hours         12:30-2 pm MW, 1-2 pm TR (and by appointment)
Office location      A435 Langford
Textbook and Resource Material

Required Text:

Recommended Texts:

References:
- ACI 318-14 Code and Commentary
- AISC 14th ed. Steel Construction Manual
- Masonry Joint Structural Code
- National Design Specifications for Wood

Grading Policies

Students should refer to the Academic section in Student Rules and Regulations http://student-rules.tamu.edu.

Assignments:
- Due as stated on the assignment statements.
- Only one assignment without University excuse may be turned in for credit no later than one week after the due date and before final exams begin. All other assignments will receive no credit if late without a recognized excuse or after final exams have begun.
- Assignments with incorrect formatting will be penalized.
- Learning portfolios cannot be submitted late without a recognized excuse.

Quizzes:
- Quizzes will be given at any time during the class period. Make-up quizzes without an excuse will not be given.
- Practice quizzes will be posted electronically.
- No quiz scores will be "dropped".
- Use of cell phones with a calculator application during quizzes and exams is prohibited.

Final Exam:
- The final exam will be comprehensive and is officially scheduled for 1-3 PM Friday, May 6.

Teaching Assistant:
- Lacey Masters (lacey_helm@email.tamu.edu)

Structures Help Desk:
- Callie Wendlandt (callie_w@email.tamu.edu)
- ARCA 400B 845-6580 Posted Hours: http://faculty.arch.tamu.edu/anichols/schedule/

Other Resources:
- The Student Learning Center provides tutoring in math and physics. (http://slc.tamu.edu/tutoring.shtml) Other tutoring services are listed at http://scs.tamu.edu/sites/default/files/tutoring.pdf The Academic Success Center offers workshops at http://us.tamu.edu/Undergraduate-Studies/Academic-Success-Center

Grievances:
- For grievances other than those listed in Part III in Texas A&M University Student Rules: http://student-rules.tamu.edu/ the instructor must be the first point of contact.
Grading Information and Rubric

The levels listed for graded work (projects, quizzes, exams) and pass-fail work (assignments) must both be met to earn the course letter grade:

<table>
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<tr>
<th>Letter Grade</th>
<th>Graded work</th>
<th>Pass-Fail work</th>
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<tbody>
<tr>
<td>A</td>
<td>A average (90-100%)</td>
<td>Pass for 90 to 100% of assignments</td>
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<tr>
<td>B</td>
<td>B average (80-89%)</td>
<td>Pass for 83 to 100% of assignments</td>
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<tr>
<td>C</td>
<td>C average (70-79%)</td>
<td>Pass for 75 to 100% of assignments</td>
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<tr>
<td>D</td>
<td>D average (60-69%)</td>
<td>Pass for 65 to 100% of assignments</td>
</tr>
<tr>
<td>F</td>
<td>F average (&lt;59%)</td>
<td>Pass for 0% to 100% of assignments</td>
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Graded work: This typically constitutes 10 quizzes, a learning portfolio (worth 1.5 quizzes) and a final exam (worth 4 quizzes). This equates to proportions of approximately 64% to quizzes, 10% to the learning portfolio, and 26% to the final exam.

Pass/fail work: This constitutes all practice assignments and projects, each with a value of 1 unit. Criteria for passing is at least 75% completeness and correctness along with every problem attempted. Percent effort expected for a problem in a practice assignment is provided on the assignment statement. This is considered a lab course and the assignments are required work with credit given for competency. The work is necessary to apply the material and prepare for the quizzes and exam. It is expected that this work will be completed with assistance or group participation, but all graded work is only by the individual.

Attendance Policies

The University views class attendance as the responsibility of an individual student. Attendance is essential to complete the course successfully. University rules related to excused and unexcused absences are located online at http://student-rules.tamu.edu/rule07. Students who request an excused absence are required to uphold the Aggie Honor Code and Student Conduct Code (See TAMU Student Rule 24).

Project due dates will be provided in the project statements. Students should contact the instructor if work is turned in late due to an absence that is excused under the University’s attendance policy. In such cases the instructor will either provide the student an opportunity to make up any quiz, exam or other graded activities or provide a satisfactory alternative to be completed within 30 calendar days from the last day of the absence. There will be no opportunity for students to make up work missed because of an unexcused absence.

Other Pertinent Attendance Information

Absences related to illness or injury must be documented according to http://shs.tamu.edu/attendance including the Explanatory Statement for Absence from class for 3 days or less. Doctor visits not related to immediate illness or injury are not excused absences.

Lecture, Lab, and Textbook:
- The lecture slides should be viewed prior to class. Class will be reserved for review of the lectures. Lab will consist of problem solving requiring the textbook. The lecture slide handouts are available on the class web page and eCampus.
- Lecture and lab are consecutive (and not separate).
- Use of electronic devices during lecture and lab is prohibited.

Notes:
- The notes and related handouts are available on the class web page at http://faculty.arch.tamu.edu/anichols/courses/elements-architectural-structures/, or on eCampus. A bound set can be purchased from Notes-n-Quotes through Textbook Solutions at 107 Walton Dr. at the intersection of New Main Dr. and Texas Ave. in the Eastgate neighborhood.

eCampus:
- eCampus is the on-line course system useful for downloading files, uploading assignments, reading messages and replying, as well as posting scores; and is accessed with your NetID. This will be used to post class materials, questions and responses by class members and the instructor, and scores. It can be accessed at http://ecampus.tamu.edu/
### Course Topics, Calendar of Activities, Major Assignment Dates

**Tentative Schedule** *(subject to change at any time throughout the semester)*

Note: Materials in the Class Note Set not specifically mentioned above are provided as references or aids.

<table>
<thead>
<tr>
<th>Week</th>
<th>Topic</th>
<th>Required Reading/Problems</th>
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</table>
| 1    | 1. Structural Performance Requirements, Systems, Math and Applied Physics | Read*: Text Introduction (pp. 1-7); note sets 1.1, 1.2 & 1.3  
Practice: *Math Worksheets*  
Solve: Assignment 1 *(start)* |
|      | 2. Forces                                                              | Read: Text 1.1-1.4; note set 2                                                                                           |
| 2    | 3. Equilibrium, Free Body Diagrams & Analysis of Planar Trusses       | Read: Text 2.6; note set 3.1  
Reference: *note set 3.2*  
Due: Assignment 1 over material from lectures 1 & 2 |
|      | 4. Response to Forces and Temperature                                  | Read: Text 2.1, 2.2, 3.1; note set 4                                                                                   |
| 3    | 5. Moments, Rotational Equilibrium & Beam Reactions                   | Read: Text 1.5-1.7; note set 5  
Due: Assignment 2 over material from lectures 2-3 & *(Cardboard Couch Swing Design) Project Part* |
|      | 6. Beam Shear and Bending                                             | Read: Text 3.2-3.5; note set 6.1  
Reference: *note set 6.2* |
| 4    | 7. Semi-graphical Method: Shear and Bending Moment Diagrams           | Read: note set 6.1  
Reference: *note set 6.3*  
Due: Assignment 3 over material from lectures 4 & 5 |
|      | 8. Beam Section Properties                                            | Read: Text Appendix A; note set 8  
Quiz 1 over material from lectures 1-3                                                                                     |
| 5    | 9. Beam Stresses                                                      | Read: Text 3.6-3.7; note set 9  
Due: Assignment 4 over material from lectures 6 & 7                                                                            |
|      | 10. Other Beams and Pinned Frames                                     | Read: Text 3.8; note sets 10.1 & 10.2  
Quiz 2 over material from lectures 4 & 5                                                                                   |
| 6    | 11. Rigid Frames - Compression & Buckling                             | Read: Text 2.8, 3.9-3.11 *(not footing pressure analysis)*; note set 11.1  
Reference: *note set 11.2*  
Due: Assignment 5 over material from lectures 8 & 9                                                                            |
|      | 12. Design Loads and Methodology                                      | Read: Text 1.8-1.11; note set 12.1  
Reference: *note sets 12.2, 12.3, 12.4 & 12.5*  
Quiz 3 over material from lectures 6 & 7                                                                                   |
| 7    | 13. Wood Construction Materials & Beam Design                         | Read: Text 4.5 & 5 *(all); note sets 13.1 & 13.2  
Due: Assignment 6 over material from lectures 10 & 11                                                                     |
|      | 14. Column Design                                                     | Read: Text 6; note set 13.2  
Quiz 4 over material from lectures 8 & 9                                                                                   |
| 8    | 15. Joints and Connection Stresses                                    | Read: Text 7; note sets 13.2 & 15  
Due: Assignment 7 over material from lectures 12 & 13                                                                        |
|      | 16. Steel Construction Materials & Beam Design                        | Read: Text 4.6 & 8 *(all); Text 9.1-9.8; note set 16  
Quiz 5 over material from lectures 9-11                                                                                   |
| 9    | 17. Trusses, Decks & Plate Girders                                    | Read: Text 9.9-9.12; note sets 16 & 17  
Due: Assignment 8 over material from lectures 14-15                                                                       |
|      | 18. Column Design                                                     | Read: Text 10; note set 16  
Quiz 6 over material from lectures 12 & 13                                                                                   |
| 10   | 19. Bolted Connections & Tension Members                              | Read: Text 11; note set 16  
Due: Assignment 9 over material from lectures 16-17                                                                        |
|      | 20. Welds and Light Gage Steel                                        | Read: Text 12; note set 16  
Quiz 7 over material from lectures 14 & 15                                                                                   |
Reference: *note set 21.2*  
Due: Assignment 10 over material from lectures 18-19                                                                     |
|      | 22. T-beams & Slabs                                                   | Read: Text 13.4-13.5; note set 21.1  
Quiz 8 over material from lectures 16 & 17                                                                                   |
| 12   | 23. Shear, Torsion, Reinforcement & Deflection                        | Read: Text 13.6-13.8; note sets 21.1 & 23  
Due: Assignment 11 over material from lectures 20-21                                                                       |
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| 24.  | Floor Systems & Continuous Beams | **Read:** Text 14; note sets 21.1 & 24.1  
**Reference:** note sets 24.2 & 24.3  
**Quiz 9** over material from lectures 18-20 |
| 13   | Columns & Frames | **Read:** Text 15; note set 21.1  
**Due:** Assignment 12 over material from lectures 22-23 & (Cardboard Couch Swing Design) Project Part II |
| 26.  | Foundation Design & Footings | **Read:** Text 3.9 (footing pressure section only), Text 16; note sets 26.1 & 26.2  
**Quiz 10** over material from lectures 21-23 |
| 14   | Masonry Construction Beams & Columns | **Read:** Text 15.4; note sets 27.1, 27.2 & 27.3  
**Due:** Assignment 13 over material from lectures 24-25 |
| 28.  | Shell Systems and Synthesis | **Read:** Text 4 & 17; note sets 28.1 & 28.2  
**Due:** Learning Portfolio |

**FINAL:**  
**1-3 PM Friday, May 6** (comprehensive)

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**Americans with Disabilities Act (ADA)**  
The Americans with Disabilities Act (ADA) is a federal anti-discrimination statute that provides comprehensive civil rights protection for persons with disabilities. Among other things, this legislation requires that all students with disabilities be guaranteed a learning environment that provides for reasonable accommodation of their disabilities. If you believe you have a disability requiring an accommodation, please contact Disability Services, currently located in the Disability Services building at the Student Services at White Creek complex on west campus or call 845-1637. For additional information visit [http://disability.tamu.edu](http://disability.tamu.edu)

**Academic Integrity**  
*“An Aggie does not lie, cheat, or steal, or tolerate those who do.”*  

Upon accepting admission to Texas A&M University, a student immediately assumes a commitment to uphold the Honor Code, to accept responsibility for learning, and to follow the philosophy and rules of the Honor System. Students will be required to state their commitment on examinations, research papers, and other academic work. Ignorance of the rules does not exclude any member of the TAMU community from the requirements or the processes of the Honor System. For additional information please visit: [http://aggiehonor.tamu.edu](http://aggiehonor.tamu.edu)

**Care of Facilities**  
Please respect your facilities in the College of Architecture (studio space, photo lab, shop, labs,...)  
The use of spray paint, spray adhesive or other surface-altering materials is not permitted in the Langford Complex, except in designated zones. Students who violate this rule will be liable for the expenses associated with repairing damaged building finishes and surfaces.

Throughout the semester and at the end of the semester, your area must be clean of all trash.

No power tools may be used in the design studio. No dust or odor producing processes may be conducted in the studio. No wet casting processes may be conducted in the studio. The college shop and spray booth facilities must be used for the above mentioned processes.

Professional behavior and conduct is expected of each student.

**All studio desks must be covered. In addition students must have at minimum an 18” x 24” cutting mat at their desk.**

**Studio Policy (required of all studios)**  
All students, faculty, administration and staff of the Department of Architecture at Texas A&M University are dedicated to the principle that the Design Studio is the central component of an effective education in architecture. They are equally dedicated to the belief that students and faculty must lead balanced lives and use time wisely, including time outside the design studio, to gain from all aspects of a university education and world experiences. They also believe that design is the integration of many parts, that process is as important as product, and that the act of design and of professional practice is inherently interdisciplinary, requiring active and respectful collaboration with others.
Students and faculty in every design studio will embody the fundamental values of optimism, respect, sharing, engagement, and innovation. Every design studio will therefore encourage the rigorous exploration of ideas, diverse viewpoints, and the integration of all aspects of architecture (practical, theoretical, scientific, spiritual, and artistic), by providing a safe and supportive environment for thoughtful innovation. Every design studio will increase skills in professional communication, through drawing, modeling, writing and speaking.

Every design studio will, as part of the syllabus introduced at the start of each class, include a clear statement on time management, and recognition of the critical importance of academic and personal growth, inside and outside the studio environment. As such it will be expected that faculty members and students devote quality time to studio activities, while respecting the need to attend to the broad spectrum of the academic life. Every design studio will establish opportunities for timely and effective review of both process and products. Studio reviews will include student and faculty peer review. Where external reviewers are introduced, the design studio instructor will ensure that the visitors are aware of the Studio Culture Statement and recognize that the design critique is an integral part of the learning experience. The design studio will be recognized as place for open communication and movement, while respecting the needs of others, and of the facilities.

**Important Links Below**

- Department of Architecture Website: [http://dept.arch.tamu.edu/](http://dept.arch.tamu.edu/)
- Department Financial Assistance: [http://dept.arch.tamu.edu/financial-assistance/](http://dept.arch.tamu.edu/financial-assistance/)
- Academic Calendar: [http://registrar.tamu.edu/general/calendar.aspx](http://registrar.tamu.edu/general/calendar.aspx)
- Final Exam Schedule Online: [http://registrar.tamu.edu/Courses,-Registration,-Scheduling/Final-Exam-Schedule](http://registrar.tamu.edu/Courses,-Registration,-Scheduling/Final-Exam-Schedule)
- On-Line Catalog: [http://catalog.tamu.edu](http://catalog.tamu.edu)
- Student Rules: [http://student-rules.tamu.edu/](http://student-rules.tamu.edu/)
- Aggie Honor System Office: [http://aggiehonor.tamu.edu/](http://aggiehonor.tamu.edu/)
### DEPARTMENT OF ARCHITECTURE

#### ARCH 614

#### SPRING 2016

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#### JANUARY

- **King Holiday**

#### FEBRUARY

- **classes begin**
- **last day to add**

#### MARCH

- **mid-term grades due**

#### APRIL

- **pre-registration begins**

#### MAY

1. **Monday classes**
2. **Friday classes**
3. **Reading Day**
4. **Final exams**
5. **Commencement (Fri. and Sat.)**
6. **Grades due**

#### ACKNOWLEDGEMENTS
ARCH 614. Student Understandings

1) I understand that there are intellectual standards in this course and that I am responsible for monitoring my own learning.

2) I understand that the class will focus on practice, not on lecture.

3) I understand that I am responsible for preparing for lecture with the assigned reading and lecture show by internalizing key concepts, recognizing key questions, and evaluating what makes sense and what doesn’t make sense to me.

4) I understand that I will be held regularly responsible for assessing my own work using criteria and standards discussed in class.

5) I understand that if at any time in the semester I feel unsure about my “grade”, I may request an assessment from the instructor.

6) I understand that there are 13 practice assignments, one due every week during the bulk of the semester.

7) I understand that there are group projects and I will be responsible to take an active part in advancing the work of the group.

8) I understand that I will occasionally be required to assess the work of my classmates in an objective manor using the same criteria and standards used to assess my own work.

9) I understand that there are 10 graded quizzes, one given every week during the bulk of the semester.

10) I understand that there is a final exam in the course.

11) I understand that I must do a Learning Portfolio, which is a self-evaluation that makes my “case” for receiving a particular grade using criteria provided in class and citing evidence from my work across the semester.

12) I understand that the work of the course requires Consistent classroom attendance and active participation.

13) I understand that I will regularly be required to demonstrate that I have prepared for lecture.

14) I understand that the class will not be graded on a curve. I understand that it is theoretically possible for the whole class to get an A or an F.

15) I understand the basis of the final grade as outlined in the syllabus.

16) I understand that since the final grade is based on percentages from graded work and competency on assignments as outlined in the syllabus, that the minimum level of both must be satisfied to obtain the letter grade. The criteria for assignments that are considered “passing” is outlined in the syllabus section on Learning Objectives.

NAME ____________________________ DATE __________________________

signature

______________________________

printed name
List of Symbol Definitions

\(a\)  
long dimension for a section subjected to torsion (in, mm);  
acceleration due to gravity, 32.17 ft/sec\(^2\), 9.81 m/sec\(^2\);  
unit area (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\));  
distance used in beam formulas (ft, m);  
depth of the effective compression block in a concrete beam (in, mm)

\(a\)  
area bounded by the centerline of a thin walled section subjected to torsion (in\(^2\), mm\(^2\))

\(A\)  
area, often cross-sectional (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\))

\(A_{e}\)  
net effective area, equal to the total area ignoring any holes and modified by the lag factor, \(U\), (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\)) (see \(A_{net}\))

\(A_{g}\)  
gross area, equal to the total area ignoring any holes (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\))

\(A_{gv}\)  
gross area subjected to shear for block shear rupture (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\))

\(A_{net}\)  
net effective area, equal to the gross area subtracting any holes (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\)) (see \(A_{e}\))

\(A_{nt}\)  
net area subjected to tension for block shear rupture (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\))

\(A_{nv}\)  
net area subjected to shear for block shear rupture (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\))

\(A_{p}\)  
bearing area (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\))

\(A_{throat}\)  
area across the throat of a weld (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\))

\(A_{s}\)  
area of steel reinforcement in concrete beam design (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\))

\(A_{s'}\)  
area of compression steel reinforcement in concrete beam design (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\))

\(A_{v}\)  
area of concrete shear stirrup reinforcement (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\))

\(A_{web}\)  
web area in a steel beam equal to the depth \(x\) web thickness (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\))

\(A_{l}\)  
area of column in spread footing design (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\))

\(A_{2}\)  
projected bearing area of column load in spread footing design (in\(^2\), ft\(^2\), mm\(^2\), m\(^2\))

\(ASD\)  
Allowable Stress Design

\(b\)  
width, often cross-sectional (in, ft, mm, m);  
narrow dimension for a section subjected to torsion (in, mm);  
number of truss members;  
rectangular column dimension in concrete footing design (in, mm, m);  
distance used in beam formulas (ft, m)

\(b_{E}\)  
effective width of the flange of a concrete T beam cross section (in, mm)

\(b_{f}\)  
width of the flange of a steel or concrete T beam cross section (in, mm)

\(b_{o}\)  
perimeter length for two-way shear in concrete footing design (in, mm, m)

\(b_{w}\)  
width of the stem of a concrete T beam cross section (in, mm)

\(B\)  
spread footing dimension in concrete design (ft, m);  
dimension of a steel base plate for concrete footing design (in, mm, m)

\(B_{l}\)  
factor for determining \(M_{u}\) for combined bending and compression

\(B_{s}\)  
width within the longer dimension of a rectangular spread footing that reinforcement must be concentrated within for concrete design (ft, m)
\( c \) distance from the neutral axis to the top or bottom edge of a beam (in, mm, m);
rectangular column dimension in concrete footing design (in, mm, m);
the distance from the top of a masonry or concrete beam to the neutral axis (in, mm, m) \((\text{see } x)\);
buckling and crushing interaction factor for wood columns \((\text{see } C_p)\)

\( c_i \) distance from the center of a circular shape to the inner surface under torsional shear strain (in, mm, m)

\( c_o \) distance from the center of a circular shape to the outer surface under torsional shear strain (in, mm, m)

\( c_1 \) coefficient for shear stress for a rectangular bar in torsion

\( c_2 \) coefficient for shear twist for a rectangular bar in torsion

\( CL, \ell \) center line

\( C \) compression label;
compression force (lb, kips, N, kN);
dimension of a steel base plate for concrete footing design (in, mm, m)

\( C_b \) modification factor for LRFD steel beam design

\( C_c \) column slenderness classification constant for steel column design;
compressive force in the concrete of a doubly reinforced concrete beam (lb, k, N, kN)

\( C_D \) load duration factor for wood design

\( C_F \) size factor for wood design

\( C_f \) form factor for circular sections or or square sections loaded in plane of diagonal for wood design

\( C_{fu} \) flat use factor for wood design

\( C_F \) size factor for wood design

\( C_H \) shear stress factor for wood design

\( C_i \) incising factor for wood design

\( C_L \) beam stability factor for wood design

\( C_m \) modification factor for combined stress in steel design

\( C_M \) wet service factor for wood design

\( C_p \) column stability factor for wood design

\( C_r \) repetitive member factor for wood design

\( C_s \) compressive force in the compression steel of a doubly reinforced concrete beam (lb, k, N, kN)

\( C_t \) temperature factor for wood design

\( C_T \) buckling stiffness factor for wood truss design

\( C_v \) web shear coefficient for steel design

\( C_V \) glulam volume factor for wood design
\(d\) diameter of a circle (in, mm, m);
depth, often cross-sectional (in, mm, m);
perpendicular distance from a force to a point in a moment calculation (in, mm, m);
critical cross section dimension of a rectangular timber column cross section related to the profile (axis) for buckling (in, mm, m);
effective depth from the top of a reinforced concrete beam to the centroid of the tensile steel (in, mm);
symbol in calculus to represent a very small change (like the greek letters for \(d\), see \(\delta\) & \(\Delta\))
\(d'\) effective depth from the top of a reinforced concrete beam to the centroid of the compression steel (in, mm)

\(d_b\) depth of a steel W beam section (in, mm);
bar diameter of concrete reinforcement (in, mm)
nominal bolt diameter (in, mm)

\(d_f\) depth of a steel W column flange (in, mm)

\(d_x\) difference in the x direction between an area centroid and the centroid of the composite shape (in, mm)

\(d_y\) difference in the y direction between an area centroid and the centroid of the composite shape (in, mm)

\(D\) diameter of a circle (in, mm, m);
dead load for LRFD design

\(DL\) dead load

\(e\) dimensional change to determine strain (see \(s\) or \(\varepsilon\)) (in, mm);
eccentric distance of application of a force (\(P\)) from the centroid of a cross section (in, mm)

\(E\) modulus of elasticity (psi, ksi, kPa, MPa, GPa);
earthquake load for LRFD design

\(E_c\) modulus of elasticity of concrete (psi, ksi, kPa, MPa, GPa)

\(E_{\text{min}}\) reference modulus of elasticity for stability (psi, ksi, kPa, MPa, GPa)

\(E_{\text{min n}}\) reference nominal modulus of elasticity for stability with LRFD (psi, ksi, kPa, MPa, GPa)

\(E_s\) modulus of elasticity of steel (psi, ksi, kPa, MPa, GPa)

\(E'_{\text{min}}\) adjusted modulus of elasticity for stability (psi, ksi, kPa, MPa, GPa)

\(E'_{\text{min n}}\) adjusted nominal modulus of elasticity for stability with LRFD (psi, ksi, kPa, MPa, GPa)

\(f\) symbol for stress (psi, ksi, kPa, MPa)
symbol for function with respect to some variable; ie. \(f(t)\)

\(f_a\) calculated axial stress (psi, ksi, kPa, MPa)

\(f_b\) calculated bending stress (psi, ksi, kPa, MPa)

\(f_c\) calculated compressive stress (psi, ksi, kPa, MPa)

\(f'_{c}\) concrete design compressive stress (psi, ksi, kPa, MPa)

\(f_{cr}\) calculated column stress based on the critical column load \(P_{cr}\) (psi, ksi, kPa, MPa)

\(f_m\) calculated compressive stress in masonry (psi, ksi, kPa, MPa)
\( f'_m \) masonry design compressive stress (psi, ksi, kPa, MPa)
\( f_p \) calculated bearing stress (psi, ksi, kPa, MPa)
\( f_s \) calculated steel stress for reinforced masonry (psi, ksi, kPa, MPa)
\( f_t \) calculated tensile stress (psi, ksi, kPa, MPa)
\( f_x \) combined stress in the direction of the major axis of a column (psi, ksi, kPa, MPa)
\( f_v \) calculated shearing stress (psi, ksi, kPa, MPa)
\( f_y \) yield stress (psi, ksi, kPa, MPa)
\( f_{yt} \) yield stress of transverse reinforcement (psi, ksi, kPa, MPa)

\( F \) force (lb, kip, N, kN);
capacity of a nail in shear (lb, kip, N, kN);
hydraulic fluid load for LRFD design

\( F_a \) allowable axial stress (psi, ksi, kPa, MPa)
\( F_b \) allowable bending stress (psi, ksi, kPa, MPa)
\( F_b' \) allowable bending stress for ASD wood design (psi, ksi, kPa, MPa)
\( F_{in} \) nominal bending stress for LRFD wood design (psi, ksi, kPa, MPa)
\( F_c \) allowable compressive stress (psi, ksi, kPa, MPa);
critical unfactored compressive stress for LRFD steel design
\( F_{c,L} \) allowable compressive stress perpendicular to the wood grain (psi, ksi, kPa, MPa)

\( F_{connector} \) resistance capacity of a connector (lb, kips, N, kN)
\( F_{c,E} \) intermediate compressive stress for ASD wood column design dependant on material (psi, ksi, kPa, MPa)
\( F_{cr} \) flexural buckling (column) stress in ASD and LRFD (psi, ksi, kPa, MPa)
\( F_c' \) allowable compressive stress for ASD wood column design (psi, ksi, kPa, MPa)
\( F_{cn}' \) nominal compressive stress for LRFD wood design (psi, ksi, kPa, MPa)
\( F_{c,E}' \) intermediate compressive stress for ASD wood column design dependant on load duration (psi, ksi, kPa, MPa)
\( F_e \) elastic critical buckling stress is steel design
\( F_h \) force component in the horizontal direction (lb, kip, N, kN)
\( F_{horizontal-resist} \) resultant frictional force resisting sliding in a footing or retaining wall (lb, kip, N, kN)
\( F_n \) nominal strength in LRFD steel design (psi, ksi, kPa, MPa)
nominal tension or shear strength of a bolt (psi, ksi, kPa, MPa)
\( F_p \) allowable bearing stress parallel to the wood grain (psi, ksi, kPa, MPa)
\( F_{sliding} \) resultant force causing sliding in a footing or retaining wall (lb, kip, N, kN)
\( F_t \) allowable tensile stress (psi, ksi, kPa, MPa)
\( F_v \) allowable shear stress (psi, ksi, kPa, MPa); allowable shear stress in a welded connection; force component in the vertical direction (lb, kip, N, kN)

\( F'_v \) allowable shear stress for ASD wood beam design (psi, ksi, kPa, MPa)

\( F'_w \) nominal shear stress for LRFD wood beam design (psi, ksi, kPa, MPa)

\( F_x \) force component in the x coordinate direction (lb, kip, N, kN)

\( F_w \) allowable weld stress (psi, ksi, kPa, MPa)

\( F_y \) force component in the y coordinate direction (lb, kip, N, kN); yield stress (psi, ksi, kPa, MPa)

\( F_u \) ultimate stress a material can sustain prior to failure (psi, ksi, kPa, MPa)

\( F.S. \) factor of safety (also see SF)

\( g \) acceleration due to gravity, 32.17 ft/sec\(^2\), 9.807 m/sec\(^2\); transverse center-to-center spacing (gage) between fastener gage lines (in, mm,)

\( G \) shear modulus (psi, ksi, kPa, MPa, GPa); relative stiffness of columns to beams in a rigid connection (see \( \Psi \))

\( h \) depth, often cross-sectional (in, ft, mm, m); height (in, ft, mm, m); sag of a cable structure (ft, m); effective height of a wall or column (see \( \ell_e \))

\( h' \) effective height of a wall or column (see \( \ell_e \))

\( h_w \) height of the web in a W section (in, ft, mm, m) (also see \( t_w \))

\( h_f \) depth of a flange in a T section (in, ft, mm, m); height of a concrete spread footing (in, ft, mm, m)

\( H \) hydraulic soil load for LRFD design

\( H_A \) horizontal load from active soil or water pressure (lb, k, N, kN)

\( I \) moment of inertia (in\(^4\), mm\(^4\), m\(^4\))

\( \bar{I} \) moment of inertia about the centroid (in\(^4\), mm\(^4\), m\(^4\))

\( \hat{I} \) moment of inertia about the centroid of a composite shape (in\(^4\), mm\(^4\), m\(^4\))

\( I_c \) moment of inertia about the centroid of a composite shape (in\(^4\), mm\(^4\), m\(^4\))

\( I_{min} \) minimum moment of inertia of \( I_x \) and \( I_y \) (in\(^4\), mm\(^4\), m\(^4\))

\( I_o \) moment of inertia about the centroid (in\(^4\), mm\(^4\), m\(^4\))

\( I_{transformed} \) moment of inertia of a multi-material section transformed to one material (in\(^4\), mm\(^4\), m\(^4\))

\( I_x \) moment of inertia with respect to an x-axis (in\(^4\), mm\(^4\), m\(^4\))

\( I_y \) moment of inertia with respect to a y-axis (in\(^4\), mm\(^4\), m\(^4\))

\( j \) multiplier by effective depth of masonry section for moment arm, \( jd \) (see \( d \))

\( J, J_o \) polar moment of inertia (in\(^4\), mm\(^4\), m\(^4\))
$k$  
- kips (1000 lb);  
- shape factor for steel beams, $M_p/M_y$;  
- effective length factor for columns (*also* $K$);  
- distance from outer face of W flange to the web toe of fillet (in, mm);  
- multiplier by effective depth of masonry section for neutral axis, $kd$

$kg$  
- kilograms

$k/lf$  
- kips per linear foot (k/ft)

$ksf$  
- kips per square foot (k/ft$^2$)

$k/si$  
- kips per square inch (k/in$^2$)

$kN$  
- kiloNewtons ($10^3$ N)

$kPa$  
- kiloPascals ($10^3$ Pa)

$K$  
- effective length factor with respect to column end conditions;  
- masonry mortar strength designation

$K_{CE}$  
- material factor for wood column design

$K_F$  
- format conversion factor for timber LRFD design

$l$  
- length (in, ft, mm, m);  
- cable span (ft, m)

$l_d$  
- development length of concrete reinforcement (in, ft, mm, m)

$l_{dc}$  
- development length of compression reinforcement in concrete footing design (in, ft, mm, m)

$l_{dh}$  
- development length for hooks (in, ft, mm, m)

$l_e$  
- effective length that can buckle for wood column design (in, ft, mm, m)

$l_n$  
- effective clear span for concrete one-way slab design (ft, m)

$l_{sc}$  
- lap splice length in compression for reinforcement (in, ft, mm, m)

$lb$  
- pound force

$L$  
- length (in, ft, mm, m);  
- live load for LRFD design;  
- spread footing dimension in concrete design (ft, m)

$L_b$  
- unbraced length of a steel beam in LRFD design (ft, m)

$L_c$  
- maximum unbraced length of a steel beam in ASD design for compression buckling limit (ft, m);  
- clear distance between the edge of a hole and edge of next hole or edge of the connected steel plate (in, ft, mm, m)

$L_d$  
- development length of reinforcement in concrete (ft, m)

$L_e$  
- effective length that can buckle for column design (ft, m)

$L_{lm}$  
- projected length for bending in concrete footing design (ft, m)

$L_{lp}$  
- limiting length of a steel beam in LRFD design for full plastic strength (ft, m)

$L_r$  
- roof live load in LRFD design;  
- limiting length of a steel beam in LRFD design for inelastic lateral-torsional buckling (ft, m)

$L_{ut}$  
- maximum unbraced length of a steel beam in ASD design for stress limit of $0.6F_y$
$L'$ length of the one-way shear area in concrete footing design (ft, m)

$LL$ live load

$LRFD$ Load and Resistance Factor Design

$m$ mass (lb-mass, g, kg);
meters

$mm$ millimeters

$M$ moment of a force or couple (lb-ft, kip-ft, N-m, kN-m);
bending moment (lb-ft, kip-ft, N-m, kN-m);
masonry mortar strength designation

$M_a$ required bending moment in steel ASD beam design (unified) (lb-ft, kip-ft, N-m, kN-m)

$M_A$ moment value at quarter point of unbraced beam length for LRFD beam design (lb-ft, kip-ft, N-m, kN-m)

$M_B$ nominal moment capacity of a reinforced concrete beam at the balanced steel ratio ($\rho_b$) for limiting strains in both concrete and steel (lb-ft, kip-ft, N-m, kN-m)

$M_C$ moment value at half point of unbraced beam length for LRFD beam design (lb-ft, kip-ft, N-m, kN-m)

$M_c$ nominal moment capacity of a reinforced concrete beam based on compression force in a concrete section (lb-ft, kip-ft, N-m, kN-m) *(also see $M_n$)*

$M_C$ moment value at three quarter point of unbraced beam length for LRFD beam design (lb-ft, kip-ft, N-m, kN-m)

$M_m$ moment capacity of a reinforced masonry beam (lb-ft, kip-ft, N-m, kN-m)

$M_n$ nominal moment capacity of a reinforced concrete beam based on steel yielding and concrete design strength (lb-ft, kip-ft, N-m, kN-m)

$M_{overturning}$ resulting moment from all forces on a footing or retaining wall causing overturning (lb-ft, kip-ft, N-m, kN-m)

$M_P$ internal bending moment when all fibers in a cross section reach the yield stress (lb-ft, kip-ft, N-m, kN-m) *(also see $M_{ult}$)*

$M_r$ required nominal moment capacity based on design moment for reinforced concrete (lb-ft, kip-ft, N-m, kN-m) *(also see $M_n$)*

$M_{resis}$ resulting moment from all forces on a footing or retaining wall resisting overturning (lb-ft, kip-ft, N-m, kN-m)

$M_t$ nominal moment capacity of a reinforced concrete beam based on tensile force in the steel reinforcement (lb-ft, kip-ft, N-m, kN-m) *(also see $M_n$)*

$M_u$ factored moment calculated in concrete design from load factors (lb-ft, kip-ft, N-m, kN-m)

$M_{ult}$ internal bending moment when all fibers in a cross section reach the yield stress (lb-ft, kip-ft, N-m, kN-m) *(also see $M_P$)*

$M_y$ internal bending moment when the extreme fibers in a cross section reach the yield stress (lb-ft, kip-ft, N-m, kN-m)

$n$ number of truss joints, nails or bolts;

modulus of elasticity transformation coefficient from steel to concrete

$n.a.$ neutral axis (axis connecting beam cross-section centroids)
\( N \)  
Newtons; 
-bearing-type connection with bolt threads included in shear plane; 
normal load (lb, kip, N, kN); 
bearing length on a wide flange steel section (in, mm) 
masonry mortar strength designation

\( o.c. \)  
on-center

\( O \)  
point of origin; 
masonry mortar strength designation

\( p \)  
pitch of nail or bolt spacing (in, mm) \((\text{also see } s)\); 
pressure (lb/in\(^2\), lb/ft\(^2\), kip/in\(^2\), kip/ft\(^2\), Pa, MPa); 
reinforcement ratio in concrete beam design = \( A_s/bd \) (or possibly \( A_s/bt, A_s/bh \)) (no units) \((\text{see } \rho)\)

\( p_A \)  
active soil pressure (lb/ft\(^2\), kN/m\(^3\))

\( p_b \)  
balanced reinforcement ratio in concrete beam design \((\text{see } \rho_b)\)

\( plf \)  
pounds per linear foot (lb/ft)

\( psf \)  
pounds per square foot (lb/ft\(^2\))

\( psi \)  
pounds per square inch (lb/in\(^2\))

\( P \)  
force, concentrated (point) load (lb, kip, N, kN)

\( P_a \)  
required axial force in ASD steel design (unified) (lb, kip, N, kN)

\( P_e \)  
available axial strength for steel unified design (lb, kip, N, kN)

\( P_{cr} \)  
critical (failure) load in column calculations (lb, kip, N, kN)

\( P_{el} \)  
Euler buckling strength in steel unified design (lb, kip, N, kN)

\( P_n \)  
maximum column load capacity in LRFD steel and concrete design (lb, kip, N, kN)

\( P_o \)  
maximum axial force with no concurrent bending moment in a reinforced concrete column (lb, kip, N, kN)

\( P_r \)  
required axial force in steel unified design (lb, kip, N, kN)

\( P_u \)  
factored column load calculated from load factors in LRFD steel and concrete design (lb, kip, N, kN)

\( Pa \)  
Pascals (N/m\(^2\))

\( q \)  
shear flow (lb/in, kips/ft, N/m, kN/m)

\( q_{allowed} \)  
allowable soil bearing pressure (lb/ft\(^2\), kips/ft\(^2\), N/m\(^2\), Pa, MPa)

\( q_{net} \)  
net allowed soil bearing pressure (lb/ft\(^2\), kips/ft\(^2\), N/m, Pa, MPa)

\( q_u \)  
factored soil bearing pressure in concrete design from load factors (lb/ft\(^2\), kips/ft\(^2\), N/m, Pa, MPa)

\( Q \)  
first moment area used in shearing stress calculations (in\(^3\), mm\(^3\), m\(^3\))

\( Q_{connected} \)  
first moment area used in shear calculations for built-up beams (in\(^3\), mm\(^3\), m\(^3\))

\( Q_x \)  
first moment area about an x axis (using y distances) (in\(^3\), mm\(^3\), m\(^3\))

\( Q_y \)  
first moment area about an y axis (using x distances) (in\(^3\), mm\(^3\), m\(^3\))

\( r \)  
radius of a circle (in, mm, m); 
radius of gyration (in, mm, m)
\( r_o \)  
- polar radius of gyration (in, mm, m)

\( r_x \)  
- radius of gyration with respect to an x-axis (in, mm, m)

\( r_y \)  
- radius of gyration with respect to a y-axis (in, mm, m)

\( R \)  
- force, reaction or resultant (lb, kip, N, kN); 
- radius of curvature of a beam (ft, m); 
- rainwater or ice load for LRFD design

\( R_a \)  
- required strength (ASD-unified) \((\text{also see } V_a, M_a)\)

\( R_n \)  
- concrete beam design ratio \(= \frac{M_o}{bd^2}\) (lb/in\(^2\), MPa) 
- nominal value for LRFD design to be multiplied by \(\phi\) \((\text{also see } P_n, M_n)\) 
- nominal value for ASD design to be divided by the safety factor \(\Omega\)

\( R_x \)  
- reaction or resultant component in the x coordinate direction (lb, kip, N, kN)

\( R_y \)  
- reaction or resultant component in the y coordinate direction (lb, kip, N, kN)

\( s \)  
- strain (change in length divided by length (no units); 
- displacement with respect to time (ft, m); 
- length of a segment of a thin walled section (in, mm); 
- pitch of nail spacing (in, mm) \((\text{also see } p)\); 
- spacing of stirrups in reinforced concrete beams (in, mm); 
- longitudinal center-to-center spacing of any two consecutive holes (in, mm)

\( s.w. \)  
- self-weight

\( S \)  
- section modulus \((\text{in}^3, \text{mm}^3, \text{m}^3)\); 
- snow load for LRFD design; 
- allowable strength of a weld for a given size (lb/in, kips/in, N/mm, kN/m); 
- masonry mortar strength designation

\( S_{\text{required}} \)  
- section modulus required to not exceed allowable bending stress \((\text{in}^3, \text{mm}^3, \text{m}^3)\)

\( S_x \)  
- section modulus with respect to the x-centroidal axis \((\text{in}^3, \text{mm}^3, \text{m}^3)\)

\( S_y \)  
- section modulus with respect to the y-centroidal axis \((\text{in}^3, \text{mm}^3, \text{m}^3)\)

\( SC \)  
- slip critical bolted connection

\( SF \)  
- safety factor \((\text{see } F.S.)\)

\( S4S \)  
- surface-four-sided

\( t \)  
- thickness (in, mm, m); 
- time (sec, hrs)

\( t_f \)  
- thickness of the flange of a steel beam cross section (in, mm, m)

\( t_w \)  
- thickness of the web of a steel beam cross section (in, mm, m)

\( T \)  
- tension label; 
- tensile force (lb, kip, N, kN); 
- torque (lb-ft, kip-ft, N-m, kN-m); 
- throat size of a weld (in, mm); 
- effect of thermal load for LRFD design

\( U \)  
- shear lag factor for bolted connections

\( U_{bs} \)  
- reduction coefficient for block shear rupture
\( v \) velocity (ft/sec, m/sec, mi/h); shear force per unit length (lb/ft, k/ft, N/m, kN/m) \((\text{see } q)\)

\( V \) shear force (lb, kip, N, kN)

\( V_a \) required shear in steel ASD design (unified) (lb, kip, N, kN)

\( V_c \) shear force capacity in concrete (lb, kip, N, kN)

\( V_n \) nominal shear force capacity for concrete design (lb, kip, N, kN)

\( V_s \) shear force capacity in steel (lb, kip, N, kN)

\( V_u \) factored shear calculated in concrete design from load factors (lb, kip, N, kN)

\( V_{u1} \) factored one-way shear calculated in concrete footing design from load factors (lb, kip, N, kN)

\( V_{u2} \) factored two-way shear calculated in concrete footing design from load factors (lb, kip, N, kN)

\( w \) load per unit length on a beam (lb/ft, kip/ft, N/m, kN/m); load per unit area on a surface (lb/ft\(^2\), kip/ft\(^2\), N/m\(^2\), kN/m\(^2\)); width dimension (in, ft, mm, m)

\( w_c \) weight of reinforced concrete per unit volume (lb/ft\(^3\), N/m\(^3\))

\( w_u \) factored load per unit length on a beam from load factors (lb/ft, kip/ft, N/m, kN/m); factored load per unit area on a surface from load factors (lb/ft\(^2\), kip/ft\(^2\), N/m\(^2\), kN/m\(^2\))

\( W \) weight (lb, kip, N, kN); total load from a uniform distribution (lb, kip, N, kN); wind load for LRFD design

\( x \) a distance in the \( x \) direction (in, ft, mm, m); the distance from the top of a masonry or concrete beam to the neutral axis (in, mm, m) \((\text{see } c)\)

\( \bar{x} \) the distance in the \( x \) direction from a reference axis to the centroid of a shape (in, mm)

\( \hat{x} \) the distance in the \( x \) direction from a reference axis to the centroid of a composite shape (in, mm)

\( X \) bearing-type connection with bolt threads excluded from shear plane

\( y \) a distance in the \( y \) direction (in, ft, mm, m); distance from the neutral axis to the \( y \)-level of a beam cross section (in, mm)

\( \bar{y} \) the distance in the \( y \) direction from a reference axis to the centroid of a shape (in, mm)

\( \hat{y} \) the distance in the \( y \) direction from a reference axis to the centroid of a composite shape (in, mm)

\( z \) the distance from a unit area to a reference axis (in, ft, mm, m) \((\text{also see } d_x \text{ and } d_z)\)

\( Z \) plastic section modulus of a steel beam (in\(^3\), mm\(^3\)) lateral design value for a single fastener in a timber connection (lb/nail, k/bolt)

\( ' \) symbol for feet

\( " \) symbol for inches

\( \# \) symbol for pounds

\( = \) symbol for equal to

\( \approx \) symbol for approximately equal to

\( \propto \) symbol for proportional to

\( \leq \) symbol for less than or equal to

\( \int \) symbol for integration
\( \alpha \) coefficient of thermal expansion (\(^\circ\)C, \(^\circ\)F);
angle, in a math equation (degrees, radians)

\( \beta \) angle, in a math equation (degrees, radians)

\( \beta_c \) ratio of long side to short side of the column in concrete footing design

\( \beta_i \) coefficient to determine the stress block height in concrete beam design

\( \delta \) elongation (in, mm)  
\( \text{(also see } e) \)

\( \delta_P \) elongation due to axial load (in, mm)

\( \delta_s \) shear deformation (in, mm)

\( \delta_T \) elongation due to change in temperature (in, mm)

\( \Delta \) beam deflection (in, mm);
\( \text{an increment} \)

\( \Delta_{LL} \) beam deflection due to live load (in, mm)

\( \Delta_{max} \) maximum calculated beam deflection (in, mm)

\( \Delta_{TL} \) beam deflection due to total load (in, mm)

\( \Delta_x \) beam deflection in beam diagrams and formulas (in, mm)

\( \Delta T \) change in temperature (\(^\circ\)C, \(^\circ\)F)

\( \varepsilon \) strain  
\( \text{(also see } s) \)

\( \varepsilon_t \) thermal strain

\( \phi \) diameter symbol;
angle of twist (degrees, radians);
resistance factor in LRFD steel design and reinforced concrete design

\( \phi_b \) resistance factor for flexure in LRFD design

\( \phi_c \) resistance factor for compression in LRFD design

\( \phi_s \) resistance factor for stability in timber LRFD design

\( \phi_t \) resistance factor for tension in LRFD design

\( \phi_v \) resistance factor for shear in LRFD design

\( \lambda \) time effect factor in LRFD timber design;
modification factor for reinforced concrete shear for lightweight materials

\( \lambda_c \) design constant for slenderness evaluation for steel columns in LRFD design

\( \mu \) Poisson’s ratio;
coefficient of static friction
\( \gamma \) specific gravity of a material (lb/in\(^3\), lb/ft\(^3\), N/m\(^3\), kN/m\(^3\));
angle, in a math equation (degrees, radians);
shearing strain (no units);
load factor in LRFD design
ratio of reinforcement width to width of column

\( \gamma_D \) dead load factor in LRFD steel design

\( \gamma_L \) live load factor in LRFD steel design

\( \theta \) angle, in a trig equation (degrees, radians);
slope of the deflection of a beam at a point (degrees, radians)

\( \pi \) pi (180°)

\( \rho \) radial distance (in, mm);
radius of curvature in beam deflection relationships (ft, m);
reinforcement ratio in concrete beam design = \( A_s/bd \) (or possibly \( A_s/bt, A_s/bh \)) (no units)

\( \rho_b \) balanced reinforcement ratio in concrete beam design

\( \rho_g \) reinforcement ratio in concrete column design = \( A_{sl}/A_g \)

\( \rho_{max} \) maximum reinforcement ratio allowed in concrete beam design for ductile behavior

\( \sigma \) engineering symbol for normal stress (axial or bending)

\( \tau \) engineering symbol for shearing stress

\( \nu_c \) shearing stress capacity in concrete design (psi, ksi, kPa, MPa)

\( \omega \) load per unit length on a beam (lb/ft, kip/ft, N/m, kN/m)  \((see\ w)\);
load per unit area (lb/ft\(^2\), kips/ft\(^2\), N/m\(^2\), Pa, MPa)

\( \Sigma \) summation symbol

\( \Omega \) safety factor for ASD of steel (unified)

\( \Psi \) relative stiffness of columns to beams in a rigid connection \((see\ G)\)
Structural Glossary

Allowable strength: Nominal strength divided by the safety factor.

Allowable stress: Allowable strength divided by the appropriate section property, such as section modulus or cross section area.

Applicable building code: Building code und which the structure is designed.

ASD (Allowable Strength Design): Method of proportioning structural components such that the allowable strength equals or exceeds the required strength of the component under the action of the ASD load combinations.

ASD load combination: Load combination in the applicable building code intended for allowable strength design (allowable stress design).


Axial force: A force that is acting along the longitudinal axis of a structural member.

Base shear: A lateral (wind or seismic) force acting at the base of a structure.

Beam: Structural member that has the primary function of resisting bending moments.

Beam-column: Structural member that resists both axial force and bending moment.

Bearing (local compressive yielding): Limit state of local compressive yielding due to the action of a member bearing against another member or surface.

Bending moment: A force rotating about a point; causes bending in beams, etc.

Block shear rupture: In a connection, limit state of tension fracture along one path and shear yielding or shear fracture along another path.

Bracing: Diagonal members that are used to stiffen a structure, by utilizing the inherent in-plane stiffness of a triangular framework.

Braced frame: An essentially vertical truss system that provides resistance to lateral forces and provides stability for the structural system.

Buckling: Limit state of sudden change in the geometry of a structure or any of its elements under a critical loading condition.

Buckling strength: Nominal strength for buckling or instability limit states.

Built-up member, cross-section, section, shape: Member, cross-section, section or shape fabricated from elements that are nailed, welded, glued or bolted together.

Camber: Curvature fabricated into a beam or truss so as to compensate for deflection induced by loads.

Cantilevers: Structural elements or systems that are supported only at one end.

Cement: The generic name for cementitious (binder) materials used in concrete, which is a commonly used building material.

Center of gravity: The location of resultant gravity forces on an object or objects.

Centroid: The center of mass of a shape or object.
**Chord member:** Primary member that extends, usually horizontally, through a truss connection.

**Cold-rolled steel structural member:** Shape manufactured by roll forming cold-or hot-rolled coils or sheets without manifest addition of heat such as would be required for hot forming.

**Collector:** An element that transfers load from a diaphragm to a resisting element.

**Column:** Structural member that has the primary function of resisting axial force.

**Component (of vector):** One of several vectors combined to a resultant vector.

**Composite:** Condition in which steel and concrete elements and members work as a unit in the distribution of internal forces.

**Composite materials:** Those consisting of a combination of two or more distinct materials, together yielding superior characteristics to those of the individual constituents.

**Compression:** A force that tends to shorten or crush a member or material.

**Concentrated force:** A force acting on a single point.

**Concentrated load:** An external concentrated force (also known as a point load).

**Concrete:** Material composed mainly of cement, crushed rock or gravel, sand and water.

**Concrete crushing:** Limit state of compressive failure in concrete having reached the ultimate strain.

**Connection:** A connection joins members to transfer forces or moments from one to the other.

**Cope:** Cutout made in a structural member to remove a flange and conform to the shape of an intersecting member.

**Couple:** A couple is a system of two equal forces of opposite direction offset by a distance. A couple causes a moment whose magnitude equals the magnitude of the force times the offset distance.

**Cover plate:** Plate welded or bolted to the flange of a member to increase cross-sectional area, section modulus or moment of inertia.

**Creep:** Plastic deformation that proceeds with time.

**Curvature:** The geometric quantity defined by the inverse of the radius of curvature, \( 1/R \)

**Damping:** Reduces vibration amplitude, like amplitude seismic vibration.

**Dead load:** The weight of a structure or anything permanently attached to it.

**Deflection:** Deflection is the vertical moment under gravity load of beams for example, while lateral movement under wind of seismic load is called drift.

**Deformation:** A change of the shape of an object or material.

**Design load:** Applied load determined in accordance with either LRFD load combinations or ASD load combinations, whichever is applicable.

**Design strength:** Resistance factor multiplied by the nominal strength, \( \sigma Rn \).

**Design stress range:** Magnitude of change in stress due to the repeated application and removal of service live loads. For locations subject to stress reversal it is the algebraic difference of the peak stresses.
Design stress: Design strength divided by the appropriate section property, such as section modulus or cross section area.

Determinate structure: A structure with the number of reactions equal to the number of static equations (3).

Diagonal Bracing: Inclined structural member carrying primarily axial force in a braced fame.

Diaphragm plate: Plate possessing in-plane shear stiffness and strength, used to transfer forces to the supporting elements.

Diaphragm: Roof, floor or other membrane or bracing system that transfers in-plane forces to the lateral force resisting system.

Displacement: May be a translation, a rotation, or a combination of both.

Distributed load: An external force which acts over a length or an area.

Double curvature: Deformed shape of a beam with one or more inflection points within the span.

Double-concentrated forces: Two equal and opposite forces that form a couple on the same side of the loaded member.

Drift: Lateral deflection of structure due to lateral wind or seismic load.

Ductility: The capacity of a material to deform without breaking; it is measured as the ratio of total strain at failure, divided by the strain at the elastic limit.

Durability: Ability of a material, element or structure to perform its intended function for its required life without the need for replacement or significant repair, but subject to normal maintenance.

Dynamic equilibrium: Equilibrium of a moving object without change of motion.

Dynamic load: Cyclic load, such as gusty wind or seismic loads.

Effective length factor, $K$: Ratio between the effective length and the unbraced length of the member.

Effective length: Length of an otherwise identical column with the same strength when analyzed with pinned end conditions.

Effective net area: Net area modified to account for the effect of shear lag.

Effective section modulus: Section modulus reduced to account for buckling of slender compression elements.

Effective width: Reduced width of a plate or slab with an assumed uniform stress distribution which produces the same effect on the behavior of a structural member as the actual plate or slab with its nonuniform stress distribution.

Elastic: A material or structure is elastic if it returns to its original geometry upon unloading.

Elastic/plastic: Materials that have both an elastic zone and a plastic zone (i.e. steel).

Elastic limit: The point of a stress/strain graph beyond which deformation of a material is plastic, i.e. remains permanently deformed.

Elastic modulus: The linear slope value relating material stress to strain.

End-bearing pile: A pile supported on firm soil or rock.
Energy: The work to move a body a distance; energy is the product of forces times distance.
Epicenter: The point on the Earth’s surface above the hypocenter where an earthquake originates.
Equilibrium: An object is in equilibrium if the resultant of all forces acting on it has zero magnitude.
External force: A force acting on an object; external forces are also called applied forces.
Factored load: Product of a load factor and the nominal load.
Fatigue: Limit state of crack initiation and growth resulting from repeated application of live loads, usually by reversing the loading direction.
Fillet weld: Weld of generally triangular cross section made between intersecting surfaces of elements.
Fitted bearing stiffener: Stiffener used at a support or concentrated load that fits tightly against one or both flanges of a beam so as to transmit load through bearing.
Fixed connection: A connection that resists axial and shear forces and bending moments.
Flexure: Bending deformation (of increasing curvature).
Flexural buckling: Buckling mode in which a compression member deflects laterally without twist or change in cross-sectional shape.
Flexural-torsional buckling: Buckling mode in which a compression member bends and twists simultaneously without change in cross-sectional shape.
Force: Resultant of distribution of stress over a prescribed area, or an action that tends to change the shape of an object, move an object, or change the motion of an object.
Foundations: There are two basic types: ‘shallow,’ which includes pad footing, strip footings and rafts and ‘deep’ i.e. piles. The choice is a function of the strength and stiffness of the underlying strata and the load to be carried, the aim being to limit differential settlement on the structure and more importantly the finishes.
Fully restrained moment connection: Connection capable of transferring moment with negligible rotation between connected members.
Funicular: The shape of a chain or string suspended form two points under any load.
Gravity: An attractive force between objects; each object accelerates at the attractive force divided by its mass.
Groove weld: Weld in a groove between connection elements.
Gusset plate: Plate element connecting truss members of a strut or brace to a beam or column.
Hertz: Cycles per second.
Horizontal diaphragm: A floor or roof deck to resist lateral load.
Horizontal shear: Force at the interface between steel and concrete surfaces in a composite beam.
Indeterminate structure: A structure with more unknown reactions than static equations (3).
Inelastic: Inelastic (plastic) strain implies permanent deformation.
Inertia: Tendency of objects at rest to remain at rest and objects in motion to remain in motion.
In-plane instability: Limit state of a beam-column bent about its major axis while lateral buckling or lateral-torsional buckling is prevented by lateral bracing.

Instability: Limit state reached in the loading of a structural component, frame or structure in which a slight disturbance in the loads or geometry produces large displacements.

Internal force: The force within an object that resists external forces, also called resisting force.

Joint: Area where two or more ends, surfaces, or edges are attached. Categorized by type of fastener or weld used and method of force transfer.

Joist: A repetitive light beam.

K-connection: Connection in which forces in branch members or connecting elements transverse to the main member are primarily equilibrated by forces in other branch members or connecting elements on the same side of the main member.

Kern: The core of a post or other compression member which limits eccentric stresses being tensile.

Lacing: Plate, angle or other steel shape, in a lattice configuration, that connects two steel shapes together.

Lap joint: Joint between two overlapping connection elements in parallel planes.

Lateral bracing: Diagonal bracing, shear walls or equivalent means for providing in-plane lateral stability.

Lateral load resisting system: Structural system designed to resist lateral loads and provide stability for the structure as a whole.

Lateral load: Load, such as that produced by wind or earthquake effects, acting in a lateral direction.

Lateral-torsional buckling: Buckling mode of a flexural member involving deflection normal to the plane of bending occurring simultaneously with twist about the shear center of the cross-section.

Length effects: Consideration of the reduction in strength of a member based on its unbraced length.

Limit state: Condition in which a structure or component becomes unfit for service and is judged either to be no longer useful for its intended function (serviceability limit state) or to have reached its ultimate load-carrying capacity (strength limit state).

Linear: A structural or material behavior is linear if its deformation is directly proportional to the loading.

Line of action: The line of action defines the location and incline of a vector.

Linear elastic: A force-displacement relationship which is both linear and elastic.

Live load: Any load not permanently attached to the structure.

Load: Force or other action that results from the weight of building materials, occupants and their possessions, environmental effects, differential movement, or restrained dimensional changes.

Load effect: Forces, stresses and deformations produced in a structural component by the applied loads.
Load factor: Factor that accounts for deviations of the nominal load from the actual load, for uncertainties in the analysis that transforms the load into a load effect and for the probability that more than one extreme load will occur simultaneously.

Local bending: Limit state of large deformation of a flange under a concentrated tensile force.

Local buckling: Limit state of buckling of a compression element within a cross section.

Local crippling: Limit state of local failure of web plate in the immediate vicinity of a concentrated load or reaction.

Local yielding: Yielding that occurs in a local area of an element.

LRFD (Load and Resistance Factor Design): Method of proportioning structural components such that the design strength equals or exceeds the required strength of the component under the action of the LRFD load combinations.

LRFD load combination: Load combination in the applicable building code intended for strength design (load and resistance factor design).

Main member: Chord member or column to which branch members or other connecting elements are attached.

Mass: Mass is the property of an object to resist acceleration.

Magnitude: a scalar value of physical units, such as force or displacement.

Modulus of elasticity: The proportional constant relating stress/strain of material in the linear elastic range: calculated as stress divided by strain. The modulus of elasticity is the slope of the stress-strain graph, usually denoted as E, also as Young’s Modulus Y, or E-Modulus.

Moment: A force causing rotation without translation; defined as force times lever arm.

Moment of inertia: Moment of inertia is the capacity of an object to resist bending or buckling, defined as the sum of all parts of the object times the square of their distance from the centroid.

Moment connection: Connection that transmits bending moment between connected members.

Moment frame: Framing system that provides resistance to lateral loads and provides stability to the structural system, primarily by shear and flexure of the framing members and their connections.

Net area: Gross area reduced to account for removed material.

Nominal dimension: Designated or theoretical dimension, as in the tables of section properties.

Nominal load: Magnitude of the load specified by the applicable building code.

Nominal strength: Strength of a structure or component (without the resistance factor or safety factor applied) to resist load effects, as determined in accordance with this Specification.

Normal stress: Stress acting parallel to the axis of an object in compression and tension; normal stress is usually simply called stress.

Out-of-plane buckling: Limit state of a beam-column bent about its major axis while lateral buckling or lateral-torsional buckling is not prevented by lateral bracing.

Overlap connection: Connection in which intersecting branch members overlap.

Overturn: Topping, or tipping over under lateral load.
Permanent load: Load in which variations over time are rare or of small magnitude. All other loads are variable loads.

Pin connection: A pin connection transfers axial and shear forces but no bending moment.

Pin support: A pin support resists axial and shear forces but no bending moment.

Pitch: Longitudinal center-to-center spacing of fasteners. Center-to-center spacing bolt threads along axis of bolt.

Plastic: Material may be elastic or plastic. Plastic deformation of a structure or material under load remains after the load is removed.

Plastic analysis: Structural analysis based on the assumption of rigid-plastic behavior, in other words, that equilibrium is satisfied throughout the structure and the stress is at or below the yield stress.

Plastic hinge: Yielded zone that forms in a structural member when the plastic moment is attained. The member is assumed to rotate further as if hinged, except that such rotation is restrained by the plastic moment.

Plastic moment: Theoretical resisting moment developed within a fully yielded cross section.

Plastic stress distribution method: Method for determining the stresses in a composite member assuming that the steel section and the concrete in the cross section are fully plastic.

Plate girder: Built-up beam.

Plug weld: Weld made in a circular hole in one element of a joint fusing that element to another element.

Post-buckling strength: Load or force that can be carried by an element, member, or frame after initial buckling has occurred.

Pressure: Similar to stress, the force intensity at a point, except that pressure is acting on the surface of an object rather than within the object.

Prying action: Amplification of the tension force in a bolt caused by leverage between the point of applied load, the bolt and the reaction of the connected elements.

Punching load: Component of branch member force perpendicular to a chord.

P-δ effect: Effect of loads acting on the deflected shape of a member between joints or nodes.

P-Δ effect: Effect of loads acting on the displaced location of joints or nodes in a structure. In tiered building structures, this is the effect of loads acting on the laterally displaced location of floors and roofs.

Radius of gyration: A mathematical property, determining the stability of a cross section, defined as: \( r = \sqrt{\frac{I}{A}} \), where \( I \) = moment of inertia and \( A \) = cross section area.

Reaction: The response of a structure to resist applied load.

Required strength: Forces, stresses and deformations acting on the structural component, determined by either structural analysis, for the LRFD or ASD load combinations, as appropriate, or as specified by the Specification or Standard.

Resilience: The property of structures to absorb energy without permanent deformation of fracture.
**Resistance factor $\phi$:** Factor that accounts for unavoidable deviations of the *nominal strength* from the actual strength and for the manner and consequences of failure.

**Resultant:** The resultant of a system of forces is a single force or moment whose magnitude, direction, and location make it statically equivalent to the system of forces.

**Retaining wall:** Wall used to hold back soil or other materials.

**Roller support:** In two dimensions, a roller support restrains one translation degree of freedom.

**Rupture strength:** In a connection, strength limited by tension or shear rupture.

**Safety factor:** Factor that accounts for deviations of the actual strength from the nominal strength, deviations of the actual *load* from the *nominal load*, uncertainties in the analysis that transforms the load into a *load effect*, and for the manner and consequence of failure.

**Scalar:** A mathematical entity with a numeric value but no direction (in contrast to a vector).

**Section modulus:** The property of a cross section defined by its shape and orientation; section modulus is denoted $S$, and $S = \frac{I}{c}$, where $I$ = moment of inertia about the centroid and $c$ is the distance from the centroid to the edge of the section.

**Service load combination:** Load combination under which serviceability limit states are evaluated.

**Service load:** Load under which *serviceability limit states* are evaluated.

**Serviceability limit state:** Limiting condition affecting the ability of a structure to preserve its appearance, maintainability, durability or the comfort of its occupants or function of machinery, under normal usage.

**Shear:** A sliding force, pushing and pulling in opposite directions.

**Shear buckling:** Buckling mode in which a plate element, such as the web of a beam, deforms under pure shear applied in the plane of the plate.

**Shear connector:** Headed stud, cannell, plate or other shape welded to a steel member and embedded in concrete of a composite member to transmit shear forces at the interface between the two materials.

**Shear connector strength:** Limit state of reaching the strength of a shear connector, as governed by the connector bearing against the concrete in the slab or by the *tensile strength* of the connector.

**Shear modulus:** The ratio of shear stress divided by the shear strain in a linear elastic material.

**Shear rupture:** Limit state of rupture (fracture) due to shear.

**Shear strain:** Strain measuring the intensity of racking in a material. Shear strain is measured as the change in angle of a small square part of a material.

**Shear stress:** Stress acting parallel to an imaginary plane cut through an object.

**Shear wall:** Wall that provides resistance to lateral loads in the plane of the wall and provides stability for the structural system.

**Shear yielding:** Yielding that occurs due to shear.

**Shear yielding (punching):** In a connection, *limit state* based on out-of-plane shear strength of the chord wall to which branch members are attached.
Slip: In a bolted connection, *limit state* of relative motion of connected parts prior to the attainment of the *available strength* of the connection.

*Slip-critical connection*: Bolted connection designed to resist movement by friction on the faying surface of the connection under the clamping forces of the bolts.

*Slot weld*: Weld made in an elongated hole fusing an element to another element.

*Splice: Connection* between two structural elements joined at their ends to form a single, longer element.

*Stability*: Condition reached in the loading of a structural component, frame or structure in which a slight disturbance in the *loads* or geometry does not produce large displacements.

*Static equilibrium*: Equilibrium of an object at rest.

*Stiffness*: The capacity of a material to resist deformation.

*Stiffened element*: Flat compression element with adjoining out-of-plane elements along both edges parallel to the direction of loading.

*Stiffener*: Structural element, usually an angle or plate, attached to a *member* to distribute *load*, transfer shear or prevent buckling.

*Stiffness*: Resistance to deformation of a member or structure, measured by the ratio of the applied force (or moment) to the corresponding displacement (or rotation).

*Strain*: Change of length along an axis, calculated as $\epsilon = \Delta L / L$, where $L$ is the original length and $\Delta L$ is the change of length.

*Strength*: The capacity of a material to resist breaking.

*Strength design*: A design method based on factored load and ultimate strength for concrete design.

*Strength limit state*: Limiting condition affecting the safety of the structure, in which the ultimate load-carrying capacity is reached.

*Stress*: Force per unit area caused by axial force, moment, shear or torsion.

*Stress concentration*: Localized stress considerably higher than average (even in uniformly loaded cross sections of uniform thickness) due to abrupt changes in geometry or localized loading.

*Stress resultant*: A system of forces which is statically equivalent to a stress distribution over an area.

*Stress*: The internal reaction to an applied force, measured in force per unit area.

*Structure*: Composition of elements that define form and resist applied loads.

*Structural Aluminum*: Elements manufactured of aluminum for structural purposes, generally 50% larger than comparable steel elements due to the lower *modulus of elasticity*.

*Structural Steel*: Elements manufactured of steel with properties designated by *ASTM standards*, including A36, A992 & A572.

*Strong axis*: Major principal centroidal axis of a cross section.

*Structural analysis*: Determination of *load effects* on members and *connections* based on principles of structural mechanics.
Structural component: Member, connector, connecting element or assemblage.

Structural system: An assemblage of load-carrying components that are joined together to provide interaction or interdependence.

T-connection: Connection in which the branch member or connecting element is perpendicular to the main member and in which forces transverse to the main member are primarily equilibrated by shear in the main member.

Tensile rupture: Limit state of rupture (fracture) due to tension.

Tensile strength (of material): Maximum tensile stress that a material is capable of sustaining as defined by ASTM.

Tensile strength (of member): Maximum tension force that a member is capable of sustaining.

Tensile yielding: Yielding that occurs due to tension.

Tension: A force that tends to elongate or enlarge an object.

Tension and shear rupture: In a bolt, limit state of rupture (fracture) due to simultaneous tension and shear force.

Tie plate: Plate element used to join parallel components of a built-up column, girder or strut rigidly connected to the parallel components and designed to transmit shear between them.

Torsion: A twisting moment.

Torsional bracing: Bracing resisting twist of a beam or column.

Torsional buckling: Buckling mode in which a compression member twists about its shear center axis.

Torsional yielding: Yielding that occurs due to torsion.

Translation: Motion of an object along a straight line path without rotation.

Transverse reinforcement: Steel reinforcement in the form of closed ties or welded wire fabric providing confinement for the concrete surrounding the steel shape core in an encased concrete composite column.

Transverse stiffener: Web stiffener oriented perpendicular to the flanges, attached to the web.

Truss: A linear support system consisting of triangular panels usually with pin joints.

Ultimate strength: The utmost strength reached by a material before breaking.

Unbraced length: Distance between braced points of a member, measured between the centers of gravity of the bracing members.

Uneven load distribution: In a connection, condition in which the load is not distributed through the cross section of connected elements in a manner that can be readily determined.

Unframed end: The end of a member not restrained against rotation by stiffeners of connection elements.

Unstiffened elements: Flat compression element with an adjoining out-of-plane element along one edge parallel to the direction of loading.

Uplift: Upward force, usually wind uplift.

Variable load: Load not classified as permanent load.
Vector: A mathematical entity having a magnitude, line of action, and a direction in space.

Vertical bracing system: System of shear walls, braced frames or both, extending through one or more floors of a building.

Vertical diaphragm: A wall to resist lateral load.

Vibration: The cyclic motion of an object.

Wall: A vertical element to resist load and define space; shear walls also resist lateral loads.

Weak axis: Minor principal centroidal axis of a cross section.

Web buckling: Limit state of lateral instability of a web.

Web compression buckling: Limit state of out-of-plane compression buckling of the web due to a concentrated compression force.

Web sideways buckling: Limit state of lateral buckling of the tension flange opposite the location of a concentrated compression force.

Weld metal: Portion of a fusion weld that has been completely melted during welding. Weld metal has elements of filler metal and base metal melted in the weld thermal cycle.

Working stress: The same as allowable stress.

Yield moment: In a member subjected to bending, the moment at which the extreme outer fiber first attains the yield stress.

Yield point: First stress in a material at which an increase in strain occurs without an increase in stress as defined by ASTM.

Yield strength: Stress at which a material exhibits a specified limiting deviation from the proportionality of stress to strain as defined by ASTM.

Yield strain: The strain of a material which occurs at the level of yield stress.

Yield stress: Generic term to denote either yield point or yield strength, as appropriate for the material.

Yielding: Limit state of inelastic deformation that occurs after the yield stress is reached.

Yielding (plastic moment): Yielding throughout the cross section of a member as the bending moment reaches the plastic moment.

Yielding (yield moment): Yielding at the extreme fiber on the cross section of a member when the bending moment reached the yield moment.

References:
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Compiled by Charles S. Tritt, Ph.D.
Last revised 11/5/02
http://people.msoe.edu/~tritt/greek.html
The graph below represents a conversion chart that can be used to convert gallons to liters or liters to gallons. We know that 1 gallon = 3.8 liters, and 10 gallons = 38 liters. The line was made by connecting the two points (1, 3.8) and (10, 38).

Notice that on the vertical scale for liters each line represents 1 liter, whereas on the horizontal scale for gallons each line represents \( \frac{1}{2} \), or 0.2, gallon. These scales were used to make the graph easier to read.

Use the graph to find the following conversions:

1. 11 gallons = ____ liters  
2. 6 gallons = ____ liters  
3. 2 gallons = ____ liters  
4. 5 gallons = ____ liters  
5. 19 liters = ____ gallons  
6. 26 liters = ____ gallons  
7. 30 liters = ____ gallons  
8. 42 liters = ____ gallons  

(continued)
To find the area of the shape in Figure 4.26, cover part of it with squares and half squares. The area of the rest can be figured out by completing the rectangle around the remaining triangle. The problem is worked out step by step.

**Step 1:** Fit whole squares (marked by Xs) into the shape. There are 4 whole squares.

**Step 2:** Place half squares in the shape. There are 2 half squares.

**Step 3:** The remaining area can be measured by completing the rectangle. The whole rectangle measures 4 square units; therefore, the triangle measures 2 square units.

**Step 4:** The shape has a total area of 7 square units: 4 whole squares, 2 half squares, and a triangle that measures 2 square units.
Find the areas of the following shapes. Work them out on a geoboard, and record the areas under each shape.
You will find the methods we’ve been using to find area don’t work for the triangle in Figure 4.31. Whole squares and half squares don’t fit. There are no right angles to help in making rectangles. We need to learn one more method—called the subtractive method—to help us measure this area.

**Step 1:** Make a rectangle or square around the triangle. The square measures 4 square units. If we can find the area of the shaded parts and subtract that from 4 (the area of the square), we will have the area of the triangle.

**Step 2:** One half square fits into the bottom corner. That leaves two right triangles.

**Step 3:** Make rectangles around the right triangles. These rectangles each have an area of 2 square units; therefore, these triangles each have an area of 1 square unit.

**Step 4:** We have two right triangles, each with an area of 1 square unit. We also know that there is a half square unit in the corner. This makes a total area of 2½ square units that are inside the square but outside the original triangle. Subtract this 2½ from 4. The area of the original triangle is 1½ square units.
Worksheet 5.2

Draw pictures to help you solve these problems.

1. If \( \frac{3}{6} \) bag of potting soil weighs 10 pounds, how much does the whole bag weigh?

2. After 18 walnuts were used to bake some brownies, \( \frac{3}{4} \) package was left. About how many walnuts does the package hold?

3. The pickle barrel was \( \frac{3}{5} \) full. After 23 pickles were sold, it was \( \frac{1}{3} \) full. About how many pickles does the barrel hold?

4. If 1\( \frac{1}{2} \) pounds of nails cost $1.74, how much does 1 pound cost?

5. Alice bought a pair of cross-country skis for $90 at a \( \frac{1}{4} \)-off sale. What was the original price?
Worksheet 5.3

Solve these problems, drawing a picture for each one. Use a calculator if you wish.

1. A board 21 feet long is cut into two pieces. One piece is 5 feet longer than the other. How long is each piece?

2. A certain number is 6 more than another number. Their sum is 42. Find the numbers.

3. A certain number is 6 times as big as another number. Their sum is 42. Find the numbers.

4. A notebook costs $1.50 more than a pencil. Together they cost $2.10. How much does the pencil cost?
Worksheet 5.4

Solve these problems. Use a calculator if you wish.

1. Two numbers differ by 5. (This means that one number is 5 more than the other.) Their sum is 55.
   a. Draw and label a picture to represent this information.
   
   b. Find the two numbers.

2. The sum of two numbers is 55. The smaller number divides the larger evenly, with an answer (quotient) of 10. (In other words, the larger number is 10 times as big as the smaller.)
   a. Draw and label a picture to represent this information.
   
   b. Find the two numbers.

3. The Lopez family lives on a triangular plot of land with a perimeter of 180 feet. The first side of their lot is 10 feet shorter than the second side. The third side is 10 feet longer than the second side.
   a. Which side is the shortest?
   
   b. How much longer is the third side than the first?
   
   c. Draw a picture to represent the information in this problem.
   
   d. Find the length of each side of the lot.

4. Two times a number is 4 less than 20.
   a. If we use a box to represent the number in this problem, why do we need to add four circles to the picture to make the total equal to 20?

   b. What is the number?

(continued)
Solve these problems.

1. Sue's age is 5 years less than Ann's age.
   a. If Sue is 35, how old is Ann?
   b. If Sue is 15, how old is Ann?
   c. If Sue is 42, how old is Ann?
   d. If Ann is 36, how old is Sue?

2. Paul's income is $2,500 less than Mary's income.
   a. If Paul makes $8,000, how much does Mary earn?
   b. If Paul makes $20,000, how much does Mary make?
   c. If Paul makes $13,500, what is Mary's salary?
   d. If Mary makes $25,000, what does Paul earn?

3. Diane's income is $2,000 less than Ted's.
   a. If Diane earns $17,000, how much does Ted earn?
   b. If Diane earns $32,000, how much is Ted's income?
   c. If Ted's income is $13,000, how much is Diane's?

4. Bill's income today is $3,000 more than it was two years ago.
   a. If he makes $27,000 now, what did he make then?
   b. If he makes $19,000 now, what did he make then?
   c. If he made $15,000 two years ago, what does he make now?
   d. If he made $32,000 two years ago, what does he make now?

5. Angela's age is three times more than Robert's age.
   a. If Robert is 17, how old is Angela?
   b. If Robert is 25, how old is Angela?
   c. If Angela is 24, how old is Robert?
   d. If Angela is 48, how old is Robert?

6. Peter's salary is exactly half of David's.
   a. If Peter earns $800, how much does David earn?
   b. If Peter earns $1,200, how much does David earn?
   c. If David earns $14,000, how much does Peter earn?
   d. If David earns $20,000, how much does Peter earn?
Math & Physics in Architectural Structures

1. Parallel lines never intersect.

2. Two lines are perpendicular (or normal) when they intersect at a right angle = 90°.

3. Intersecting (or concurrent) lines cross or meet at a point.

4. If two lines cross, the opposite angles are identical:

5. If a line crosses two parallel lines, the intersection angles with the same orientation are identical:

6. If the sides of two angles are parallel and intersect in the same fashion, the angles are identical.

7. If the sides of two angles are parallel, but intersect in the opposite fashion, the angles are supplementary: \( \alpha + \beta = 180^\circ \).

8. If the sides of two angles are perpendicular and intersect in the same fashion, the angles are identical.
9. If the sides of two angles are perpendicular, but intersect in the opposite fashion, the angles are \textit{supplementary}: \(\alpha + \beta = 180^\circ\).

10. If the side of two angles bisects a right angle, the angles are \textit{complimentary}: \(\alpha + \gamma = 90^\circ\).

11. If a right angle bisects a straight line, the remaining angles are \textit{complimentary}: \(\alpha + \gamma = 90^\circ\).

12. The sum of the interior angles of a triangle = 180°.

13. For a right triangle, that has one angle of 90°, the sum of the other angles = 90°. These angles are \textit{complimentary}.

14. For a right triangle, the sum of the squares of the sides equals the square of the hypotenuse:

\[ AB^2 + AC^2 = CB^2 \]

15. Similar triangles have identical angles in the same orientation. Their sides are related by:

\begin{align*}
\text{Case 1:} & \quad \frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE} \\
\text{Case 2:} & \quad \frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}
\end{align*}
16. For right triangles:

\[
\begin{align*}
\sin \alpha &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AB}{CB} \\
\cos \alpha &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AC}{CB} \\
\tan \alpha &= \frac{\text{opposite}}{\text{adjacent}} = \frac{AB}{AC}
\end{align*}
\]

(SOHCAHTOA)

17. If an angle is greater than 180° and less than 360°, \( \sin \) will be less than 0.
   If an angle is greater than 90° and less than 270°, \( \cos \) will be less than 0.
   If an angle is greater than 90° and less than 180°, \( \tan \) will be less than 0.
   If an angle is greater than 270° and less than 360°, \( \tan \) will be less than 0.

18. LAW of SINES (any triangle)

\[
\frac{\sin \alpha}{A} = \frac{\sin \beta}{B} = \frac{\sin \gamma}{C}
\]

19. LAW of COSINES (any triangle)

\[
A^2 = B^2 + C^2 - 2BC \cos \alpha
\]

20. Surfaces or areas have dimensions of width and length and units of length squared (ex. in² or inches x inches).

21. Solids or volumes have dimension of width, length and height or thickness and units of length cubed (ex. m³ or m x m x m)

22. Force is defined as mass times acceleration. So a weight due to a mass is accelerated upon by gravity:

\[
F = mg \quad g = 9.81 \frac{m}{\text{sec}^2} = 32.17 \frac{ft}{\text{sec}^2}
\]

23. Weight can be determined by volume if the unit weight or \textit{density} is known: 

\[
W = V \cdot \gamma
\]

where \( V \) is in units of length³ and \( \gamma \) is in units of force/unit volume

24. Algebra: If \( a \cdot b = c \cdot d \) then it can be rewritten:

\[
\begin{align*}
a \cdot b + k &= c \cdot d + k & \text{add a constant} \\
c \cdot d &= a \cdot b & \text{switch sides} \\
a &= \frac{c \cdot d}{b} & \text{divide both sides by} \ b \\
a \cdot \frac{d}{c} &= \frac{d}{b} & \text{divide both sides by} \ b \cdot c
\end{align*}
\]
25. Cartesian Coordinate System

![Diagram of Cartesian Coordinate System]

26. Solving equations with one unknown:

1\textsuperscript{st} order polynomial:
\[ 2x - 1 = 0 \quad \cdots \quad 2x = 1 \quad \cdots \quad x = \frac{1}{2} \]
\[ ax + b = 0 \quad \cdots \quad x = -\frac{b}{a} \]

2\textsuperscript{nd} order polynomial:
\[ ax^2 + bx + c = 0 \quad \cdots \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{(two answers)} \]
\[ x^2 - 1 = 0 \quad \cdots \quad x = \frac{-0 \pm \sqrt{0^2 - 4(-1)}}{2 \times 1} \quad x = \pm 1 \]

27. Solving 2 linear equations simultaneously: \textit{also see item #38}

One equation consisting only of variables can be rearranged and then substituted into the second equation:

\textbf{ex:} \quad 5x - 3y = 0 \quad \text{add 3y to both sides to rearrange} \quad 5x = 3y \quad 5x = 3y
\quad 4x - y = 2 \quad \text{divide both sides by 5} \quad x = \frac{3}{5} y \quad x = \frac{3}{5} y
\quad \text{substitute x into the other equation} \quad 4\left(\frac{3}{5} y\right) - y = 2 \quad 4\left(\frac{3}{5} y\right) - y = 2
\quad \text{add like terms} \quad \frac{7}{5} y = 2 \quad \frac{7}{5} y = 2
\quad \text{simplify} \quad y = \frac{10}{7} = 1.43 \quad y = \frac{10}{7} = 1.43

Equations can be added and factored to eliminate one variable:

\textbf{ex:} \quad 2x + 3y = 8 \quad 2x + 3y = 8
\quad 4x - y = 2 \quad 12x - 3y = 6 \quad 12x - 3y = 6
\quad \text{and add} \quad 14x + 0 = 14 \quad 14x + 0 = 14
\quad \text{simplify} \quad |x = 1| \quad |x = 1|
\quad \text{put x=1 in an equation for y} \quad 2.1 + 3y = 8 \quad 2.1 + 3y = 8
\quad \text{simplify} \quad 3y = 6 \quad 3y = 6
\quad |y = 2| \quad |y = 2|
28. Derivatives of polynomials

\[ \begin{align*}
y = \text{constant} & \quad \frac{dy}{dx} = 0 \\
y = x & \quad \frac{dy}{dx} = 1 \\
y = ax & \quad \frac{dy}{dx} = a \\
y = x^2 & \quad \frac{dy}{dx} = 2x \\
y = x^3 & \quad \frac{dy}{dx} = 3x^2 
\end{align*} \]

29. The minimum and maximum of a function can be found by setting the derivative = 0 and solving for the unknown variable.

30. Calculators (and software) process equations by an “order of operations”, which typically means they process functions like exponentials and square roots before simpler functions such as + or -. BE SURE to specify with parenthesis what order you want, or you’ll get the wrong answers. It is also important to have degrees set in your calculator for trig functions.

For instance, Excel uses – for sign (like -1) first, then will process exponents and square roots, times and divide, followed by plus and minus. If you type 4x10^2 and really mean (4x10)^2 you will get an answer of 400 instead of 1600.

31. Measures in physics include length, force, mass, and temperature. US Customary units include inches, feet, lb (force), kips (1000 lb), and Fahrenheit. SI (Standards International) units include millimeters, centimeters, meters, grams, kilograms, Newtons, kилоNewtons, and Celcius.

32. Vectors have direction and magnitude.

33. Particles want to move when pushed. They can translate, rotate, speed, and accelerate.

34. Gravity accounts for masses being attracted to the ground and force due to gravity on a mass.

35. Equilibrium is the state of forces in balance, such that there is no movement.

36. Fluids (liquids and gasses) can cause pressure and resulting forces over an area.

37. Structural materials can change length or volume with a change in temperature.

38. Using a TI-83 to solve a system of linear equations in a matrix form with \texttt{rref}:

Matrices of linear equations expect the coefficients in front of variable to be put in the same order in each row, and the numerical solution (= to) as the last value. So for the 2\textsuperscript{nd} set of equations in item #27 (2x + 3y = 8 and 4x − y = 2), the matrix to enter would look like

\[
\begin{bmatrix}
2 & 3 & 8 \\
4 & -1 & 2 \\
\end{bmatrix}
\]
1. Press \texttt{2nd [MATRIX]. Press \texttt{[ to display the MATRIX EDIT menu. Press 1 to select \texttt{1:[A]},} \begin{center}
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\textbf{MATRIX[A]} & 2 x3 \\
\hline
\texttt{[0 0 0 ]} & \texttt{[0 0 0 ]} \\
\hline
\end{tabular}
\end{center}

2. Press \texttt{2 ENTER 3 ENTER} to define a 2 x 3 matrix. The rectangular cursor indicates the current element. Ellipses (\ldots) indicate additional columns beyond the screen.

3. Press \texttt{2 ENTER} to enter the first element. The rectangular cursor moves to the second column of the first row.

4. Press \texttt{3 ENTER 8 ENTER} to complete the first row for \(2x + 3y = 8\)

5. Press \texttt{4 ENTER -1 ENTER 2 ENTER} to enter the second row for \(4x - y = 2\)

6. Press \texttt{2nd [QUIT]} to return to the home screen. If necessary, press \texttt{CLEAR} to clear the home screen. Press \texttt{2nd [MATRIX] \texttt{[ to display the MATRIX MATH menu. Press \texttt{[ to wrap to the end of the menu. Select \texttt{B:rref(} to copy \texttt{rref(} to the home screen. \begin{center}
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\textbf{MATRIX[A]} & 2 x3 \\
\hline
\texttt{[2 0 0 ]} & \texttt{[0 0 0 ]} \\
\hline
\end{tabular}
\end{center}

7. Press \texttt{2nd [MATRIX] 1} to select \texttt{1:[A]} from the MATRIX NAMES menu. Press \texttt{1 ENTER}. The reduced row-echelon form of the matrix is displayed and stored in \texttt{Ans}. \begin{center}
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\textbf{rref(}\texttt{[[1 0 1 \]}} & \texttt{[0 1 2]]} \\
\hline
\end{tabular}
\end{center}

\begin{align*} 
1x + 0y &= 1 \quad \text{therefore} \quad x = 1 \\
0x + 1y &= 2 \quad \text{therefore} \quad y = 2 
\end{align*}
Numerical Computations

Accuracy

The accuracy of a numerical value is often expressed in terms of the number of significant digits that the value contains. What are significant digits? Any nonzero digit is considered significant; zeroes that appear to the left or right of a digit sequence are used to locate the decimal point and are not considered significant. Thus the numbers 0.00345, 3.45, 3450, and 3,450,000 all contain three significant digits represented by the sequence 3–4–5. Zeros bounded on both sides by nonzero digits are also significant; 0.005067, 5.067, 50.67, and 506,700 each contain four significant digits, as represented by the numerical sequence 5–0–6–7.

The accuracy of a solution can be no greater than the accuracy of the data on which the solution is based. For example, the length of one side of a right triangle may be given as 20 ft. Without knowing the possible error in the length measurement, it is impossible to determine the error in the answer obtained from it. We will usually assume that the data are known with an accuracy of 0.2 percent. The possible error in the 20-ft length would therefore be 0.04 ft.

To maintain an accuracy of approximately 0.2 percent in our calculations, we will use the following practical rule: use four digits to record numbers beginning with 1 and three digits to record numbers beginning with 2 through 9. Thus a length of 19 ft becomes 19.00 ft, a length of 20 ft becomes 20.0 ft, and a length of 43 ft becomes 43.0 ft.

You will notice one exception to this rule throughout the text: values of the trigonometric functions are traditionally written to four decimal places, and that practice will be followed here, not for increased accuracy, but to clarify the computations used in worked examples.

Rounding Off Numbers*

If the data are given with greater accuracy than we wish to maintain (see Fig. 1.1), the following rules may be used to round off their values:

1. When the digit dropped is greater than 5, increase the digit to the left by 1. Example: 23.56 ft becomes 23.6 ft.
2. When the digit dropped is less than 5, drop it without changing the digit to the left. Example: 23.34 ft becomes 23.3 ft.
3. When the digit dropped is 5 followed only by zeros, increase the digit to the left by 1 only if it becomes even. If the digit to the left becomes odd, drop the 5 without changing the digit to the left. Example: 23.5500 ft rounded to three numbers becomes 23.6 ft, and 23.4500 ft becomes 23.4 ft. (This practice is often referred to as the round-even rule.)

*American Society of Mechanical Engineers (ASME) Orientation and Guide for Use of SI (Metric) Units, 9th edition, 1982, p 11. By increasing the digit to the left for a final 5 followed by zeros only if the digit becomes even, we are dividing the rounding process evenly between increasing the digit to the left and leaving the digit to the left unchanged.
Calculators

Electronic calculators and computers are widely available for use in engineering. Their speed and accuracy make it possible to do difficult numerical computations in a routine manner. However, because of the large number of digits appearing in solutions, their accuracy is often misleading. As pointed out previously, the accuracy of the solution can be no greater than the accuracy of the data on which the solution is based. Care should be taken to retain sufficient digits in the intermediate steps of the calculations to ensure the required accuracy of the final answer. Answers with more significant digits than are reasonable should not be recorded as the final answer. An accuracy greater than 0.2 percent is rarely justified.
Problem Solving, Units and Numerical Accuracy

Problem Solution Method:

1. Inputs
   Outputs
   “Critical Path”

### GIVEN:

- on graph paper

### FIND:

### SOLUTION

2. Draw simple diagram of body/bodies & forces acting on it/them.

3. Choose a reference system for the forces.

4. Identify key geometry and constraints.

5. Write the basic equations for force components.

6. Count the equations & unknowns.

7. **SOLVE**

8. “Feel” the validity of the answer. (Use common sense. Check units…)

---

Example: Two forces, A & B, act on a particle. What is the resultant?

1. **GIVEN:** Two forces on a particle and a diagram with size and orientation

2. **FIND:** The “resultant” of the two forces

3. **SOLUTION:**

   2. Draw what you know (the diagram, any other numbers in the problem statement that could be put on the drawing….)

   3. Choose a reference system. What would be the easiest? Cartesian, radian?

   4. Key geometry: the location of the particle as the origin of all the forces
      Key constraints: the particle is “free” in space

   5. Write equations:

      \[
      \text{sizeof } A^2 + \text{sizeof } B^2 = \text{sizeof resultant} \\
      \sin \alpha = \frac{\text{sizeof } B}{\text{sizeof } A + B}
      \]

   6. Count: Ununknowns: 2, magnitude and direction ≤ Equations: 2 \therefore can solve

   7. Solve: graphically or with equations

   8. “Feel”: Is the result bigger than A and bigger than B? Is it in the right direction? (like A & B)
Units

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<td>kg</td>
<td>m</td>
<td>s</td>
<td>$N = \frac{kg \cdot m}{s^2}$</td>
</tr>
<tr>
<td>Absolute</td>
<td>lb</td>
<td>ft</td>
<td>s</td>
<td>$Poundal = \frac{lb \cdot ft}{s^2}$</td>
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</table>

$lb_{force} = lb_{(mass)} \times 32.17 \frac{ft}{s^2}$

Gravitational constant

$\text{English}$

$g_e = 32.17 \frac{ft}{s^2}$

$\text{SI}$

$g_e = 9.81 \frac{m}{s^2}$

Conversions

(English)

$1 \text{ in} = 25.4 \text{ mm}$

$1 \text{ lb} = 4.448 \text{ N}$

Numerical Accuracy

Depends on

1) accuracy of data you are given
2) accuracy of the calculations performed

*The solution CANNOT be more accurate than the less accurate of #1 and #2 above!*

**Definitions:**

- Precision: the number of significant digits
- Accuracy: the possible error

Relative error measures the degree of accuracy:

$$\frac{\text{relative error}}{\text{measurement}} \times 100 = \text{degree of accuracy} (%)$$

For engineering problems, accuracy *rarely* is less than 0.2%. 
# Forces and Vectors

## Notation:

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$F$</td>
<td>name for force vectors, as is $A$, $B$, $C$, $T$ and $P$</td>
</tr>
<tr>
<td>$F_x$</td>
<td>force component in the x direction</td>
</tr>
<tr>
<td>$F_y$</td>
<td>force component in the y direction</td>
</tr>
<tr>
<td>$h$</td>
<td>cable sag height</td>
</tr>
<tr>
<td>$L$</td>
<td>span length</td>
</tr>
<tr>
<td>$R$</td>
<td>name for resultant vectors</td>
</tr>
<tr>
<td>$R_x$</td>
<td>resultant component in the x direction</td>
</tr>
<tr>
<td>$R_y$</td>
<td>resultant component in the y direction</td>
</tr>
<tr>
<td>tail</td>
<td>start of a vector (without arrowhead)</td>
</tr>
<tr>
<td>tip</td>
<td>direction end of a vector (with arrowhead)</td>
</tr>
<tr>
<td>$T$</td>
<td>name for a tension force</td>
</tr>
<tr>
<td>$x$</td>
<td>x axis direction</td>
</tr>
<tr>
<td>$y$</td>
<td>y axis direction</td>
</tr>
<tr>
<td>$W$</td>
<td>name for force due to weight</td>
</tr>
<tr>
<td>$\phi$</td>
<td>angle</td>
</tr>
<tr>
<td>$\theta$</td>
<td>angle, in a trig equation, ex. $\sin \theta$, that is measured between the x axis and tail of a vector</td>
</tr>
</tbody>
</table>

## Force Characteristics

- Forces have *a point of application* -
  - *size* – units of lb, K, N, kN
  - *direction* – to a reference system,
  - *sense* - indicated by an arrow, or by sign convention (+/-)
- Classifications include: *Static & Dynamic*
- Structural types separated primarily into *Dead Load* and *Live Load* with further identification as wind, earthquake (seismic), impact, etc.

## Rigid Body

- *Ideal* material that doesn’t deform
- Forces on rigid bodies can be *internal* - within or at connections
  - or *external* - applied
- Rigid bodies can *translate* (move in a straight line)
  - or *rotate* (change angle)

- Weight of truck is external (gravity)
- Push by driver is external
- Reaction of the ground on wheels is external

If the truck moves forward: *it translates*

If the truck gets put up on a jack: *it rotates*
- **Transmissibility:** We can replace a force at a point on a body by that force on another point on the body **along the line of action of the force.**

  ![Diagram of transmissibility](image)

  External conditions haven’t changed

  For the truck:

  ![Diagram of truck](image)

  - The same external forces will result in the same conditions for motion
  - Transmissibility applies to EXTERNAL forces. INTERNAL forces respond differently when an external force is moved.
  - **DEFINITION:** 2D Structure - A structure that is flat and may contain a plane of symmetry. All forces on this structure are in the same plane as the structure.

**Internal and External Forces**

- **Internal forces** occur within a member or between bodies within a system
- **External forces** represent the action of other bodies or gravity on the rigid body

**Force System Types**

- **Collinear** – all forces along the same **line**
- **Coplanar** – all forces in the same **plane**
• Space – not all concurrent or coplanar (all out there in 3 dimensions)

Further classification as

• Concurrent – all forces go through the same point

• Parallel – all forces are parallel

Static Equilibrium

Equilibrium exists when the force system on a body or object produces no rotation or translation.

Graphical Addition of Forces and Resultants

• Parallelogram law: when adding two vectors acting at a point, the result is the diagonal of the parallelogram

• The tip-to-tail method is another graphical way to add vectors.

• With 3 (three) or more vectors, successive application of the parallelogram law will find the resultant OR drawing all the vectors tip-to-tail in any order will find the resultant.

Rectangular Force Components and Addition

• It is convenient to resolve forces into perpendicular components (at 90°).

• Parallelogram law results in a rectangle.

• Triangle rule results in a right triangle.
\[ \theta \text{ is: } \text{between } x \text{ & } F \]
\[ F_x = F \cdot \cos \theta \]
\[ F_y = F \cdot \sin \theta \]
\[ F = \sqrt{F_x^2 + F_y^2} \]
\[ \tan \theta = \frac{F_y}{F_x} \]

When \( 90^\circ < \theta < 270^\circ \), \( F_x \) is negative
When \( 180^\circ < \theta < 360^\circ \), \( F_y \) is negative
When \( 0^\circ < \theta < 90^\circ \) and \( 180^\circ < \theta < 270^\circ \), \( \tan \theta \) is positive
When \( 90^\circ < \theta < 180^\circ \) and \( 270^\circ < \theta < 360^\circ \), \( \tan \theta \) is negative

- Addition (analytically) can be done by adding all the \( x \) components for a resultant \( x \) component and adding all the \( y \) components for a resultant \( y \) component.

\[ R_x = \sum F_x, \quad R_y = \sum F_y \quad \text{and} \quad R = \sqrt{R_x^2 + R_y^2} \quad \tan \theta = \frac{R_y}{R_x} \]

**CAUTION:** An interior angle, \( \phi \), between a vector and either coordinate axis can be used in the trig functions. BUT _No sign_ will be provided by the trig function, which means you must give a sign and determine if the component is in the \( x \) or \( y \) direction.

*For example, \( F \sin \phi = \text{opposite side, which should be negative in } x! *
Example 1 (page 18)

Example 2. The angle between two forces of 40 and 90 lb, as shown in Figure 1.4e, is 60°. Determine the resultant.

Steps:
1. GIVEN: Write down what’s given (drawing and numbers).
2. FIND: Write down what you need to find. (resultant graphically)
3. SOLUTION:
   4. Draw the 40 lb and 90 lb forces to scale with tails at O. (If the scale isn’t given, you must choose one that fits on your paper; ie. 1 inch = 30 lb.)
   5. Draw parallel reference lines at the ends of the vectors.
   6. Draw a line from O to the intersection of the reference lines.
   7. Measure the length of the line.
   8. Convert the line length by the scale into pounds (by multiplying by the force measure and dividing by the scale value, i.e. X inches * 30 lb / 1 inch).
   9. Measure the angle with respect to the positive x axis.
Alternate solution:

4. Draw one vector
5. Draw the other vector at the TIP of the first one (away from the tip).
6. Draw a line from 0 to the tip of the final vector and continue at step 7

Equilibrant
The force equal and opposite to a resultant, that allows a system to be in equilibrium, is called an equilibrant.

Example 2 (pg 22)

Example 4. Let it be required to find the resultant of the four concurrent forces $P_1$, $P_2$, $P_3$, and $P_4$ shown in Figure 1.9a. $P_1 = 12\ N$, $P_2 = 17\ N$, $P_3 = 20\ N$, $P_4 = 17\ N$, 10 mm = 5 N.
Example 3 (pg 18) Determine the resultant vector analytically with the component method.

Cable Structures

Cables have the same tension all along the length if they are not cut. The force magnitude is the same everywhere in the cable *even if it changes angles*. Cables CANNOT be in compression. (They flex instead.)

*High-strength steel* is the most common material used for cable structures because it has a high strength to weight ratio.

Cables must be supported by vertical supports or towers and must be anchored at the ends. Flexing or unwanted movement should be resisted. (Remember the Tacoma Narrows Bridge?)

Cables with a single load have a **concurrent** force system. It will only be in equilibrium if the cable is **symmetric** when the load can slide and come to rest.

The forces anywhere in a **straight segment** can be resolved into x and y components of $T_x = T \cos \theta$ and $T_y = T \sin \theta$.

The shape of a cable having a **uniform distributed load** is almost parabolic, which means the geometry and cable length can be found with:

$$y = 4h \left( \frac{Lx - x^2}{L^2} \right)$$

where $y$ is the vertical distance from the straight line from cable start to end.

$h$ is the vertical sag (maximum $y$).

$x$ is the distance from one end to the location of $y$.

$L$ is the horizontal span.
\[ L_{\text{total}} = L \left( 1 + \frac{9/2 h^2}{L^2} - \frac{32/5 h^4}{L^4} \right) \]

where \( L_{\text{total}} \) is the total length of parabolic cable
h and L are defined above.

Example 4 (pg 24) Using force polygons and component relationships, determine the magnitudes in cables BC and CA.
**Point Equilibrium & Truss Analysis**

**Notation:**

- \( b \) = number of members in a truss
- \( (C) \) = shorthand for compression
- \( F \) = name for force vectors, as is \( X, T, \) and \( P \)
- \( F_{AB} \) = name of a truss force between joints named \( A \) and \( B \), ex.
- \( FBD \) = free body diagram
- \( F_x \) = force component in the x direction, as is \( T_x \)
- \( F_y \) = force component in the y direction, as is \( T_y \)
- \( n \) = number of joints in a truss
- \( N \) = normal force (perpendicular to something)
- \( R \) = name for resultant vectors
- \( R_x \) = resultant component in the x direction
- \( R_y \) = resultant component in the y direction
- \( T \) = name for a tension force
- \( (T) \) = shorthand for tension
- \( x \) = x axis direction, or horizontal dimension
- \( y \) = y axis direction, or vertical dimension
- \( \mu \) = coefficient of static friction
- \( \theta \) = angle, in a trig equation, ex. \( \sin \theta \), that is measured between the x axis and tail of a vector
- \( \Sigma \) = summation symbol

- **EQUILIBRIUM** is the state where the resultant of the forces on a particle or a rigid body is zero. There will be no rotation or translation. The forces are referred to as balanced.

  ex: 2 forces of same size, opposite direction

  ex: 4 forces, polygon rule shows that it closes

- Analytically, for a point:

  \[
  R_x = \sum F_{x(\text{or}h)} = 0 \quad R_y = \sum F_{y(\text{or}v)} = 0 \quad \text{(scalar addition)}
  \]

- **NEWTON’S FIRST LAW:** If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

  \[
  R_x = \sum F_x = 0 \quad R_y = \sum F_y = 0
  \]
• It is ABSOLUTELY NECESSARY to consider all the forces acting on a body (applied directly and indirectly) using a FREE BODY DIAGRAM. Omission of a force would ruin the conditions for equilibrium.

• FREE BODY DIAGRAM (aka FBD): Sketch of a significant isolated particle of a body or structure showing all the forces acting on it. Forces can be from
  - externally applied forces
  - weight of the rigid body
  - reaction forces or constraints
  - forces developed within a section member

Collinear Force System

• All forces act along the same line. Only one equilibrium equation is needed: \( \sum F_{\text{in-line}} = 0 \)

• Equivalently: \( R_x = \sum F_x = 0 \) and \( R_y = \sum F_y = 0 \)

Concurrent Force System

• All forces act through the same point. Only two equilibrium equations are needed:
  \( R_x = \sum F_x = 0 \) and \( R_y = \sum F_y = 0 \)

• How to solve when there are more than three forces on a free body:
  1. Resolve all forces into x and y components using known and unknown forces and angles. (Tables are helpful.)
  2. Determine if any unknown forces are related to other forces and write an equation.
  3. Write the two equilibrium equations (in x and y).
  4. Solve the equations simultaneously when there are the same number of equations as unknown quantities. (see math handout)

• Common problems have unknowns of:
  1) magnitude of force
  2) direction of force

FREE BODY DIAGRAM STEPS FOR A POINT:

1. Determine the point of interest. (What point is in equilibrium?)

2. Detach the point from and all other bodies (“free” it).
3. Indicate all external forces which include:
   - action on the point by the supports & connections
   - action on the point by other bodies
   - the weigh effect (=force) of any body attached to the point (force due to gravity)

4. All forces should be clearly marked with magnitudes and direction. The sense of forces should be those acting on the point not from the point.

5. Dimensions/angles should be included for force component computations.

6. Indicate the unknown forces, such as those reactions or constraining forces where the body is supported or connected.

   - **Force Reactions** can be categorized by the type of connections or supports. A force reaction is a force with known line of action, or a force of unknown direction. The line of action of the force is directly related to the motion that is prevented.

   ![Diagram](image)

   prevents motion: prevents motion:
   up and down vertical & horizontal

   - The line of action should be indicated on the FBD. The sense of direction is determined by the type of support. (Cables are in tension, etc…) If the sense isn’t obvious, assume a sense. When the reaction value comes out positive, the assumption was correct. When the reaction value comes out negative, the assumption was opposite the actual sense. DON’T CHANGE THE ARROWS ON YOUR FBD OR SIGNS IN YOUR EQUATIONS.

   - With the 2 equations of equilibrium for a point, there can be no more than 2 unknowns.

**Friction**

- There will be a force of resistance to movement developed at the contact face between objects when one is made to slide against the other. This is known as static friction and is determined from the reactive force, \(N\), which is normal to the surface, and a coefficient of friction, \(\mu\), which is based on the materials in contact.

\[ F = \mu N \]

- If the static friction force is exceeded by the pushing force, there will still be friction, but it is called kinetic friction, and it is smaller than static friction, so it is moving.

- The friction resistance is independent of the amount of contact area.
<table>
<thead>
<tr>
<th>Materials</th>
<th>μ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal on ice</td>
<td>0.03-0.05</td>
</tr>
<tr>
<td>Metal on metal</td>
<td>0.15-0.60</td>
</tr>
<tr>
<td>Metal on wood</td>
<td>0.20-0.60</td>
</tr>
<tr>
<td>Metal on stone</td>
<td>0.30-0.70</td>
</tr>
<tr>
<td>Wood on wood</td>
<td>0.30-0.70</td>
</tr>
<tr>
<td>Steel on steel</td>
<td>0.75</td>
</tr>
<tr>
<td>Stone on stone</td>
<td>0.40-0.70</td>
</tr>
<tr>
<td>Rubber on concrete</td>
<td>0.60-0.90</td>
</tr>
<tr>
<td>Aluminum on aluminum</td>
<td>1.10-1.70</td>
</tr>
</tbody>
</table>

- **CABLE STRUCTURES:**

**Cables with Several Concentrated Loads or Fixed Geometry**

- In order to completely constrain cables, the number of unknown support reactions will be more than the available number of equilibrium equations. We can solve because we have additional equations from geometry due to the slope of the cable.

- The tension in the cable IS NOT the same everywhere, but the horizontal component in a cable segment WILL BE.

**Truss Structures**

- A truss is made up of straight two-force members connected at its ends. The triangular arrangement produces stable geometry. Loads on a truss are applied at the joints only.
- Joints are pin-type connections (resist translation, not rotation).
- Forces of action and reaction on a joint must be equal and opposite.
- Members in TENSION are being pulled.
- Members in COMPRESSION are being squeezed.
- External forces act on the joints.
**Truss configuration:**

Three members form a rigid assembly with **3 (three)** connections. To add members and still have a rigid assembly, **2 (two)** more must be added with one connection between.

For rigidity: \( b = 2n - 3 \), where \( b \) is number of members and \( n \) is number of joints.

---

**Method of Joints**

- The method takes advantage of the conditions of equilibrium at each joint.
  1. Determine support reaction forces.
  2. Draw a FBD of each member AND each joint.
  3. Identify geometry of angled members.
  4. Identify zero force members and other special (easy to solve) cases.
  5. Each pin is in equilibrium (\( \sum F_x = 0 \) and \( \sum F_y = 0 \) for a concurrent force system).
  6. Total equations = \( 2n = b + 3 \) (one force per member + 3 support reactions).

Advantages: Can find every member force
Disadvantages: Lots of equations, easy to lose track of forces found.

**Tools available:**
- Tip-to-tail method for 3 joint forces must close
- Analytically, there will be at most 2 unknowns with 2 equilibrium equations.
Joint Configurations  (special cases to recognize for faster solutions)

Case 1)  Two Bodies Connected

\[ \text{F}_{AB} \text{ has to be equal (=} \text{ to } \text{F}_{BC} \]

Case 2)  Three Bodies Connected with Two Bodies in Line

\[ \text{F}_{AB} \text{ and } \text{F}_{BC} \text{ have to be equal, and } \text{F}_{BD} \text{ has to be 0 (zero).} \]

Case 3)  Three Bodies Connected and a Force – 2 Bodies aligned & 1 Body and a Force are Aligned

Four Bodies Connected - 2 Bodies Aligned and the Other 2 Bodies Aligned

\[ \text{F}_{AB} \text{ has to equal } \text{F}_{BC}, \text{ and [F}_{BD} \text{ has to equal P] or [F}_{BD} \text{ has to equal F}_{BE}] \]
Graphical Analysis

The method utilizes what we know about force triangles and plotting force magnitudes to scale.

1. Draw an accurate form diagram of the truss at a convenient scale with the loads and support reaction forces.

2. Determine the support reaction forces.

3. Working clockwise and from left to right, apply interval notation to the diagram, assigning capital letters to the spaces between external forces and numbers to internal spaces.

4. Construct a load line to a convenient scale of length to force by using the interval notation and working clockwise around the truss from the upper left plotting the lengths of the vertical and horizontal loads.

5. Starting at a left joint where we know there are fewer than three forces, we draw reference lines in the direction of the unknown members so that they intersect. Label the intersection with the number of the internal space.

6. Go to the next joint (clockwise and left to right) with two unknown forces and repeat for all joints. The diagram should close.

7. Measure the line segments and apply interval notation to determine their sense: Proceeding clockwise around the joint, follow the notation. The direction toward the joint is compressive. The direction away from the joint is tensile.

Example 1 (pg 72 & 77) Using the method of joints, determine all member forces.
Example 2
Using the method of joint, determine all member forces.

SOLUTION:

Find the joints with 2 (or less unknowns) for FBD’s: A and H, while looking for any special cases like E, which has “crossed” members and forces. \( F_{DE} = F_{EF} \) and \( F_{BC} = 500 \text{ lb (tension)} \).

(Check off members found: AB, BD, AD, BC, DC, DE, EC, EF, CG, CF, FG, GH, FH)

Let’s use A first (but H is just as acceptable). Draw the point, adding the known force, and draw the unknown member forces away from the point, assuming tension (shown as dashed). Find the geometry of member AB from the rise of 10 ft and the run of 15 ft. The hypotenuse will be \( \sqrt{10^2 + 15^2} = 18.03 \):

\[
\Sigma F_x = F_{AD} + F_{AD} \frac{15}{18.03} = 0
\]

\[
\Sigma F_y = 412.5^\circ + F_{AD} \frac{10}{18.03} = 0
\]

and substituting the (negative) value of \( F_{AD} \) into the \( \Sigma F_x \), \( F_{AD} = 618.75 \text{ lb (T)} \)

(Check off members found: AB, BD, AD, BC, DC, DE, EC, EF, CG, CF, FG, GH, FH)

Review which joints have 2 (or less) unknowns: B and H.

Let’s use B. Because we know \( F_{AB} \) is in compression we draw the force into the point.

We need the geometry of member BC with a rise of 5 (15-10) and a run of 15 with a hypotenuse of \( \sqrt{5^2 + 15^2} = 15.81 \):

\[
\Sigma F_x = 743.7^\circ + F_{BC} \frac{15}{15.81} = 0
\]

\[
\Sigma F_y = 743.7^\circ + F_{BC} \frac{10}{15.81} - F_{AD} = 0
\]

(substituting the negative value of \( F_{BC} \))

\( F_{BC} = 652.1 \text{ lb (C)} \)

FBD = 206.2 lb (T)

(Check off members found: AB, BD, AD, BC, DC, DE, EC, EF, CG, CF, EF, FG, GH, FH)

Review which joints have 2 (or less) unknowns: D and H.

Let’s use D. Both \( F_{AD} \) and \( F_{BD} \) are tensile, so we can draw them away. The geometry of DE is familiar with the rise the same as the run for an angle of 45°:

\[
\Sigma F_x = -618.75^\circ + F_{DE} \cos 45^\circ + F_{DE} = 0
\]

\[
\Sigma F_y = -150^\circ + 206.2^\circ + F_{DE} \sin 45^\circ = 0
\]

\( F_{DE} = 79.5 \text{ lb (C)} \)

and substituting the (negative) value of \( F_{BC} \) into the \( \Sigma F_y \), \( F_{DE} = 675.0 \text{ lb (T)} = F_{EF} \) (from above)

(Check off members found: AB, BD, AD, BC, DC, DE, EC, EF, CG, CF, FG, GH, FH)

Review which joints have 2 (or less) unknowns: C and H.

Let’s use C. Draw \( F_{DC} \) and \( F_{BC} \) as compressive forces. And the geometry has been found due to symmetry, with the angle of \( F_{CF} \) a negative 45°:

\[
F_C = 652.1^\circ \frac{15}{15.81} + 79.5^\circ \cos 45^\circ + F_{CF} \cos(-45^\circ) + F_{CG} \frac{15}{15.81} = 0
\]

\[
\Sigma F_y = 652.1^\circ \frac{5}{15.81} + 79.5^\circ \sin 45^\circ - 500^\circ + F_{CF} \sin(-45^\circ) - F_{CG} \frac{5}{15.81} = 0
\]

Solve simultaneously because there isn’t an easy way to find one unknown equal to a value multiplied by the other unknown:

\[
\Sigma F_x = 674.9^\circ + 0.707 F_{CF} + 0.949 F_{CG} = 0
\]

\[
\Sigma F_y = -237.6^\circ - 0.707 F_{CF} - 0.316 F_{CG} = 0
\]

add: \( 437.5^\circ + 0.633 F_{CF} = 0 \)

\( F_{CG} = -690.8 \text{ lb (C)} \) and substituting, \( F_{CF} = -27.6 \text{ lb (C)} \)

(Check off members found: AB, BD, AD, BC, DC, DE, EC, EF, CG, CF, FG, GH, FH)
Example 2 (continued)

Review which joints have 2 (or less) unknowns: G, F, and H. Let's use F because H really looks like A mirrored. Draw F_{CF} as compressive and F_{EF} in tension. The angle from for F_{CF} is negative 45°:
\[ \Sigma F_x = -675.0 + 27.6 \cos(-45°) + F_{FH} = 0 \quad F_{FH} = 655.5 \text{ lb (T)} \]
\[ \Sigma F_y = 27.6 \sin(-45°) - 200 + F_{FG} = 0 \quad F_{FG} = 219.5 \text{ lb (T)} \]

(Check off members found: AB, BD, AD, BC, DC, DE, EC, EF, CG, CF, FG, GH, FH)

Review which joints have 2 (or less) unknowns; which are G and H. Let's use G and pretend that we have only found F_{GF} (and not F_{CG}) in order to show a set of equations that use substitution with the algebra. The geometry has been found due to symmetry:
\[ \Sigma F_x = -F_{CG} \frac{15}{15.81} + F_{GH} \frac{15}{18.03} = 0 \quad \text{rearranging:} \quad F_{CG} = F_{GH} \frac{15}{18.03} \frac{15}{15} = F_{GH} \frac{15.81}{18.03} \]
\[ \Sigma F_y = F_{CG} \frac{5}{15.81} - F_{GH} \frac{10}{18.03} = 219.5 = 0 \]

Substituting:
\[ \Sigma F_x = (F_{GH} \frac{15}{18.03} \frac{15.81}{18.03}) - F_{GH} \frac{10}{18.03} - 219.5 = 0 \]
Simplifying \[ -0.277 F_{GH} = 219.5 \quad F_{GH} = -791.6 \text{ lb (C)} \]
and \[ F_{CG} = -694.1 \text{ lb (C)} \] (which validates the earlier answer found of 690.8 lb (C) with respect to rounding errors in all fractions and trig functions)
(Check off members found: AB, BD, AD, BC, DC, DE, EC, EF, CG, CF, FG, GH, FH)

(Typically, the last joint left will verify that the joint is in equilibrium with values found, but in this exercise the last joint was used to show the algebraic method of substitution.)
Truss Analysis using Multiframe

1. The software is on the teaching computers in the College of Architecture in Programs under the Windows Start menu. Multiframe is under the Bentley Engineering menu. It is also available at the Open Access Labs (OAL) and the Virtual OAL.

2. There are tutorials available on line at http://www.daystarsoftware.com/support/mftutorials that list the tasks and order in greater detail. The first task is to define the unit system:
   - Choose Units… from the View menu. Unit sets are available, but specific units can also be selected by double clicking on a unit or format and making a selection from the menu.

3. To see the scale of the geometry, a grid option is available:
   - Choose Grid… from the View menu

4. To create the geometry, you must be in the Frame window (default). The symbol is the frame in the window toolbar:

The Member toolbar shows ways to create members:

The Generate toolbar has convenient tools to create typical structural shapes:
   - To create a truss, use the add connected members button:
- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window. The ESC button will end the segmented drawing. Continue to use the add connected members button. Any time the cursor is over an existing joint, the joint will be highlighted by a red circle.

- The geometry can be set precisely by selecting the joint (drag), and bringing up the joint properties menu (right click) to set the coordinates.

- The support types can be set by selecting the joint (drag) and using the Joint Toolbar (pin shown), or the Frame / Joint Restraint ... menu (right click).

**NOTE:** If the support appears at both ends of the member, you had the member selected rather than the joint. Select the joint to change support for and right click to select the joint restraints menu or select the correct support on the joint toolbar.

The support forces will be determined in the analysis.

5. All members must have sections assigned (see section 6.) in order to calculate reactions and deflections. To use a standard steel section **proceed to step 6.** For custom sections, the section information must be entered. To define a section:

- Choose Edit Sections / Add Section... from the Edit menu
- Type a name for your new section
- Choose group Frame from the group names provided so that the section will remain with the file data
- Choose a shape. The Flat Bar shape is a rectangular section.
- Enter the cross section data.
Table values 1-9 must have values for a Flat Bar, but not all are used for every analysis. A recommendation is to put the value of 1 for those properties you don’t know or care about. Properties like $t_f$, $t_w$, etc. refer to wide flange sections.

- Answer any query. If the message says there is an error, the section will not be created until the error is corrected.

6. The standard sections library loaded is for the United States. If another section library is needed, use the Open Sections Library... command under the file menu, choose the library folder, and select the SectionsLibrary.slb file.

Select the members (drag to make bold) and assign sections with the Section button on the Member toolbar:

- Choose the group name and section name:

7. In order for Multiframe to recognize that the truss members are two-force bodies, all joints must be highlighted and assigned as pins with the Pinned Joints button on the Joint toolbar:

8. The truss geometry is complete, and in order to define the load conditions you must be in the Load window represented by the green arrow:

9. The Load toolbar allows a joint to be loaded with a force or a moment in global coordinates, shown by the first two buttons after the display numbers button. It allows a member to be loaded with a distributed load, concentrated load or moment (next three buttons) in global coordinates, as well as loading with distributed or single force or moment in the local coordinate system (next three buttons). It allows a load panel to be loaded with a distributed load in global or local coordinates (last two buttons).

- Choose the joint to be loaded (drag) and select the load type (here shown for point loading):
• Choose the direction by the arrow shown. There is no need to put in negative values for downward loading.
• Enter the values of the load

**NOTE: Do not put support reactions as applied loads. The analysis will determine the reaction values**

Multiframe will automatically generate a grouping called a Load Case named Load Case 1 when a load is created. All additional loads will be added to this load case unless a new load case is defined (Add case under the Case menu).

10. In order to run the analysis after the geometry, member properties and loading has been defined:
• Choose Linear from the Analyze menu

11. If the analysis is successful, you can view the results in the Plot window represented by the red moment diagram:

12. The Plot toolbar allows the numerical values to be shown (1.0 button), the reaction arrows to be shown (brown up arrow) and reaction moments to be shown (brown curved arrow):

• To show the axial force diagram, Choose the purple Axial Force button. Tensile members will have “T” by the value (if turned on), while compression members will have “C” by the value
• To show the deflection diagram, Choose the blue Deflection button
• To animate the deflection diagram, Choose Animate... from the Display menu. You can also save the animation to a .avi file by checking the box.
- To see exact values of axial load and deflection, double click on the member and move the vertical cross hair with the mouse. The ESC key will return you to the window.

13. The Data window (D) allows you to view all data “entered” for the geometry, sections and loading. These values can be edited.

14. The Results window (R) allows you to view all results of the analysis including displacements, reactions, member forces (actions) and stresses. These values can be cut and pasted into other Windows programs such as Word or Excel.

NOTE: Px’ refers to the axial load (P) in the local axis x direction (x’).

15. To save the file Choose Save from the File menu.
16. To load an existing file Choose Open... from the File menu.
17. To print a plot Choose Print Window... from the File menu. As an alternative, you may copy the plot (Ctrl+c) and paste it in a word processing document (Ctrl+v).
Mechanics of Materials

Notation:

\( a \) = acceleration
\( A \) = area (net = with holes, bearing = in contact, etc...)
\( ASD \) = allowable stress design
\( d \) = diameter of a hole
\( e \) = change in length
\( E \) = modulus of elasticity or Young’s modulus
\( f \) = symbol for stress
\( f_{\text{allowable}} \) = allowable stress
\( f_{\tau} \) = shear stress
\( f_{\rho} \) = bearing stress (see P)
\( F_{\text{allowed}} \) = allowable stress
\( F.S. \) = factor of safety
\( g \) = acceleration due to gravity
\( GPa \) = giga Pascals (10^9 Pa)
\( kPa \) = kilopascals or 1 kN/m^2
\( ksi \) = kips per square inch
\( L \) = length
\( LRFD \) = load and resistance factor design
\( m \) = mass
\( MPa \) = megapascals or 10^6 N/m^2 or 1 N/mm^2
\( psi \) = pounds per square inch
\( psf \) = pounds per square foot
\( P \) = name for axial force vector
\( P' \) = name for internal axial force vector
\( Pa \) = Pascal (N/m^2)
\( R \) = name for reaction force vector
\( R_{\text{L}}, R_{D}, or R_{n} \) = name for design quantity (force or moment) for LRFD
\( s \) = normal strain
\( t \) = thickness
\( v \) = velocity
\( \alpha \) = coefficient of thermal expansion for a material
\( \varepsilon \) = strain
\( \varepsilon_{t} \) = thermal strain (no units)
\( \delta \) = elongation or length change
\( \delta_{\rho} \) = elongation due to axial load
\( \delta_{\text{restr}} \) = restrained length change
\( \delta_{T} \) = elongation due or length change due to temperature
\( \Delta T \) = change in temperature
\( \phi \) = angle of twist
\( \mu \) = lateral strain ratio or Poisson’s ratio
\( \gamma \) = shear strain
\( \gamma_{D} \) = dead load factor for LRFD
\( \gamma_{L} \) = live load factor for LRFD
\( \theta \) = angle of principle stress
\( \rho \) = radial distance
\( \sigma \) = engineering symbol for normal stress
\( \tau \) = engineering symbol for shearing stress

Mechanics of Materials is a basic engineering science that deals with the relation between externally applied load and its effect on deformable bodies. The main purpose of Mechanics of Materials is to answer the question of which requirements have to be met to assure STRENGTH, RIGIDITY, AND STABILITY of engineering structures.
To solve a problem in Mechanics of Materials, one has to consider THREE ASPECTS OF THE PROBLEM:

1. **STATICS**: equilibrium of external forces, internal forces, stresses
2. **GEOMETRY**: deformations and conditions of geometric fit, strains
3. **MATERIAL PROPERTIES**: stress-strain relationship for each material, obtained from material testing.

- **STRESS** – The intensity of a force acting over an area

**Normal Stress (or Direct Stress)**

Stress that acts along an *axis* of a member; can be internal or external; can be compressive or tensile.

\[
f = \sigma = \frac{P}{A_{net}}
\]

Strength condition: \( f = \frac{P}{A_{net}} < f_{allowable} \) or \( F_{allowed} \)

**Shear Stress (or Direct Shear Stress)**

Stress that acts perpendicular to an *axis* or *length* of a member, or that is parallel to the cross section is called shear stress.

Shear stress cannot be assumed to be uniform, so we refer to *average shearing stress*.

\[
f_v = \tau = \frac{P}{A_{net}}
\]

Strength condition: \( f_v = \frac{P}{A_{net}} < \tau_{allowable} \) or \( F_{allowed} \)
Shear Stress in Beams

A stress that acts transversely to the cross section, or along the beam length. It happens when there are vertical loads on a horizontal beam, and is larger than direct shear stress from the internal vertical force over the cross section.

Bearing Stress

A compressive normal stress acting between two bodies.

\[ f_p = \frac{P}{A_{\text{bearing}}} \]

Bending Stress in Beams

A normal stress caused by bending; can be compressive or tensile.

Torsional Stress

A shear stress caused by torsion (moment around the axis).
Bolts in Shear and Bearing

**Single shear** - forces cause only one shear “drop” across the bolt.

(a) Two steel plates bolted using one bolt.  
(b) Elevation showing the bolt in shear.

c. \[ f_s = \frac{P}{A} \]  
d. \[ f_v = \text{Average shear stress through bolt cross section} \]
\[ A = \text{Bolt cross-sectional area} \]

Figure 5.11 A bolted connection—single shear.

**Double shear** - forces cause two shear changes across the bolt.

\[ f_s = \frac{P}{2A} \]  
(two shear planes)

Free-body diagram of middle section of the bolt in shear.

Figure 5.12 A bolted connection in double shear.
Bearing of a bolt on a bolt hole – The bearing surface can be represented by projecting the cross section of the bolt hole on a plane (into a rectangle).

\[
f_p = \frac{P}{A} = \frac{P}{td}
\]

- STRAIN – The relative change in geometry (length, angle, etc.) with respect to the original geometry. Deformation is the change in the size or shape of a structural member under loading. It can also occur due to heating or cooling of a material.

Normal Strain

In an axially loaded member, normal strain, \(s\) (or \(\varepsilon\)) is the change in the length, \(e\) (or \(\delta\)) with respect to the original length, \(L\).

\[
s = \frac{e}{L} \quad \text{or} \quad \varepsilon = \frac{\delta}{L}
\]

It is UNITLESS, but may be called strain or microstrain (\(\mu\)).
**Shearing Strain**

In a member loaded with shear forces, shear strain, \( \gamma \) is the change in the sheared side, \( \delta s \) with respect to the original height, \( L \). For small angles: \( \tan \phi \approx \phi \).

\[
\gamma = \frac{\delta s}{L} = \tan \phi \approx \phi
\]

In a member subjected to twisting, the shearing strain is a measure of the angle of twist with respect to the length and distance from the center, \( \rho \):

\[
\gamma = \frac{\rho \phi}{L}
\]

**Testing of Load vs. Strain**

Behavior of materials can be measured by recording deformation with respect to the size of the load. For members with constant cross section area, we can plot stress vs. strain.

**BRITTLE MATERIALS** - ceramics, glass, stone, cast iron; show abrupt fracture at small strains.

**DUCTILE MATERIALS** – plastics, steel; show a yield point and large strains (considered plastic) and “necking” (give warning of failure)

**SEMI-BRITTLE MATERIALS** – concrete; show no real yield point, small strains, but have some “strain-hardening”.

**Linear-Elastic Behavior**

In the straight portion of the stress-strain diagram, the materials are *elastic*, which means if they are loaded and unloaded no permanent deformation occurs.
**True Stress & Engineering Stress**

True stress takes into account that the area of the cross section changes with loading. Engineering stress uses the original area of the cross section.

**Hooke’s Law – Modulus of Elasticity**

In the linear-elastic range, the slope of the stress-strain diagram is constant, and has a value of \( E \), called Modulus of Elasticity or Young’s Modulus.  
\[
f = E \cdot s = E \cdot \varepsilon
\]

**Isotropic Materials** – have the same \( E \) with any direction of loading.

**Anisotropic Materials** – have different \( E \)’s with the direction of loading.

**Orthotropic Materials** – have directionally based \( E \)’s

---

**Table D-1 Elastic moduli of selected materials**

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus of elasticity ( E )</th>
<th>Shear modulus ( G )</th>
<th>Poisson’s ratio ( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 10^6 ) psi</td>
<td>GPa</td>
<td>( 10^6 ) psi</td>
</tr>
<tr>
<td>Aluminum</td>
<td>10-12</td>
<td>70</td>
<td>3.8</td>
</tr>
<tr>
<td>Aluminum alloys</td>
<td>10.6</td>
<td>73</td>
<td>4</td>
</tr>
<tr>
<td>2014-T6</td>
<td>10</td>
<td>70</td>
<td>3.8</td>
</tr>
<tr>
<td>7075-T6</td>
<td>10.4</td>
<td>72</td>
<td>3.9</td>
</tr>
<tr>
<td>Brick (compression)</td>
<td>1.5–3.5</td>
<td>10–24</td>
<td>4.5–10</td>
</tr>
<tr>
<td>Cast iron</td>
<td>12–25</td>
<td>80–170</td>
<td>5.6</td>
</tr>
<tr>
<td>Gray cast iron</td>
<td>14</td>
<td>115</td>
<td>6.2</td>
</tr>
<tr>
<td>Concrete (compression)</td>
<td>2.6–4.4</td>
<td>18–30</td>
<td>5.5</td>
</tr>
<tr>
<td>Copper</td>
<td>5</td>
<td>115</td>
<td>6.2</td>
</tr>
<tr>
<td>Copper alloys</td>
<td>14–18</td>
<td>96–120</td>
<td>5.2–6.8</td>
</tr>
<tr>
<td>Brass</td>
<td>14–16</td>
<td>96–110</td>
<td>5.2–6</td>
</tr>
<tr>
<td>80% Cu, 20% Zn</td>
<td>15</td>
<td>100</td>
<td>5.5</td>
</tr>
<tr>
<td>Naval brass</td>
<td>15</td>
<td>100</td>
<td>5.5</td>
</tr>
<tr>
<td>Bronze</td>
<td>14–17</td>
<td>96–120</td>
<td>5.2–6.3</td>
</tr>
<tr>
<td>Manganese bronze</td>
<td>15</td>
<td>100</td>
<td>5.6</td>
</tr>
<tr>
<td>Glass</td>
<td>7–12</td>
<td>50–80</td>
<td>2.9–5</td>
</tr>
<tr>
<td>Magnesium</td>
<td>5.8</td>
<td>40</td>
<td>2.2</td>
</tr>
<tr>
<td>Nickel</td>
<td>30</td>
<td>210</td>
<td>11.4</td>
</tr>
<tr>
<td>Nylon</td>
<td>0.3–0.4</td>
<td>2–3</td>
<td>0.4</td>
</tr>
<tr>
<td>Rubber</td>
<td>0.0001–0.0006</td>
<td>0.001–0.004</td>
<td>0.00004–0.00002</td>
</tr>
<tr>
<td>Steel</td>
<td>28–32</td>
<td>190–220</td>
<td>10.8–12.3</td>
</tr>
<tr>
<td>Stone (compression)</td>
<td>6–10</td>
<td>40–70</td>
<td>0.2–0.3</td>
</tr>
<tr>
<td>Granite</td>
<td>7–14</td>
<td>50–100</td>
<td>0.2–0.3</td>
</tr>
<tr>
<td>Marble</td>
<td>16</td>
<td>110</td>
<td>5.8</td>
</tr>
<tr>
<td>Titanium</td>
<td>15–18</td>
<td>100–124</td>
<td>5.6–6.8</td>
</tr>
<tr>
<td>Titanium alloys</td>
<td>52</td>
<td>360</td>
<td>22</td>
</tr>
<tr>
<td>Tungsten</td>
<td>52</td>
<td>360</td>
<td>22</td>
</tr>
<tr>
<td>Wood (bending)</td>
<td>1.5–1.6</td>
<td>10–11</td>
<td>11–12</td>
</tr>
<tr>
<td>Ash</td>
<td>1.6–1.8</td>
<td>11–12</td>
<td>11–14</td>
</tr>
<tr>
<td>Oak</td>
<td>2</td>
<td>190</td>
<td>10.9</td>
</tr>
</tbody>
</table>
Plastic Behavior & Fatigue

Permanent deformations happen outside the linear-elastic range and are called **plastic** deformations. Fatigue is damage caused by reversal of loading.

- **The proportional limit** (at the end of the **elastic** range) is the greatest stress valid using Hooke’s law.

- **The elastic limit** is the maximum stress that can be applied before permanent deformation would appear upon unloading.

- **The yield point** (at the **yield stress**) is where a ductile material continues to elongate without an increase of load. (May not be well defined on the stress-strain plot.)

- **The ultimate strength** is the largest stress a material will see before rupturing, also called the **tensile strength**.

- **The rupture strength** is the stress at the point of rupture or failure. It may not coincide with the ultimate strength in ductile materials. In brittle materials, it will be the same as the ultimate strength.

- **The fatigue strength** is the stress at failure when a member is subjected to reverse cycles of stress (up & down or compression & tension). This can happen at much lower values than the ultimate strength of a material.

- **Toughness** of a material is how much work (a combination of stress and strain) us used for fracture. It is the area under the stress-strain curve.

Concrete does not respond well to tension and is tested in compression. The strength at crushing is called the **compression strength**.

Materials that have time dependent elongations when loaded are said to have **creep**. Concrete and wood experience creep. Concrete also has the property of shrinking over time.
Poisson’s Ratio

For an isometric material that is homogeneous, the properties are the same for the cross section:

\[ \varepsilon_y = \varepsilon_z \]

There exists a linear relationship while in the linear-elastic range of the material between longitudinal strain and lateral strain:

\[ \mu = - \frac{\varepsilon_y}{\varepsilon_x} = - \frac{\varepsilon_z}{\varepsilon_x} \]

Positive strain results from an increase in length with respect to overall length.

Negative strain results from a decrease in length with respect to overall length.

\( \mu \) is the Poisson’s ratio and has a value between 0 and \( \frac{1}{2} \), depending on the material.

Relation of Stress to Strain

\[ f = \frac{P}{A}; \quad \varepsilon = \frac{\delta}{L} \quad \text{and} \quad E = \frac{P}{\delta} \quad \frac{A}{L} \quad \text{which rearranges to:} \quad \delta = \frac{PL}{AE} \quad \text{or} \quad \varepsilon = \frac{PL}{AE} \]

Orthotropic Materials

One class of non-isotropic materials is orthotropic materials that have directionally based values of modulus of elasticity and Poisson’s ratio (\( E, \mu \)).

Ex: plywood, laminates, fiber reinforced polymers with direction fibers
Stress Concentrations

In some sudden changes of cross section, the stress concentration changes (and is why we used average normal stress). Examples are sharp notches, or holes or corners.

(Think about airplane window shapes...)

Maximum Stress

When both normal stress and shear stress occur in a structural member, the maximum stresses can occur at some other planes (angle of $\theta$).

Maximum Normal Stress happens when $\theta = 0^\circ$ AND

Maximum Shearing Stress happens when $\theta = 45^\circ$ with only normal stress in the $x$ direction.

Thermal Strains

Physical restraints limit deformations to be the same, or sum to zero, or to be proportional with respect to the rotation of a rigid body.

We know axial stress relates to axial strain: $e = \frac{\delta}{AE} = \frac{PL}{AE}$ which relates $e$ or $\delta$ to $P$.

Deformations can be caused by the material reacting to a change in energy with temperature. In general (there are some exceptions):

- Solid materials can contract with a decrease in temperature.
- Solid materials can expand with an increase in temperature.

The change in length per unit temperature change is the coefficient of thermal expansion, $\alpha$. It has units of $^\circ F$ or $^\circ C$ and the deformation is related by:

$$\delta_T = \alpha(\Delta T)L$$

Thermal Strain: $\epsilon_T = \alpha \Delta T$
Coefficient of Thermal Expansion

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficients ((\alpha)) [in./in./°F]</th>
<th>Coefficients ((\alpha)) [mm/mm/°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td>3.0 x 10^{-6}</td>
<td>5.4 x 10^{-6}</td>
</tr>
<tr>
<td>Glass</td>
<td>4.4 x 10^{-6}</td>
<td>8.0 x 10^{-6}</td>
</tr>
<tr>
<td>Concrete</td>
<td>5.5 x 10^{-6}</td>
<td>9.9 x 10^{-6}</td>
</tr>
<tr>
<td>Cast Iron</td>
<td>5.9 x 10^{-6}</td>
<td>10.6 x 10^{-6}</td>
</tr>
<tr>
<td>Steel</td>
<td>6.5 x 10^{-6}</td>
<td>11.7 x 10^{-6}</td>
</tr>
<tr>
<td>Wrought Iron</td>
<td>6.7 x 10^{-6}</td>
<td>12.0 x 10^{-6}</td>
</tr>
<tr>
<td>Copper</td>
<td>9.3 x 10^{-6}</td>
<td>16.8 x 10^{-6}</td>
</tr>
<tr>
<td>Bronze</td>
<td>10.1 x 10^{-6}</td>
<td>18.1 x 10^{-6}</td>
</tr>
<tr>
<td>Brass</td>
<td>10.4 x 10^{-6}</td>
<td>18.8 x 10^{-6}</td>
</tr>
<tr>
<td>Aluminum</td>
<td>12.8 x 10^{-6}</td>
<td>23.1 x 10^{-6}</td>
</tr>
</tbody>
</table>

There is no stress associated with the length change with free movement, BUT if there are restraints, thermal deformations or strains can cause internal forces and stresses.

How A Restrained Bar Feels with Thermal Strain

1. Bar pushes on supports because the material needs to expand with an increase in temperature.
2. Supports push back.

Bar is restrained, can’t move and the reaction causes internal stress.

Superposition Method

If we want to solve a statically indeterminate problem that has extra support forces, we can:

- Remove a support or supports that makes the problem look statically determinate
- Replace it with a reaction and treat it like it is an applied force
- Impose the geometry restrictions that the support imposes

For Example:

\[ \delta_T = \alpha(\Delta T)L \]
\[ \delta_p = -\frac{PL}{AE} \]
\[ \delta_p + \delta_T = 0 \quad -\frac{PL}{AE} + \alpha(\Delta T)L = 0 \]
\[ P = \alpha(\Delta T)L \frac{AE}{L} = \alpha(\Delta T)AE \quad f = -\frac{P}{A} = -\alpha(\Delta T)E \]
Dynamics

The study of bodies in motion due to forces and accelerations is called dynamics, in which time is an important consideration.

A particle or body can have a displacement that is a function of time. The velocity is the change in displacement as a function of time. The acceleration is the change in velocity as a function of time.

Calculus is helpful to work backward from acceleration to the distance traveled at a certain time. Because \( a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \), \( s(t) = v(0)t + \frac{1}{2}at^2 \)

Acceleration, from gravity or earthquakes, when acting on a mass, results in a force: \( f = ma \) and weight due to gravity on a mass is: \( W = mg \)

Work is defined as the product of the force by the distance traveled. Potential energy is the stored energy (like from the height of water in an elevated tank). Kinetic energy is from bodies in motion. Conservation of energy is the principal that work can change form, but not diminish or increase, or “work in” = “work out”.

Harmonic motion is a cyclic motion, meaning it swings one way then back the other with a certain amplitude or height, and period or frequency (time per cycle or number of cycles per unit time). Buildings and massed structures have a fundamental period, which is extremely important for their behavior under earthquake motion. Resonance causes the harmonic motion amplitude to be enhanced or magnified, which is a serious problem in structures.

Allowable Stress Design (ASD) and Factor of Safety (F.S.)(aka safety factor)

There are uncertainties in material strengths: \( F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}} \)

Allowable stress design determines the allowable stress by: \( \text{allowable stress} = \frac{\text{ultimate stress}}{F.S} \)

Load and Resistance Factor Design – LRFD

There are uncertainties in material strengths and in structural loadings.

\( R_n = \gamma_D R_D + \gamma_L R_L \leq \phi R_n \)

where \( \gamma = \text{load factor for Dead and Live loads} \)
\( R = \text{load (dead or live)} \)
\( \phi = \text{resistance factor} \)
\( R_n = \text{ultimate load (nominal capacity)} \)
Example 1 (pg 58)

Example 3*. A running track in a gymnasium is hung from the roof trusses by 9.84 ft [3 m] long steel rods, each of which supports a tensile load of 11,200 lb [49,818 N]. The round rods have a diameter of 7/8 in. [22.23 mm] with the ends upset, that is, made larger by forging. This upset allows the full cross-sectional area of the rod (0.601 in.²) [388 mm²] to be utilized; otherwise the cutting of the threads will reduce the cross section of the rod. Investigate this design to determine whether it is safe. Also, determine the elongation of the steel rods when the load at capacity is applied.
Example 2

A W8x67 steel beam, 20 ft. in length, is rigidly attached at one end of a concrete wall. If a gap of 0.010 in. exists at the opposite end when the temperature is 45°F, what results when the temperature rises to 95°F?

ALSO: If the beam is anchored to a concrete slab, and the steel sees a temperature change of 50°F while the concrete only sees a change of 30°F, determine the compressive stress in the beam.

\[ \alpha_c = 5.5 \times 10^{-6} /°F \quad E_c = 3 \times 10^6 \text{ psi} \]
\[ \alpha_s = 6.5 \times 10^{-6} /°F \quad E_s = 29 \times 10^6 \text{ psi} \]
Example 3

A short concrete column measuring 12 in. square is reinforced with four #8 bars ($A_s = 4 \times 0.79 \text{ in.}^2 = 3.14 \text{ in.}^2$) and supports an axial load of 250k. Steel bearing plates are used top and bottom to ensure equal deformations of steel and concrete. Calculate the stress developed in each material if:

$$E_s = 3 \times 10^6 \text{ psi and } E_c = 29 \times 10^6 \text{ psi}$$

Solution:

From equilibrium:

$$[\Sigma F_y = 0] - 250 \text{ k} + f_s A_s + f_c A_c = 0$$

$$A_s = 3.14 \text{ in.}^2$$

$$A_c = (12'' \times 12'') - 3.14 \text{ in.}^2 \equiv 141 \text{ in.}^2$$

$$3.14 f_s + 141 f_c = 250 \text{ k}$$

From the deformation relationship:

$$\delta_s = \delta_c \cdot \frac{L_s}{L_c}$$

$$\therefore \frac{\delta_s}{L_s} = \frac{\delta_c}{L_c}$$

and

$$\varepsilon_s = \varepsilon_c$$

Since

$$E = \frac{f}{\varepsilon}$$

and

$$\frac{f_s}{E_s} = \frac{f_c}{E_c}$$

$$f_s = f_c \frac{E_s}{E_c} = \frac{29 \times 10^3 (f_c)}{3 \times 10^3} = 9.67 f_c$$

Substituting into the equilibrium equation:

$$3.14 (9.76 f_c) + 141 f_c = 250$$

$$30.4 f_c + 141 f_c = 250$$

$$171.4 f_c = 250$$

$$f_c = 1.46 \text{ ksi}$$

$$\therefore f_c = 9.67 (1.46) \text{ ksi}$$

$$f_s = 14.1 \text{ ksi}$$
Moments & Supports

Notation:

\[ d = \text{perpendicular distance to a force from a point} \]
\[ F = \text{name for force vectors or magnitude of a force, as is } P, Q, R \]
\[ F_x = \text{force component in the x direction} \]
\[ F_y = \text{force component in the y direction} \]
\[ FBD = \text{free body diagram} \]
\[ L = \text{beam span length} \]
\[ M = \text{moment due to a force} \]
\[ R_x = \text{resultant component in the x direction} \]
\[ R_y = \text{resultant component in the y direction} \]
\[ w = \text{name for distributed load} \]
\[ W = \text{name for total force due to distributed load} \]
\[ x = \text{horizontal distance} \]
\[ x = \text{x axis direction} \]
\[ y = \text{y axis direction} \]
\[ \theta = \text{angle, in a trig equation, ex. } \sin \theta, \text{ that is measured between the x axis and tail of a vector} \]

Moment of a Force About an Axis

- Two forces of the same size and direction acting at different points are not equivalent. They may cause the same translation, but they cause different rotation.

- DEFINITION: Moment – A moment is the tendency of a force to make a body rotate about an axis. It is measured by \( F \times d \), where \( d \) is the distance perpendicular to the line of action of the force and through the axis of rotation.

For the same force, the bigger the lever arm (or moment arm), the bigger the moment magnitude, i.e. \( M_A = F \cdot d_1 < M_A = F \cdot d_2 \).
- Units: SI: N·m, KN·m
  Engr. English: lb·ft, kip·ft

- Sign conventions: Moments have magnitude and rotational direction:
  - OUR TEXT: positive - CW + negative – CCW -
  - MOST OTHER TEXTS, INCLUDING PHYSICS TEXTS:
    positive - CCW + negative – CW -

- Moments can be added as scalar quantities when there is a sign convention.

- Repositioning a force along its line of action results in the same moment about any axis.

- A force is completely defined (except for its exact position on the line of action) by $F_x$, $F_y$, and $M_A$ about A (size and direction).
• The sign of the moment is determined by which side of the axis the force is on.

![Diagram](image)

- Varignon’s Theorem: The moment of a force about any axis is equal to the sum of moments of the components about that axis.

\[ M = F \cdot d = P \cdot d_1 + Q \cdot d_2 \]

**Proof 1:** Resolve F into components along line BA and perpendicular to it (90°).

- d from A to line AB = 0
- d from A to F_\perp = d_{BA} = \frac{d}{\cos \theta}
- F_{BA} = F \sin \theta
- F_\perp = F \cos \theta
- \sum M = F \cdot d = F_{BA} \cdot 0 + F_\perp \cdot d_{BA} = F \cos \theta \cdot \frac{d}{\cos \theta} = F \cdot d

**Proof 2:** Resolve P and Q into P_{BA} & P_\perp, and Q_{BA} & Q_\perp.

- d from A to line AB = 0
- M_A by P = P_\perp \cdot d_{BA}
- M_A by Q = Q_\perp \cdot d_{BA}
- \sum M = P_\perp \cdot d_{BA} + Q_\perp \cdot d_{BA}

and we know d_{BA} from Proof 1, and by definition: P_\perp + Q_\perp = F_\perp. We know d_{BA} and F_\perp from above, so again M = F_\perp \cdot d_{BA} = F \cdot d
- By choosing component directions such that $d = 0$ to one of the components, we can simplify many problems.

- **EQUILIBRIUM** is the state where the resultant of the forces on a particle or a rigid is zero. There will be no rotation or translation. The forces are referred to as balanced.

- **NEWTON’S FIRST LAW**: If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

\[
R_x = \sum F_x = \sum H = 0 \quad R_y = \sum F_y = \sum V = 0 \quad \text{AND} \quad \sum M = 0
\]

**Moment Couples**

- **Moment Couple**: Two forces with equal magnitude, parallel lines of action and opposite sense tend to make our body rotate even though the sum of forces is 0. The sum of the moment of the forces about any axis is not zero.

\[
\sum M = -F \cdot d_2 + F \cdot d_1 = M
\]

\[
M = F(d_1 - d_2)
\]

\[
M = F \cdot d : \text{ moment of the couple (CW)}
\]

- $M$ does not depend on where $A$ is. $M$ depends on the perpendicular distance between the line of action of the parallel forces.

- $M$ for a couple (defined by $F$ and $d$) is a constant. And the sense (+/-) is obtained by observation.

- Just as there are equivalent moments (other values of $F$ and $d$ that result in $M$) there are equivalent couples. The magnitude is the same for different values of $F$ and resulting $d$ or different values of $d$ and resulting $F$. 

\[
M = F \cdot d \\
F = \frac{M}{d} \\
d = \frac{M}{F}
\]
Equivalent Force Systems

- Two systems of forces are equivalent if we can transform one of them into the other with:

  1.) replacing *two forces on a point* by their **resultant**
  2.) resolving a *force* into two components
  3.) canceling two equal and opposite forces on a point
  4.) attaching two equal and opposite forces to a point
  5.) moving a force along its line of action
  6.) replacing a force and moment on a point with a force on another (specific) point
  7.) replacing a force on point with a force and moment on another (specific) point
  *based on the parallelogram rule and the principle of transmissibility*

- The **size and direction** are important for a moment. The location on a body doesn’t matter because couples with the same moment will have the same effect on the rigid body.

Addition of Couples

- Couples can be added as *scalars*.

- Two couples can be *replaced* by a single couple with the magnitude of the algebraic sum of the two couples.

Resolution of a Force into a Force and a Couple

- The equivalent action of a force on a body can be reproduced by that force and a force couple:

  If we’d rather have F acting at A’ which isn’t in the line of action, we can instead add F and –F at A’ with no change of action by F. Now it becomes a couple of F separated by d and the force F moved to A’. The size is F·d=M

The couple can be represented by a moment symbol:

- Any force can be replaced by itself at another point and the moment equal to the force multiplied by the distance between the original line of action and new line of action.
Resolution of a Force into a Force and a Moment

- **Principle:** Any force \( F \) acting on a rigid body (say the one at A) may be moved to any given point \( A' \), provided that a couple \( M \) is added: the moment \( M \) of the couple must equal the moment of \( F \) (in its original position at A) about \( A' \).

IN REVERSE: A force \( F \) acting at \( A' \) and a couple \( M \) may be combined into a single resultant force \( F \) acting at \( A \) (a distance \( d \) away) where the moment of \( F \) about \( A' \) is equal to \( M \).

Resultant of Two Parallel Forces

- Gravity loads act in one direction, so we may have parallel forces on our structural elements. We know how to find the resultant force, but the location of the resultant must provide the equivalent total moment from each individual force.

\[ R = A + B \quad M_c = A \cdot a + B \cdot b = R \cdot x \Rightarrow x = \frac{A \cdot a + B \cdot b}{R} \]

Equilibrium for a Rigid Body

**FREE BODY DIAGRAM STEPS:**

1. Determine the free body of interest. (What body is in equilibrium?)

2. Detach the body from the ground and all other bodies ("free" it).

3. Indicate all external forces and moments which include:
   - action on the free body by the **supports & connections**
   - action on the free body by other bodies
   - the weigh effect (=force) of the free body itself (force due to gravity)
4. All forces and moments should be clearly marked with magnitudes and direction. The sense of forces and moments should be those acting on the body not by the body.

5. Dimensions/angles should be included for moment computations and force component computations.

6. Indicate the unknown angles, distances, forces or moments, such as those reactions or constraining forces where the body is supported or connected.

- **Reactions** can be categorized by the type of connections or supports. A reaction is a force with known line of action, or a force of unknown direction, or a moment. The line of action of the force or direction of the moment is directly related to the motion that is prevented.

- The line of action should be indicated on the FBD. The sense of direction is determined by the type of support. (Cables are in tension, etc…) *If the sense isn’t obvious, assume a sense.* When the reaction value comes out positive, the assumption was correct. When the reaction value comes out negative, the assumption was opposite the actual sense. *DON’T CHANGE THE ARROWS ON YOUR FBD OR SIGNS IN YOUR EQUATIONS.*

- With the 3 equations of equilibrium, there can be no more than 3 unknowns. *COUNT THE NUMBER OF UNKNOWN REACTIONS.*
### Reactions and Support Connections

#### Table 2-1 Supports for Coplanar Structures

<table>
<thead>
<tr>
<th>Type of Connection</th>
<th>Idealized Symbol</th>
<th>Reaction</th>
<th>Number of Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) light cable</td>
<td><img src="image1" alt="Light Cable" /></td>
<td><img src="image2" alt="Reaction" /></td>
<td>One unknown. The reaction is a force that acts in the direction of the cable or link.</td>
</tr>
<tr>
<td>weightless link</td>
<td><img src="image3" alt="Weightless Link" /></td>
<td><img src="image4" alt="Reaction" /></td>
<td>One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.</td>
</tr>
<tr>
<td>rollers</td>
<td><img src="image5" alt="Rollers" /></td>
<td><img src="image6" alt="Reaction" /></td>
<td>One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.</td>
</tr>
<tr>
<td>rocker</td>
<td><img src="image7" alt="Rocker" /></td>
<td><img src="image8" alt="Reaction" /></td>
<td>One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.</td>
</tr>
<tr>
<td>smooth contacting surface</td>
<td><img src="image9" alt="Smooth Contacting Surface" /></td>
<td><img src="image10" alt="Reaction" /></td>
<td>One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.</td>
</tr>
<tr>
<td>smooth pin-connected collar</td>
<td><img src="image11" alt="Smooth Pin-Connected Collar" /></td>
<td><img src="image12" alt="Reaction" /></td>
<td>One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.</td>
</tr>
<tr>
<td>smooth pin or hinge</td>
<td><img src="image13" alt="Smooth Pin or Hinge" /></td>
<td><img src="image14" alt="Reactions" /></td>
<td>Two unknowns. The reactions are two force components.</td>
</tr>
<tr>
<td>slider</td>
<td><img src="image15" alt="Slider" /></td>
<td><img src="image16" alt="Reactions" /></td>
<td>Two unknowns. The reactions are a force and a moment.</td>
</tr>
<tr>
<td>fixed-connected collar</td>
<td><img src="image17" alt="Fixed-Connected Collar" /></td>
<td><img src="image18" alt="Reactions" /></td>
<td>Three unknowns. The reactions are the moment and the two force components.</td>
</tr>
</tbody>
</table>

---

**Ref:** Structural Analysis, 4th ed., R.C. Hibbeler
Load, Support Conditions & Reactions for Beams

Types of Forces

Concentrated – single load at one point
Distributed – loading spread over a distance or area

Types of supports:

- Statically determinate
  (number of unknowns \( \leq \) number of equilibrium equations)

- Statically indeterminate: (need more equations from somewhere

Distributed Loads

Distributed loads may be replaced by concentrated loads acting through the balance/center of the distribution or load area: THIS IS AN EQUIVALENT FORCE SYSTEM.

- \( w \) is the symbol used to describe the load per unit length.

- \( W \) is the symbol used to describe the total load.
Example 1 (pg 31) Verify that the beam reactions satisfy rotational equilibrium for the rigid body. Check the summation of moments at points A, B & C.

Example 2 (pg 32)

**Example 1.** A simple beam 20 ft long has three concentrated loads, as indicated in Figure 3.6. Find the magnitudes of the reactions.
Example 3 (pg 34)

Example 8. A simple beam 16 ft long carries the loading shown in Figure 3.7a. Find the reactions.

*Note: The figure has been changed to show \( w \) rather than \( W \).

Example 4

Determine the support reactions developed at \( A \) for a cantilever beam supporting a trapezoidal load and a point load (horizontal) on the bar at the free end.
Example 5

**Figure 3.45** Cantilevered beam with point force at the end.

**Figure 3.47** Resolved horizontal reaction.

**Example Problem 3**

Find the reactions for the beam in Figure 3.45.

As a first step:

- Resolve the 10 K applied force into its components.
- Replace the support by the forces and moment that it exerts on the beam (see Figure 3.46).

All of the reactions (i.e., the two reactive forces and the reactive moment) have been drawn in the directions conventionally designated as positive. This is an assumption that can be made without thinking. The machinery of the mathematics will tell whether or not these assumptions are correct. For example, simple inspection will reveal that $M$ must be in the opposite direction shown to maintain equilibrium. However, there is no need to be concerned about that at this point in working the problem. The fact that one can make an arbitrary assumption and rely on the mathematics to tell whether it was correct is fortunate, inasmuch as there will be many complex structural situations in which determining the actual direction of a force a moment by simple inspection will be impossible. The way one is going about the problem to this point is exactly the way that any computer program would solve it.

Next, begin applying the equations of equilibrium:

$$
\sum P_x = 0 \pm \\
\sum P_y = 0 \mp \\
\sum M = 0 \perp
$$

**Figure 3.46** Resolving applied force into its components.

Applying the equation for the horizontal forces,

$$
\sum P_x = 0 \pm , \text{ yields } A_x - 6 K = 0,
$$

which solves to give $A_x = 6 K$.

The *plus* sign means, **Yes, the original direction assumed for $A_x$ was correct; that is, the actual force is to the right.**

With this new piece of information, the diagram now looks like that in Figure 3.47.

Applying the equation for the vertical forces,

$$
\sum P_y = 0 \mp, \text{ yields } A_y - 8 K = 0,
$$

which solves to give $A_y = 8 K$.

The *plus* sign means, **Yes, the original direction assumed for $A_y$ was correct; that is, the actual force is upward.**

With this new piece of information, the diagram now looks like that in Figure 3.48.

Summing the moments about the point of support,

$$
\sum M = 0 \perp \text{ yields } M + 8 K(20 \text{ ft}) = 0,
$$

which solves to give $M = 160 K \text{ ft}$.

The *minus* sign means, **No, the original direction assumed for $M$ was wrong; that is, the actual moment is counterclockwise, rather than the clockwise direction assumed in the original drawing.**

**Figure 3.48** Resolved vertical reaction.
Beam Structures and Internal Forces

Notation:

- $a$ = algebraic quantity, as is $b$, $c$, $d$
- $A$ = name for area
- $b$ = intercept of a straight line
- $d$ = calculus symbol for differentiation
- $(C)$ = shorthand for compression
- $F$ = name for force vectors, as is $P$, $F'$, $P'$
- $F_x$ = force component in the x direction
- $F_y$ = force component in the y direction
- $FBD$ = free body diagram
- $L$ = beam span length
- $m$ = slope of a straight line
- $M$ = internal bending moment
- $M(x)$ = internal bending moment as a function of distance $x$

- $R$ = name for reaction force vector
- $(T)$ = shorthand for tension
- $V$ = internal shear force
- $V(x)$ = internal shear force as a function of distance $x$
- $W$ = name for total force due to distributed load
- $w$ = name for distributed load
- $x$ = horizontal distance
- $y$ = vertical distance
- $\Delta$ = calculus symbol for small quantity
- $\Sigma$ = summation symbol

- $\int$ = symbol for integration

- **BEAMS**
  - Important type of structural members (floors, bridges, roofs)
  - Usually long, straight and rectangular
  - Have loads that are usually perpendicular applied at points along the length

- **Internal Forces**
  - **Internal forces** are those that hold the parts of the member together for equilibrium
    - Truss members:

- For any member:
  
  - $F =$ internal axial force
    (perpendicular to cut across section)
  
  - $V =$ internal shear force
    (parallel to cut across section)
  
  - $M =$ internal bending moment
**Support Conditions & Loading**

- Most often loads are perpendicular to the beam and cause only internal shear forces and bending moments.
- Knowing the internal forces and moments is necessary when designing beam size & shape to resist those loads.
- Types of loads
  - Concentrated – single load, single moment
  - Distributed – loading spread over a distance, uniform or non-uniform.
- Types of supports
  - Statically determinate: simply supported, cantilever, overhang (number of unknowns < number of equilibrium equations)
  - Statically indeterminate: continuous, fixed-roller, fixed-fixed (number of unknowns < number of equilibrium equations)

**Sign Conventions for Internal Shear and Bending Moment**

When $\sum F_y$ excluding $V$ on the left hand side (LHS) section is positive, $V$ will direct down and is considered POSITIVE.

When $\sum M$ excluding $M$ about the cut on the left hand side (LHS) section causes a smile which could hold water (curl upward), $M$ will be counter clockwise (+) and is considered POSITIVE.

On the deflected shape of a beam, the point where the shape changes from smile up to frown is called the inflection point. The bending moment value at this point is zero.
Shear and Bending Moment Diagrams

The plot of shear and bending moment as they vary across a beam length are extremely important design tools: $V(x)$ is plotted on the y axis of the shear diagram, $M(x)$ is plotted on the y axis of the moment diagram.

The load diagram is essentially the free body diagram of the beam with the actual loading (not the equivalent of distributed loads.)

Maximum Shear and Bending – The maximum value, regardless of sign, is important for design.

The Equilibrium Method

Isolate FDB sections at significant points along the beam and determine $V$ and $M$ at the cut section. The values for $V$ and $M$ can also be written in equation format as functions of the distance to the cut section.

Important Places for FBD cuts
- at supports
- at concentrated loads
- at start and end of distributed loads
- distributed loads between forces or supports with reaction forces
- at concentrated moments

The Semigraphical Method

Relationships exist between the loading and shear diagrams, and between the shear and bending diagrams.

Knowing the area of the loading gives the change in shear ($V$).

Knowing the area of the shear gives the change in bending moment ($M$).

Concentrated loads and moments cause a vertical jump in the diagram.

$$\frac{AV}{Ax} = \frac{dV}{dx} = -w$$  (the negative shows it is down because we give $w$ a positive value)

$$V_D - V_C = \int_{x_C}^{x_D} wdx = \text{the area under the load curve between C & D}$$

*These shear formulas are NOT VALID at discontinuities like concentrated loads*
The MAXIMUM BENDING MOMENT from a curve that is continuous can be found when the slope is zero \( \left( \frac{dM}{dx} = 0 \right) \), which is when the value of the shear is 0.

**Basic Curve Relationships (from calculus) for \( y(x) \)**

<table>
<thead>
<tr>
<th>Curve Type</th>
<th>Equation</th>
<th>Area (change in shear)</th>
<th>Derivative of Area</th>
<th>Resulting in a:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal Line</td>
<td>( y = b ) (constant)</td>
<td>( b \cdot x )</td>
<td>( \frac{dy}{dx} \cdot \Delta x )</td>
<td>( \Delta y \cdot \Delta x = 0 )</td>
</tr>
<tr>
<td>Sloped Line</td>
<td>( y = mx + b )</td>
<td>( \frac{\Delta y \cdot \Delta x}{2} )</td>
<td>( \frac{d}{dx} )</td>
<td>( \Delta x \cdot \Delta y )</td>
</tr>
<tr>
<td>Parabolic Curve</td>
<td>( y = ax^2 + b )</td>
<td>( \frac{\Delta y \cdot \Delta x}{3} )</td>
<td>( \frac{d}{dx} \frac{dy}{dx} )</td>
<td>( \Delta x \cdot \Delta y )</td>
</tr>
<tr>
<td>3rd Degree Curve</td>
<td>( y = ax^3 + bx^2 + cx + d )</td>
<td></td>
<td></td>
<td>( \Delta x \cdot \Delta y )</td>
</tr>
</tbody>
</table>

**Free Software Site:** [http://www.rekenwonder.com/atlas.htm](http://www.rekenwonder.com/atlas.htm)

**BASIC PROCEDURE:**

1. Find all support forces.

\( V \) diagram:

2. At free ends and at simply supported ends, the shear will have a zero value.

3. At the left support, the shear will equal the reaction force.
4. The shear will not change in x until there is another load, where the shear is reduced if the load is negative. If there is a distributed load, the change in shear is the area under the loading.

5. At the right support, the reaction is treated just like the loads of step 4.

6. At the free end, the shear should go to zero.

*M diagram:*

7. At free ends and at simply supported ends, the moment will have a zero value.

8. At the left support, the moment will equal the reaction moment (if there is one).

9. The moment will not change in x until there is another load or applied moment, where the moment is reduced if the applied moment is negative. If there is a value for shear on the V diagram, the change in moment is the area under the shear diagram.

   For a triangle in the shear diagram, the width will equal the height \( \div w! \)

10. At the right support, the moment reaction is treated just like the moments of step 9.

11. At the free end, the moment should go to zero.

**Parabolic Curve Shapes Based on Triangle Orientation**

In order to tell if a parabola curves “up” or “down” from a triangular area in the preceding diagram, the orientation of the triangle is used as a reference.

**Geometry of Right Triangles**

Similar triangles show that four triangles, each with \( \frac{1}{4} \) the area of the large triangle, fit within the large triangle. This means that \( \frac{3}{4} \) of the area is on one side of the triangle, if a line is drawn though the middle of the base, and \( \frac{1}{4} \) of the area is on the other side.

By how a triangle is oriented, we can determine the curve shape in the next diagram.

**CASE 1:** Positive triangle with fat side to the left.

**CASE 2:** Positive triangle with fat side to the right.
CASE 3: *Negative* triangle with fat side to the left.

CASE 4: *Negative* triangle with fat side to the right.
Example 1 (pg 199) *Equilibrium Method*

*Example 3.* The load diagram in Figure 3.14 shows a simple beam with two concentrated loads. Draw the shear and bending moment diagrams.
Example 2 (pg 101) *Equilibrium Method*

**Example 4.** Draw the shear and bending moment diagrams for the beam shown in Figure 3.15, which carries a uniformly distributed load of 400 lb per lin ft and a concentrated load of 21,000 lb located 4 ft from \( R_1 \).
Example 3 (pg 102) Semi-Graphical Method

Example 5. The load diagram in Figure 3.16 shows a beam with a concentrated load of 7000 lb, applied 4 ft from the left reaction, and a uniformly distributed load of 800 lb per lin ft extending over the full span. Compute the maximum bending moment on the beam.
Example 4 (pg 106) *Semi-Graphical Method*

*Example 7.* Compute the maximum bending moment for the overhanging beam shown in Figure 3.19.
Example 5  Semi-Graphical Method

For a cantilever beam with an upturned end, draw the load, shear, and moment diagrams.
Example 6 (changed from pg 108) Semi-Graphical Method

Example 8. Draw shear and bending moment diagrams for the beam in Figure 3.21b, which carries a uniformly distributed load of 500 lb/ft over its full length. It also has a concentrated load of 8 kips 2.5 ft from the free end.

SOLUTION:

Determine the reactions:

\[ \sum F_x = R_x = 0 \quad R_x = 0 \text{ k} \]
\[ \sum F_y = -8k - (500 \text{ lb/ft})(10 \text{ ft})(1 \text{ k/1000 lb}) + R_y = 0 \quad R_y = 13 \text{ k} \]
\[ \sum M = -(8k)(7.5 \text{ ft}) - (5k)(5 \text{ ft}) + M_R = 0 \quad M_R = 85 \text{ k-ft} \]

Draw the load diagram with the distributed load as given with the reactions.

Shear Diagram:

Label the load areas and calculate:

Area I = (-0.5 k/ft)(2.5 ft) = -1.25 k
Area II = (-0.5 k/ft)(7.5 ft) = -3.75 k

\[ V_A = 0 \]
\[ V_B = V_A + \text{Area I} = 0 - 1.25 k = -1.25 k \text{ and} \]
\[ V_B = V_B + \text{force at B} = -1.25 k - 8 k = -9.25 k \]
\[ V_C = V_B + \text{Area II} = -9.25 k - 3.75 k = -13 k \text{ and} \]
\[ V_C = V_C + \text{force at C} = -13 k + 13 k = 0 k \]

Bending Moment Diagram:

Label the load areas and calculate:

Area III = (-1.25 k)(2.5 ft)/2 = -1.5625 k-ft
Area IV = (-9.25 k)(7.5 ft) = -69.375 k-ft
Area V = (-13 - 9.25 k)(7.5 ft)/2 = -14.0625 k-ft

\[ M_A = 0 \]
\[ M_B = M_A + \text{Area III} = 0 - 1.5625 \text{ k-ft} = -1.5625 \text{ k-ft} \]
\[ M_C = M_B + \text{Area IV} + \text{Area V} = -1.5625 \text{ k-ft} - 69.375 \text{ k-ft} - 14.0625 \text{ k-ft} = -85 \text{ k-ft} \text{ and} \]
\[ M_C = M_C + \text{moment at C} = -85 \text{ k-ft} + 85 \text{ k-ft} = 0 \text{ k-ft} \]

The beam also has a concentrated load of 8 kips 2.5 ft from the free end. It also has a distributed load of 500 lb/ft over its full length.
Beam Analysis using Multiframe

1. The software is on the teaching computers in the College of Architecture in Programs under the Windows Start menu. Multiframe is under the Bentley Engineering menu. It is also available at the Open Access Labs (OAL) and the Virtual OAL.

2. There are tutorials available online at [http://www.daystarsoftware.com/support/mftutorials](http://www.daystarsoftware.com/support/mftutorials) that list the tasks and order in greater detail. The first task is to define the unit system:
   - Choose Units… from the View menu. Unit sets are available, but specific units can also be selected by double clicking on a unit or format and making a selection from the menu.

3. To see the scale of the geometry, a grid option is available:
   - Choose Grid… from the View menu

4. To create the geometry, you must be in the Frame window (default). The symbol is the frame in the window toolbar:

   ![Frame symbol]

   The Member toolbar shows ways to create members:

   ![Member toolbar]

   The Generate toolbar has convenient tools to create typical structural shapes:

   ![Generate toolbar]

   - To create a beam with supports at one or both ends, use the add member button:
- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window.

- To create a beam with supports NOT at the ends, use the add connected members button to create segments between supports and ends.

- Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window. The ESC button will end the segmented drawing.

- The geometry can be set precisely by selecting the beam member, bringing up the specific menu (right click), choosing Member Properties to set the length.

- The support types can be set by selecting the joint (drag) and using the Joint Toolbar (pin shown), or the Frame / Joint Restraint ... menu (right click).

  NOTE: If the support appears at both ends of the beam, you had the beam selected rather than the joint. Select the joint to change the support for and right click to select the joint restraints menu or select the correct support on the joint toolbar.

5. All members must have sections assigned (see section 6.) in order to calculate reactions and deflections. To use a standard steel section proceed to step 6. For custom sections, the section information must be entered. To define a section:

- Choose Edit Sections / Add Section… from the Edit menu

- Type a name for your new section

- Choose group Frame from the group names provided so that the section will remain with the file data

- Choose a shape. The Flat Bar shape is a rectangular section.

- Enter the cross section data.
Table values 1-9 must have values for a Flat Bar, but not all are used for every analysis. A recommendation is to put the value of 1 for those properties you don’t know or care about. Properties like $t_f$, $t_w$, etc. refer to wide flange sections.

- Answer any query. If the message says there is an error, the section will not be created until the error is corrected.

6. The standard sections library loaded is for the United States. If another section library is needed, use the Open Sections Library... command under the file menu, choose the library folder, and select the SectionsLibrary.slb file. Select the members (drag to make bold) and assign sections with the Section button on the Member toolbar:

- Choose the group name and section name:

![Select Section](image)

7. The beam geometry is complete, and in order to define the load conditions you must be in the Load window represented by the green arrow:

8. The Load toolbar allows a joint to be loaded with a force or a moment in global coordinates, shown by the first two buttons after the display numbers button. It allows a member to be loaded with a distributed load, concentrated load or moment (next three buttons) in global coordinates, as well as loading with distributed or single force or moment in the local coordinate system (next three buttons). It allows a load panel to be loaded with a distributed load in global or local coordinates (last two buttons).

- Choose the member to be loaded (drag) and select the load type (here shown for global distributed loading):
• Choose the distribution type and direction. Note that the arrow shown is the direction of the loading. There is no need to put in negative values for downward loading.

• Enter the values of the load and distances (if any). Distances can be entered as a function of the length, i.e. L/2, L/4...

**NOTE: Do not put support reactions as applied loads. The analysis will determine the reaction values.**

Multiframe will automatically generate a grouping called a Load Case named Load Case 1 when a load is created. All additional loads will be added to this load case unless a new load case is defined (Add case under the Case menu).

9. In order to run the analysis after the geometry, member properties and loading has been defined:

• Choose Linear from the Analyze menu

10. If the analysis is successful, you can view the results in the Plot window represented by the red moment diagram:

11. The Plot toolbar allows the numerical values to be shown (1.0 button), the reaction arrows to be shown (brown up arrow) and reaction moments to be shown (brown curved arrow):

• To show the moment diagram, Choose the red Moment button

• To show the shear diagram, Choose the green Shear button

• To show the axial force diagram, Choose the purple Axial Force button

• To show the deflection diagram, Choose the blue Deflection button

• To animate the deflection diagram, Choose Animate... from the Display menu. You can also save the animation to a .avi file by checking the box.
• To plot the bending moment on the “top” choose Preferences from the Edit menu and under the Presentation tab Draw moments on the compression face

• To see exact values of shear, moment and deflection, double click on the member and move the vertical cross hair with the mouse. The ESC key will return you to the window.

12. The Data window (D) allows you to view all data “entered” for the geometry, sections and loading. These values can be edited.

13. The Results window (R) allows you to view all results of the analysis including displacements, reactions, member forces (actions) and stresses. These values can be cut and pasted into other Windows programs such as Word or Excel.

NOTE: Px’ refers to the axial load (P) in the local axis x direction (x’). Vy’ refers to the shear perpendicular to the local x axis, and Mz’ refers to the bending moment.

14. To save the file Choose Save from the File menu.

15. To load an existing file Choose Open... from the File menu.

16. To print a plot Choose Print Window... from the File menu. As an alternative, you may copy the plot (Ctrl+c) and paste it in a word processing document (Ctrl+v).
1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load \( w l \)

\( R = V \) \( = \frac{w l}{2} \)

\( V_x \) \( = \frac{w (l/2 - x)}{2} \)

\( M_{max.} \) (at center) \( = \frac{w x^2}{8} \)

\( M_x \) \( = \frac{w x^2}{2} (l-x) \)

\( \Delta_{max.} \) (at center) \( = \frac{5w l^4}{384EI} \)

\( \Delta_x \) \( = \frac{w x^2}{24EI} \left( l^3 - 2lx^2 + x^4 \right) \)

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END

Total Equiv. Uniform Load \( = \frac{16W}{9\sqrt{3}} = 1.0264W \)

\( R_1 = V_1 \) \( = \frac{W}{3} \)

\( R_2 = V_2 \) \( = \frac{2W}{3} - \frac{Wx}{24EI} \)

\( V_x \) \( = \frac{W}{3} - \frac{Wx}{24EI} \)

\( M_{max.} \) (at \( x = \frac{1}{\sqrt{3}} \)) \( = \frac{W}{1283} \)

\( M_x \) \( = \frac{Wx^2}{81EI} \)

\( \Delta_{max.} \) (at \( x = \frac{1}{\sqrt{3}} \)) \( = \frac{0.01309}{3a^2} \)

\( \Delta_x \) \( = \frac{Wx^2}{180EI} \left( 3a^4 - 10a^2x^2 + 7x^4 \right) \)

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER

Total Equiv. Uniform Load \( = \frac{4W}{3} \)

\( R = V \) \( = \frac{W}{2} \)

\( V_x \) \( \text{(when } x < \frac{l}{2} \text{)} \) \( = \frac{W}{2l} \left( l^2 - 4x^2 \right) \)

\( M_{max.} \) (at center) \( = \frac{Wl}{6} \)

\( M_x \) \( \text{(when } x < \frac{l}{2} \text{)} \) \( = \frac{Wx^2}{2} \left( \frac{3x}{2} \right)^2 \)

\( \Delta_{max.} \) (at center) \( = \frac{Wl}{60EI} \)

\( \Delta_x \) \( \text{(when } x < \frac{l}{2} \text{)} \) \( = \frac{Wx^2}{480EI} \left( 5x^4 - 4x^2 \right) \)

4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED

\( R_1 = V_1 \) \( \text{(max. when } x < a \text{)} \) \( = \frac{wa}{2l} \left( 2l - b \right) \)

\( R_2 = V_2 \) \( \text{(max. when } x > a \text{)} \) \( = \frac{wa}{2l} \left( 2b - l \right) \)

\( V_x \) \( \text{(when } x > a \text{ and } (a+b) \) \( = R_1 - w (x-a) \)

\( M_{max.} \) (at \( \frac{x}{x+a} \)) \( = R_1 \left( a + R_1 \right) \)

\( M_x \) \( \text{(when } x < a \text{)} \) \( = R_1 x \)

\( M_x \) \( \text{(when } x > a \text{ and } (a+b) \) \( = R_1 x - \frac{w}{2} (x-a)^2 \)

\( M_x \) \( \text{(when } x > (a+b) \) \( = R_2 \left( l-x \right) \)

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END

\( R_1 = V_1 \) \( \text{max.} \) \( = \frac{wa}{2l} \left( 2l - a \right) \)

\( R_2 = V_2 \) \( = \frac{wa}{2l} \left( 2l \right) \)

\( V_x \) \( \text{(when } x < a \text{)} \) \( = R_1 - wx \)

\( M_{max.} \) (at \( x = \frac{R_1}{w} \)) \( = \frac{R_1^2}{2w} \)

\( M_x \) \( \text{(when } x < a \text{)} \) \( = \frac{wx^2}{2} \)

\( M_x \) \( \text{(when } x > a \text{)} \) \( = \frac{wx^2}{24EI} \left( a^2 (2l-a) - 2ax^2 (2l-a) + x^4 \right) \)

\( M_x \) \( \text{(when } x > a \text{)} \) \( = \frac{wx^2}{24EI} \left( l-x \right) \left( 4l^2 - 2x^2 - a^2 \right) \)

6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END

\( R_1 = V_1 \) \( = \frac{wa}{2l} \left( 2l - a \right) + \frac{wa^3}{2l} \)

\( R_2 = V_2 \) \( = \frac{wa}{2l} \left( 2l \right) + \frac{wa^3}{2l} \)

\( V_x \) \( \text{(when } x < a \text{)} \) \( = \frac{wa}{2l} \left( c - w_1 a^2 \right) \)

\( V_x \) \( \text{(when } x > a \text{ and } (a+b) \) \( = R_1 - w_1 x \)

\( V_x \) \( \text{(when } x > (a+b) \) \( = \frac{w_1 x^2}{2} \)

\( M_{max.} \) (at \( x = \frac{R_1}{w} \)) \( \text{when } R_2 < w_2 \) \( = \frac{R_1^2}{2w} \)

\( M_x \) \( \text{(when } x < a \text{)} \) \( = \frac{wx^2}{2} \)

\( M_x \) \( \text{(when } x > a \text{ and } (a+b) \) \( = R_1 x - \frac{w}{2} (x-a)^2 \)

\( M_x \) \( \text{(when } x > (a+b) \) \( = R_2 \left( l-x \right) - \frac{w}{2} (l-x)^2 \)
7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load \( = 2P \)

\[
R = V = \frac{P}{2}
\]

\[
M_{\text{max.}} \text{ (at point of load)} = \frac{Pf}{4}
\]

\[
M_{x} \quad \text{(when } x < \frac{L}{2} \text{)} = \frac{Px}{2}
\]

\[
\Delta_{\text{max.}} \text{ (at point of load)} = \frac{Pf^2}{48EI}
\]

\[
\Delta_{x} \quad \text{(when } x < \frac{L}{2} \text{)} = \frac{Px}{48EI} \left(3f^2 - 4x^3\right)
\]

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT

Total Equiv. Uniform Load \( = \frac{8Pab}{l} \)

\[
R_{1} = V_{1} \text{ (max. when } a < b \text{)} = \frac{Pa}{l}
\]

\[
R_{2} = V_{2} \text{ (max. when } a > b \text{)} = \frac{Pb}{l}
\]

\[
M_{\text{max.}} \text{ (at point of load)} = \frac{Pab}{l}
\]

\[
M_{x} \quad \text{(when } x < a \text{)} = \frac{Pbx}{l}
\]

\[
\Delta_{\text{max.}} \text{ (at } x = \frac{a(a + 2b)}{3l} \text{ when } a > b \text{)} = \frac{Pab(a + 2b)^2 \sqrt{3a(a + 2b)}}{27EI}
\]

\[
\Delta_{x} \quad \text{(at point of load)} = \frac{Pa^2 b}{3EI l}
\]

\[
\Delta_{x} \quad \text{(when } x < a \text{)} = \frac{Pbx}{6EI l} \left(1 - b^2 - x^3\right)
\]

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED

Total Equiv. Uniform Load \( = \frac{8Pa}{l} \)

\[
R = V = \frac{P}{8}
\]

\[
M_{\text{max.}} \text{ (between loads)} = \frac{P}{8}
\]

\[
M_{x} \quad \text{(when } x < a \text{)} = \frac{P}{8}
\]

\[
\Delta_{\text{max.}} \text{ (at center)} = \frac{Pa}{24EI} \left(3a^2 - 4a^3\right)
\]

\[
\Delta_{x} \quad \text{(when } x < a \text{)} = \frac{P}{6EI} \left(3a - 3a^2 - a^3\right)
\]

\[
\Delta_{x} \quad \text{(when } x > a \text{ and } (l - a) \text{)} = \frac{P}{6EI} \left(31x - 3x^2 - a^3\right)
\]

10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED

\[
R_{1} = V_{1} \text{ (max. when } a < b \text{)} = \frac{P}{l} \left(1 - a + b\right)
\]

\[
R_{2} = V_{2} \text{ (max. when } a > b \text{)} = \frac{P}{l} \left(1 - b - a\right)
\]

\[
V_{x} \quad \text{(when } x > a \text{ and } (l - b) \text{)} = \frac{P}{l} \left(b - a\right)
\]

\[
M_{1} \quad \text{(max. when } a > b \text{)} = R_{1}a
\]

\[
M_{2} \quad \text{(max. when } a < b \text{)} = R_{2}b
\]

\[
M_{x} \quad \text{(when } x < a \text{)} = R_{1}x
\]

\[
M_{x} \quad \text{(when } x > a \text{ and } (l - b) \text{)} = R_{1}x - P(x - a)
\]

11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED

\[
R_{1} = V_{1} \quad \text{(max. when } R_{1} < P_{1} \text{)} \quad = \frac{P_{1} \left(l - x \right) + P_{2}b}{l}
\]

\[
R_{2} = V_{2} \quad \text{(max. when } R_{2} < P_{2} \text{)} \quad = \frac{P_{2}a + P_{2} \left(l - b\right)}{l}
\]

\[
V_{x} \quad \text{(when } x > a \text{ and } (l - b) \text{)} \quad = R_{1} - P_{1}
\]

\[
M_{1} \quad \text{(max. when } R_{1} < P_{1} \text{)} \quad = R_{1}a
\]

\[
M_{2} \quad \text{(max. when } R_{2} < P_{2} \text{)} \quad = R_{2}b
\]

\[
M_{x} \quad \text{(when } x < a \text{)} \quad = R_{1}x
\]

\[
M_{x} \quad \text{(when } x > a \text{ and } (l - b) \text{)} \quad = R_{1}x - P_{1}(x - a)
\]

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load \( = \frac{wl}{8} \)

\[
R_{1} = V_{1} \quad \text{(max. when } x = \frac{3}{8} \text{)} \quad = \frac{5wl}{8}
\]

\[
V_{x} \quad \text{(at } x = \frac{3}{8} \text{)} \quad = \frac{9}{128} \frac{w}{8}
\]

\[
M_{1} \quad \text{(at } x = \frac{3}{8} \text{)} \quad = \frac{R_{1}x - \frac{3w}{8}}{185EI}
\]

\[
M_{x} \quad \text{(at } x = \frac{1}{16} \left(1 + \sqrt{33}\right) = 0.4215f) \quad = \frac{wL^4}{48EI}
\]

\[
\Delta_{\text{max.}} \quad \text{(at } x = \frac{1}{16} \left(1 + \sqrt{33}\right) = 0.4215f) \quad = \frac{wL^4}{48EI}
\]

\[
\Delta_{x} \quad \text{(at } x = \frac{1}{16} \left(1 + \sqrt{33}\right) = 0.4215f) \quad = \frac{wL^4}{48EI}
\]
13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER — CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load = \( \frac{3P}{2} \)

\( R_1 = V_1 = \frac{5P}{16} \)

\( R_2 = V_2 \text{ max.} = \frac{11P}{16} \)

\( M_{\text{max.}} \text{ (at fixed end)} = \frac{3P}{16} \)

\( M_1 \text{ (at point of load)} = \frac{3P}{32} \)

\( M_x \text{ (when } x < \frac{1}{2} \) = \( \frac{P}{2} \left( \frac{1}{2} - \frac{11x}{16} \right) \)

\( \Delta_{\text{max.}} \text{ (at } x = \frac{1}{4} \text{)} = \frac{P}{48EI} \left( \frac{1}{5} - 0.4472 \right) = \frac{P}{48EI} \left( \frac{1}{5} - 0.4472 \right) = \frac{0.09317}{EI} \)

\( \Delta_x \text{ (when } x < \frac{1}{2} \) = \( \frac{7P}{768EI} \left( \frac{3}{8} - 5x^2 \right) \)

\( \Delta_x \text{ (when } x > \frac{1}{2} \) = \( \frac{P}{96EI} \left( \frac{1}{4} - x \right) \left( 11x - \frac{21}{2} \right) \)

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER — CONCENTRATED LOAD AT ANY POINT

\( R_1 = V_1 = \frac{Pb}{2a} \left( a + 2i \right) \)

\( R_2 = V_2 = \frac{Pb}{2a} \left( 3i^2 - a^2 \right) \)

\( M_1 \text{ (at point of load)} = \frac{Pa}{2a} \left( a + i \right) \)

\( M_2 \text{ (at fixed end)} = \frac{Pa}{2a} \left( a + i \right) \)

\( M_x \text{ (when } x < a \) = \( \frac{Pa}{2a} \left( 3i^2 - a^2 \right) \)

\( M_x \text{ (when } x > a \) = \( \frac{Pa}{2a} \left( a - i \right) \left( a - 2i \right) \)

\( \Delta_{\text{max.}} \text{ (when } a < 0.414a \text{ at } x = \frac{1}{4} \frac{a}{1.5-2a} \) = \( \frac{Pa}{3EI} \left( \frac{3a^2-a^2}{2} \right) \)

\( \Delta_{\text{max.}} \text{ (when } a > 0.414a \text{ at } x = \frac{2a}{1.5-a} \) = \( \frac{Pa}{8EI} \left( 3a^2-4a \right) \)

\( \Delta_a \text{ (at point of load)} = \frac{Pa}{24EI} \left( 3i + a \right) \)

\( \Delta_x \text{ (when } x < \frac{1}{2} \) = \( \frac{Pb}{12EI} \left( 3a^2-2ax^2-ax^2 \right) \)

\( \Delta_x \text{ (when } x > \frac{1}{2} \) = \( \frac{Pa}{12EI} \left( 1-x \right)^2 \left( 3ax^2-a^2x-2ax \right) \)

15. BEAM FIXED AT BOTH ENDS — UNIFORMLY DISTRIBUTED LOADS

Total Equiv. Uniform Load = \( \frac{2wl}{3} \)

\( R = V = \frac{w}{2} \left( \frac{l}{2} - x \right) \)

\( V_x \text{ (at ends)} = \frac{w}{l} \)

\( M_{\text{max.}} \text{ (at center)} = \frac{w}{24} \)

\( M_x \text{ (at center)} = \frac{w}{12} \left( 6lx - l^2 - 6x^2 \right) \)

\( \Delta_{\text{max.}} \text{ (at center)} = \frac{3EI}{384EI} \)

\( \Delta_x = \frac{w^2}{24EI} \left( 1-x \right)^2 \)

16. BEAM FIXED AT BOTH ENDS — CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load = \( P \)

\( R = V = \frac{P}{l} \)

\( M_{\text{max.}} \text{ (at center and ends)} = \frac{P}{l} \)

\( M_x \text{ (when } x < \frac{1}{2} \) = \( \frac{P}{8} \left( 4x-l \right) \)

\( \Delta_{\text{max.}} \text{ (at center)} = \frac{P}{192EI} \)

\( \Delta_x \text{ (when } x < \frac{1}{2} \) = \( \frac{Pb}{48EI} \left( 3l-4x \right) \)

17. BEAM FIXED AT BOTH ENDS — CONCENTRATED LOAD AT ANY POINT

\( R_1 = V_1 \text{ (max. when } a < b \) = \( \frac{Pb}{l} \left( 3a + b \right) \)

\( R_2 = V_2 \text{ (max. when } a > b \) = \( \frac{Pa}{l} \left( a + 3b \right) \)

\( M_1 \text{ (max. when } a < b \) = \( \frac{Pab}{l} \)

\( M_2 \text{ (max. when } a > b \) = \( \frac{Pa}{l} \)

\( M_a \text{ (at point of load)} = \frac{Pab}{l} \)

\( M_x \text{ (when } x < a \) = \( \frac{Pab}{l} \)

\( M_x \text{ (when } x > a \) = \( \frac{Pab}{l} \)

\( \Delta_{\text{max.}} \text{ (when } a > b \text{ at } x = \frac{2a}{3a+b} \) = \( \frac{Pb}{3EI} \left( 3a + b \right) \)

\( \Delta_{\text{max.}} \text{ (when } a < b \text{ at } x = \frac{a}{3a+b} \) = \( \frac{Pb}{6EI} \left( 3a + b - ax \right) \)
18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END

Total Equiv. Uniform Load \( = \frac{8}{3} W \)

\[ W = \frac{wl}{2} \]

Total Equiv. Uniform Load \( = wL \)

R \( = V \)

\[ V = \frac{wL^2}{12} \]

M \( \max. (\text{at fixed end}) \) \( = \frac{wL^2}{3} \)

\[ \delta \max. (\text{at free end}) = \frac{wL^2}{15EI} \]

\[ \delta_x = \frac{W}{60EI} \left(x^4 - 5/4x^4 + 4/5x^2\right) \]

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load \( = 4wl \)

R \( = V \)

\[ V = \frac{wL}{2} \]

M \( \max. (\text{at fixed end}) \) \( = \frac{wL^2}{2} \)

\[ \delta \max. (\text{at free end}) = \frac{wL^4}{8EI} \]

\[ \delta_x = \frac{w}{24EI} \left(x^4 - 4/3x^4 + 3/4\right) \]

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load \( = \frac{8}{3}wl \)

R \( = V \)

\[ V = \frac{wl}{2} \]

M \( \max. (\text{at fixed end}) \) \( = \frac{wl^2}{3} \)

M \( \max. (\text{at deflected end}) \) \( = \frac{wl^2}{6} \)

\[ \delta \max. (\text{at deflected end}) = \frac{wl^4}{24EI} \]

\[ \delta_x = \frac{w}{24EI} \left(\frac{x^2}{3} - x^2 + 3x^2\right) \]

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT

Total Equiv. Uniform Load \( = \frac{8Pb}{l} \)

R \( = \frac{b}{2P} \)

M \( \max. (\text{at fixed end}) = Pb \)

M \( \max. (\text{when } x > a) = P(x - a) \)

\[ \delta \max. (\text{at free end}) = \frac{Pb^2}{6EI} (3l - b) \]

\[ \delta \max. (\text{at point of load}) = \frac{Pb^2}{3EI} \]

\[ \delta \max. (\text{when } x < a) = \frac{Pb^2}{6EI} (3l - 3x - b) \]

\[ \delta \max. (\text{when } x > a) = \frac{Pb^2}{6EI} (-3b - l + x) \]

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END

Total Equiv. Uniform Load \( = 8P \)

R \( = \frac{P}{l} \)

M \( \max. (\text{at fixed end}) = P/l \)

M \( \max. (\text{at deflected end}) = P/l \)

\[ \delta \max. (\text{at free end}) = \frac{P}{3EI} \]

\[ \delta \max. (\text{at point of load}) = \frac{P}{6EI} (2l - 3l^2 + x^2) \]

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END

Total Equiv. Uniform Load \( = 4P \)

R \( = \frac{P}{l} \)

M \( \max. (\text{at both ends}) = P/l \)

M \( \max. (\text{at deflected end}) = P/l \)

\[ \delta \max. (\text{at deflected end}) = \frac{P}{12EI} \left(\frac{1}{2} - x\right) \]

\[ \delta \max. (\text{at point of load}) = \frac{P}{12EI} \left(l - \frac{x^2}{2}\right) \]

\[ \delta \max. (\text{at free end}) = \frac{P}{12EI} \left(\frac{1}{2}x - x^2\right) \]
24. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD

\[ R_1 = V_1 = \frac{wa}{2l} (l - a)^2 \]

\[ R_2 = V_2 + V_3 = \frac{wa}{2l} (l + a)^2 \]

\[ V_3 = \text{wa} \]

\[ V_x = \begin{cases} R_1 & \text{between supports} \\ \frac{wa}{2l} (l - a)^2 & \text{for overhang} \end{cases} \]

\[ V_{x_1} = \begin{cases} \frac{wa}{2l} & \text{at } x = \frac{l}{2} (1 - \frac{a^2}{l^2}) \\ \frac{wa}{2l} (l - a)^2 & \text{at } R_2 \end{cases} \]

\[ M_1 = \begin{cases} \frac{wa}{2l} (l - a)^2 (l - a) & \text{between supports} \\ \frac{wa^2}{2l} & \text{for overhang} \end{cases} \]

\[ M_2 = \frac{wa^2}{2l}(a-x_1)^2 \]

\[ M_x = \begin{cases} \frac{wa^2}{2l}(l - a - x_1) & \text{between supports} \\ \frac{wa^2}{2l} & \text{for overhang} \end{cases} \]

\[ \Delta_x = \begin{cases} \frac{wa^2}{24EI} (4x^3 - 4x^2 + 2x + x^2) & \text{between supports} \\ \frac{wa^2}{24EI} & \text{for overhang} \end{cases} \]

NOTE: For a negative value of \( \Delta \), deflection is upward.

25. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD ON OVERHANG

\[ R_1 = V_1 = \frac{wa^2}{2l} \]

\[ R_2 = V_2 + V_3 = \frac{wa}{2l} (2l + a) \]

\[ V_3 = \text{wa} \]

\[ V_x = \begin{cases} R_1 & \text{for overhang} \end{cases} \]

\[ V_{x_1} = \begin{cases} \frac{wa}{2l} & \text{at } x = \frac{l}{2} (1 - \frac{a^2}{l^2}) \\ \frac{wa^2}{2l} & \text{at } R_2 \end{cases} \]

\[ M_1 = \begin{cases} \frac{wa^2}{2l} (l - a - x_1) & \text{between supports} \\ \frac{wa^2}{2l} & \text{for overhang} \end{cases} \]

\[ \Delta_x = \begin{cases} \frac{wa^2}{24EI} (4x^3 - 4x^2 + 2x + x^2) & \text{between supports} \\ \frac{wa^2}{24EI} & \text{for overhang} \end{cases} \]

26. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT END OF OVERHANG

\[ R_1 = V_1 = \frac{Pa}{l} \]

\[ R_2 = V_2 + V_3 = \frac{Pa}{l} (l + a) \]

\[ V_2 = \frac{P}{b} \]

\[ M_{\text{max.}} \text{ at } R_2 = \frac{Pa}{l} \]

\[ M_x = \begin{cases} \frac{Pa}{l} & \text{between supports} \\ \frac{Pa}{l} (a - x_1) & \text{for overhang} \end{cases} \]

\[ \Delta_{\text{max.}} = \begin{cases} \frac{Pa^2}{9EI} & \text{between supports at } x = \frac{l}{\sqrt{3}} \\ \frac{Pa^2}{9EI} & \text{at point of load} \end{cases} \]

\[ \Delta_x = \begin{cases} \frac{Pa^2}{8EI} & \text{between supports} \\ \frac{Pa^2}{8EI} & \text{for overhang} \end{cases} \]

27. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD BETWEEN SUPPORTS

\[ R = V = \frac{wl}{2} \]

\[ V_x = \frac{wl}{2} - x \]

\[ V_{x_1} = \frac{wl}{2} \]

\[ M_{\text{max.}} \text{ at center} = \frac{wl^2}{8} \]

\[ M_x = \frac{wl}{2} (l - x) \]

\[ \Delta_{\text{max.}} \text{ at center} = \frac{wl^4}{384EI} \]

\[ \Delta_x = \frac{wl^4}{24EI} (2l - 2lx^2 + x^3) \]

\[ \Delta_{x_1} = \frac{wl^4}{24EI} \]

28. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS

\[ \text{Total Equiv. Uniform Load} = 8Pab \]

\[ R_1 = V_1 \text{ max. when } a < b = \frac{Pb}{l} \]

\[ R_2 = V_2 \text{ max. when } a > b = \frac{Pa}{l} \]

\[ M_{\text{max.}} \text{ at point of load } = \frac{Pab}{l} \]

\[ M_x \text{ when } x < a = \frac{Pab}{12EI} (4a + 3a - b^2 - x^2) \]

\[ \Delta_{\text{max.}} \text{ at point of load } = \frac{Pab^2}{27EI} \]

\[ \Delta_a \text{ when } x < a = \frac{Pab^2}{3EI} \]

\[ \Delta_x \text{ when } x > a = \frac{Pab^2}{6EI} (2a x^3 - a^2 + 3a - x^2) \]

\[ \Delta_{x_1} = \frac{Pab^2}{6EI} (l - a) \]
29. CONTINUOUS BEAM—TWO EQUAL SPANS—UNIFORM LOAD ON ONE SPAN

Total Equiv. Uniform Load = \( \frac{49}{64}wl \)
\( R_1 = V_1 = \frac{7}{16} wl \)
\( R_2 = V_2 + V_3 = \frac{5}{8} wl \)
\( R_3 = V_3 = \frac{1}{16} wl \)
\( V_3 = \frac{9}{16} wl \)
\( M_{max.} \), at point of load = \( \frac{49}{512} \frac{wl^2}{8} \)
\( M_1 \), at support \( R_2 \) = \( \frac{1}{16} \frac{wl^2}{8} \)
\( M_x \), when \( x < \frac{l}{2} \) = \( \frac{w}{16} (7l - 8x) \)
\( \Delta \) Max. (0.472 from \( R_1 \)) = 0.0092 \( \frac{w^4}{EI} \)

30. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT CENTER OF ONE SPAN

Total Equiv. Uniform Load = \( \frac{13}{8} P \)
\( R_1 = V_1 = \frac{13}{32} P \)
\( R_2 = V_2 + V_3 = \frac{11}{16} P \)
\( R_3 = V_3 = \frac{3}{32} P \)
\( V_2 = \frac{19}{32} P \)
\( V_3 = \frac{1}{32} P \)
\( M_{max.} \), at point of load = \( \frac{13}{64} P l \)
\( M_1 \), at support \( R_2 \) = \( \frac{3}{32} P l \)
\( \Delta \) Max. (0.480 from \( R_1 \)) = 0.015 \( \frac{P^3}{EI} \)

31. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT ANY POINT

\( R_1 = V_1 = \frac{P b}{4} + \frac{M_1 - M_2}{l} \)
\( R_2 = V_2 + V_3 = \frac{P a}{2l} + \frac{M_1 + M_2}{l} \)
\( R_3 = V_3 = \frac{P b}{4} + \frac{M_2}{l} \)
\( V_2 = \frac{P a}{4l} \)
\( V_3 = \frac{P b}{4l} \)
\( M_{max.} \), at point of load = \( \frac{P b}{4l} (4l^2 - a(l + a)) \)
\( M_1 \), at support \( R_2 \) = \( \frac{P a}{4l} (4l^2 - b(l + a)) \)
\( \Delta \) (When \( x < \frac{l}{2} \)) = \( \frac{P a}{48EI} \left( 3l^3 - 4x^3 - \frac{8l^2 - x}{P} [M_1 (2l - x) + M_2 (l + x)] \right) \)
34. CONTINUOUS BEAM—THREE EQUAL SPANS—ONE END SPAN UNLOADED

\[ R_A = 0.383 \text{wl} \quad R_B = 1.20 \text{wl} \quad R_C = 0.450 \text{wl} \quad R_D = -0.033 \text{wl} \]

\[ \text{SHEAR} \]

\[ +0.0735 \text{wl}^2 \quad -0.1167 \text{wl}^2 \quad +0.0534 \text{wl}^2 \quad -0.0333 \text{wl}^2 \]

\[ \text{MOMENT} \]

\[ 0.383 l \quad 0.583 l \quad 0.617 l \quad 0.033 l \quad 0.417 l \quad 0.033 l \]

\[ \Delta \text{Max. (0.430 from A)} = 0.0059 \text{wt}^2/\text{EI} \]

35. CONTINUOUS BEAM—THREE EQUAL SPANS—END SPANS LOADED

\[ R_A = 0.450 \text{wl} \quad R_B = 0.550 \text{wl} \quad R_C = 0.550 \text{wl} \quad R_D = 0.450 \text{wl} \]

\[ \text{SHEAR} \]

\[ +0.1013 \text{wl}^2 \quad -0.050 \text{wl}^2 \quad +0.1013 \text{wl}^2 \]

\[ \text{MOMENT} \]

\[ 0.450 l \quad 0.450 l \quad 0.450 l \]

\[ \Delta \text{Max. (0.479 from A or D)} = 0.0099 \text{wt}^2/\text{EI} \]

36. CONTINUOUS BEAM—THREE EQUAL SPANS—ALL SPANS LOADED

\[ R_A = 0.400 \text{wl} \quad R_B = 1.10 \text{wl} \quad R_C = 1.10 \text{wl} \quad R_D = 0.400 \text{wl} \]

\[ \text{SHEAR} \]

\[ +0.080 \text{wl}^2 \quad -0.100 \text{wl}^2 \quad +0.025 \text{wl}^2 \quad -0.100 \text{wl}^2 \quad +0.080 \text{wl}^2 \]

\[ \text{MOMENT} \]

\[ 0.400 l \quad 0.500 l \quad 0.500 l \quad 0.400 l \]

\[ \Delta \text{Max. (0.446 from A or D)} = 0.0069 \text{wt}^2/\text{EI} \]

37. CONTINUOUS BEAM—FOUR EQUAL SPANS—THIRD SPAN UNLOADED

\[ R_A = 0.380 \text{wl} \quad R_B = 1.223 \text{wl} \quad R_C = 0.357 \text{wl} \quad R_D = 0.598 \text{wl} \quad R_E = 0.442 \text{wl} \]

\[ \text{SHEAR} \]

\[ +0.072 \text{wl}^2 \quad -0.1205 \text{wl}^2 \quad -0.0719 \text{wl}^2 \quad -0.058 \text{wl}^2 \quad +0.0977 \text{wl}^2 \]

\[ \text{MOMENT} \]

\[ 0.380 l \quad 0.603 l \quad 0.620 l \quad 0.397 l \quad 0.558 l \quad 0.040 l \quad 0.442 l \]

\[ \Delta \text{Max. (0.475 from E)} = 0.0094 \text{wt}^2/\text{EI} \]

38. CONTINUOUS BEAM—FOUR EQUAL SPANS—LOAD FIRST AND THIRD SPANS

\[ R_A = 0.446 \text{wl} \quad R_B = 0.572 \text{wl} \quad R_C = 0.464 \text{wl} \quad R_D = 0.572 \text{wl} \quad R_E = -0.054 \text{wl} \]

\[ \text{SHEAR} \]

\[ +0.0996 \text{wl}^2 \quad -0.0536 \text{wl}^2 \quad -0.0357 \text{wl}^2 \quad -0.0536 \text{wl}^2 \quad +0.0805 \text{wl}^2 \]

\[ \text{MOMENT} \]

\[ 0.446 l \quad 0.518 l \quad 0.518 l \]

\[ \Delta \text{Max. (0.477 from A or D)} = 0.0097 \text{wt}^2/\text{EI} \]

39. CONTINUOUS BEAM—FOUR EQUAL SPANS—ALL SPANS LOADED

\[ R_A = 0.393 \text{wl} \quad R_B = 1.143 \text{wl} \quad R_C = 0.928 \text{wl} \quad R_D = 1.143 \text{wl} \quad R_E = 0.393 \text{wl} \]

\[ \text{SHEAR} \]

\[ +0.0772 \text{wl}^2 \quad -0.1071 \text{wl}^2 \quad -0.0714 \text{wl}^2 \quad -0.1071 \text{wl}^2 \]

\[ \text{MOMENT} \]

\[ 0.393 l \quad 0.536 l \quad 0.536 l \quad 0.393 l \]

\[ \Delta \text{Max. (0.440 from A and E)} = 0.0065 \text{wt}^2/\text{EI} \]
Centroids & Moments of Inertia of Beam Sections

Notation:

- \( A \) = name for area
- \( b \) = name for a (base) width
- \( C \) = designation for channel section
- \( d \) = name for centroid
- \( d \) = calculus symbol for differentiation
- \( d_s \) = difference in the \( x \) direction between an area centroid (\( \bar{y} \)) and the centroid of the composite shape (\( \hat{y} \))
- \( d_y \) = difference in the \( y \) direction between an area centroid (\( \bar{y} \)) and the centroid of the composite shape (\( \hat{y} \))
- \( F_z \) = force component in the \( z \) direction
- \( h \) = name for a height
- \( I \) = moment of inertia about the centroid
- \( I_c \) = moment of inertia about the centroid
- \( I_x \) = moment of inertia with respect to an \( x \)-axis
- \( I_y \) = moment of inertia with respect to a \( y \)-axis
- \( J_o \) = polar moment of inertia, as is \( J \)
- \( L \) = name for length
- \( O \) = name for reference origin
- \( Q_x \) = first moment area about an \( x \) axis (using \( y \) distances)
- \( Q_y \) = first moment area about an \( y \) axis (using \( x \) distances)
- \( r_o \) = polar radius of gyration
- \( r_s \) = radius of gyration with respect to an \( x \)-axis
- \( r_y \) = radius of gyration with respect to a \( y \)-axis
- \( t \) = name for thickness
- \( t_f \) = thickness of a flange
- \( t_w \) = thickness of web of wide flange
- \( W \) = name for force due to weight
- \( x \) = designation for wide flange section
- \( x \) = horizontal distance
- \( \bar{x} \) = the distance in the \( x \) direction from a reference axis to the centroid of a shape
- \( \hat{x} \) = the distance in the \( x \) direction from a reference axis to the centroid of a composite shape
- \( y \) = vertical distance
- \( \bar{y} \) = the distance in the \( y \) direction from a reference axis to the centroid of a shape
- \( \hat{y} \) = the distance in the \( y \) direction from a reference axis to the centroid of a composite shape
- \( z \) = distance perpendicular to \( x-y \) plane
- \( P \) = plate symbol
- \( \int \) = symbol for integration
- \( \Delta \) = calculus symbol for small quantity
- \( \gamma \) = density of a material (unit weight)
- \( \Sigma \) = summation symbol

- The cross section shape and how it resists bending and twisting is important to understanding beam and column behavior.

- The center of gravity is the location of the equivalent force representing the total weight of a body comprised of particles that each have a mass gravity acts upon.
Resultant force: Over a body of constant thickness in x and y

\[ \sum F_z = \sum_{i=1}^{n} \Delta W_i = W \quad \text{W} = \int dW \]

Location: \( \bar{x}, \bar{y} \) is the equivalent location of the force \( W \) from all \( \Delta W_i \)'s over all x & y locations (with respect to the moment from each force) from:

\[ \sum M_x = \sum_{i=1}^{n} x_i \Delta W_i = \bar{x}W \quad \bar{x}W = \int x dW \Rightarrow \bar{x} = \frac{\int x dW}{W} \quad \text{OR} \quad \bar{x} = \frac{\sum(x\Delta W)}{W} \]

\[ \sum M_y = \sum_{i=1}^{n} y_i \Delta W_i = \bar{y}W \quad \bar{y}W = \int y dW \Rightarrow \bar{y} = \frac{\int y dW}{W} \quad \text{OR} \quad \bar{y} = \frac{\sum(y\Delta W)}{W} \]

- The centroid of an area is the average x and y locations of the area particles.

For a shape of a uniform thickness and material:

\[ \Delta W_i = \gamma t \Delta A_i \quad \text{where:} \]

\( \gamma \) is weight per unit volume (= specific weight) with units of \( \text{N/m}^3 \) or \( \text{lb/ft}^3 \)

\( t \Delta A_i \) is the volume

So if \( W = \gamma t A \):

\[ \bar{x} \gamma tA = \int x \gamma tdA \Rightarrow \bar{x}A = \int x dA \quad \text{OR} \quad \bar{x} = \frac{\sum(x\Delta A)}{A} \quad \text{and similarly} \quad \bar{y} = \frac{\sum(y\Delta A)}{A} \]

Similarly, for a line with constant cross section, \( a \) (\( \Delta W_i = \gamma a \Delta L_i \)):

\[ \bar{x}L = \int x dL \quad \text{OR} \quad \bar{x} = \frac{\sum(x\Delta L)}{L} \quad \text{and} \quad \bar{y}L = \int y dL \quad \text{OR} \quad \bar{y} = \frac{\sum(y\Delta L)}{L} \]

- \( \bar{x}, \bar{y} \) with respect to an x, y coordinate system is the centroid of an area AND the center of gravity for a body of uniform material and thickness.

- The first moment of the area is like a force moment: and is the area multiplied by the perpendicular distance to an axis.

\[ Q_x = \int y dA = \bar{y}A \quad Q_y = \int x dA = \bar{x}A \]
**Centroids of Common Shapes**

Centroids of Common Shapes of Areas and Lines

<table>
<thead>
<tr>
<th>Shape</th>
<th>$\bar{x}$</th>
<th>$\bar{y}$</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular area</td>
<td>$\frac{b}{3}$</td>
<td>$\frac{h}{3}$</td>
<td>$\frac{bh}{2}$</td>
</tr>
<tr>
<td>Quarter-circular area</td>
<td>$\frac{4r}{3\pi}$</td>
<td>$\frac{4r}{3\pi}$</td>
<td>$\frac{\pi r^2}{4}$</td>
</tr>
<tr>
<td>Semicircular area</td>
<td>0</td>
<td>$\frac{4r}{3\pi}$</td>
<td>$\frac{\pi r^2}{2}$</td>
</tr>
<tr>
<td>Semiparabolic area</td>
<td>$\frac{3a}{8}$</td>
<td>$\frac{3h}{5}$</td>
<td>$\frac{2ah}{3}$</td>
</tr>
<tr>
<td>Parabolic area</td>
<td>0</td>
<td>$\frac{3h}{5}$</td>
<td>$\frac{4ah}{3}$</td>
</tr>
<tr>
<td>Parabolic span-drel</td>
<td>$\frac{3a}{4}$</td>
<td>$\frac{3h}{10}$</td>
<td>$\frac{ah}{3}$</td>
</tr>
<tr>
<td>Circular sector</td>
<td>$\frac{2r \sin \alpha}{3\alpha}$</td>
<td>0</td>
<td>$\alpha r^2$</td>
</tr>
<tr>
<td>Quarter-circular arc</td>
<td>$\frac{2r}{\pi}$</td>
<td>$\frac{2r}{\pi}$</td>
<td>$\frac{\pi r}{2}$</td>
</tr>
<tr>
<td>Semicircular arc</td>
<td>0</td>
<td>$\frac{2r}{\pi}$</td>
<td>$\pi r$</td>
</tr>
<tr>
<td>Arc of circle</td>
<td>$\frac{r \sin \alpha}{\alpha}$</td>
<td>0</td>
<td>$2\alpha r$</td>
</tr>
</tbody>
</table>
Symmetric Areas

- An area is symmetric with respect to a line when every point on one side is mirrored on the other. The line divides the area into equal parts and the centroid will be on that axis.

- An area can be symmetric to a center point when every \((x,y)\) point is matched by a \((-x,-y)\) point. It does not necessarily have an axis of symmetry. The center point is the centroid.

- If the symmetry line is on an axis, the centroid location is on that axis (value of 0). With double symmetry, the centroid is at the intersection.

- Symmetry can also be defined by areas that match across a line, but are \(180^\circ\) to each other.

Basic Steps (Statical Moment Method)

1. Draw a reference origin.
2. Divide the area into basic shapes
3. Label the basic shapes (components)
4. Draw a table with headers of Component, Area, \(\bar{x}, \bar{A}\), \(\bar{y}, \bar{A}\)
5. Fill in the table value
6. Draw a summation line. Sum all the areas, all the \(\bar{x}A\) terms, and all the \(\bar{y}A\) terms
7. Calculate \(\bar{x}\) and \(\bar{y}\)

Composite Shapes

If we have a shape made up of basic shapes that we know centroid locations for, we can find an “average” centroid of the areas.

\[
\bar{x}A = \bar{x} \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \bar{x}_i A_i \\
\bar{y}A = \bar{y} \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \bar{y}_i A_i
\]

Centroid values can be negative.
Area values can be negative (holes)
- **Definition: Moment of Inertia**: the second area moment
  \[ I_y = \sum x_i^2 \Delta A = \int x^2 dA \quad I_x = \sum y_i^2 \Delta A = \int y^2 dA \]
  (or \( I_{x-x} = \sum z^2 a \))

  We can define a single integral using a narrow strip:
  
  for \( I_x \), strip is parallel to \( x \)
  for \( I_y \), strip is parallel to \( y \)

  *I can be negative if the area is negative (a hole or subtraction).*

- A shape that has area at a greater distance away from an axis *through its centroid* will have a larger value of \( I \).

- Just like for center of gravity of an area, the moment of inertia can be determined with respect to *any* reference axis.

- **Definition: Polar Moment of Inertia**: the second area moment using polar coordinate axes
  \[ J_o = \int r^2 dA = \int (x^2 + y^2) dA \]
  \[ J_o = I_x + I_y \]

- **Definition: Radius of Gyration**: the distance from the moment of inertia axis for an area at which the entire area could be considered as being concentrated at.
  
  \[ I_x = r_x^2 A \Rightarrow r_x = \sqrt{\frac{I_x}{A}} \text{ radius of gyration in } x \]

  \[ r_y = \sqrt{\frac{I_y}{A}} \text{ radius of gyration in } y \]

  \[ r_o = \sqrt{\frac{J_o}{A}} \text{ polar radius of gyration, and } r_o^2 = r_x^2 + r_y^2 \]
**The Parallel-Axis Theorem**

- The moment of inertia of an area with respect to any axis not through its centroid is equal to the moment of inertia of that area with respect to its own parallel centroidal axis plus the product of the area and the square of the distance between the two axes.

\[
I = \int y'^2 dA = \int (y' - d)^2 dA \\
= \int y'^2 dA + 2d \int y' dA + d^2 \int dA
\]

but \( \int y' dA = 0 \), because the centroid is on this axis, resulting in:

\[
I = I_o + A d^2 \quad \text{(text notation)} \quad \text{or} \quad I_x = \bar{I}_x + Ad_y^2
\]

where \( I_o \) (or \( \bar{I}_x \)) is the moment of inertia about the centroid of the area about an \( x \) axis and \( d_y \) is the \( y \) distance between the parallel axes.

**Similarly**

- \( I_y = \bar{I}_y + Ad_x^2 \) Moment of inertia about a \( y \) axis
- \( J_o = \bar{J}_c + Ad^2 \) Polar moment of Inertia
- \( r_o^2 = \bar{r}_c^2 + d^2 \) Polar radius of gyration
- \( r^2 = \bar{r}^2 + d^2 \) Radius of gyration

* I can be negative again if the area is negative (a hole or subtraction).

** If \( \bar{I} \) is not given in a chart, but \( \bar{x} \) & \( \bar{y} \) are: YOU MUST CALCULATE \( \bar{I} \) WITH \( \bar{I} = I - Ad^2 \)

**Composite Areas:**

\[
I = \sum I + \sum Ad^2 \quad \text{where} \quad \bar{I} \text{ is the moment of inertia about the centroid of the component area} \\
d \text{is the distance from the centroid of the component area to the centroid of the composite area (ie. } d_y = \hat{y} - \bar{y})
\]

**Basic Steps**

1. Draw a reference origin.
2. Divide the area into basic shapes
3. Label the basic shapes (components)
4. Draw a table with headers of
   - Component, Area, \( \bar{x}, \bar{A} \), \( \bar{y}, \bar{A} \), \( \bar{I}_x, d_x, Ad_y^2 \), \( \bar{I}_y, d_y, Ad_x^2 \)
5. Fill in the table values needed to calculate \( \hat{x} \) and \( \hat{y} \) for the composite
6. Fill in the rest of the table values.
7. Sum the moment of inertia (\( \bar{I} \)'s) and \( Ad^2 \) columns and add together.
## Geometric Properties of Areas

<table>
<thead>
<tr>
<th>Shape</th>
<th>formulas</th>
<th>Area</th>
<th>( \bar{x} )</th>
<th>( \bar{y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rectangle</strong></td>
<td>( I_x = \frac{1}{12}bh^3 ), ( I_y = \frac{1}{12}b^2h )</td>
<td>( bh )</td>
<td>( \frac{b}{2} )</td>
<td>( \frac{h}{2} )</td>
</tr>
<tr>
<td><strong>Triangle</strong></td>
<td>( I_x = \frac{1}{12}bh^3 ), ( I_y = \frac{1}{12}b^2h )</td>
<td>( \frac{bh}{2} )</td>
<td>( \frac{b}{3} )</td>
<td>( \frac{h}{3} )</td>
</tr>
<tr>
<td><strong>Circle</strong></td>
<td>( I_x = I_y = \frac{1}{4}\pi r^4 ), ( J_0 = \frac{1}{2}\pi r^4 )</td>
<td>( \pi r^2 = \frac{\pi d^2}{4} )</td>
<td>( 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td><strong>Semicircle</strong></td>
<td>( I_x = 0.1098r^4 ), ( I_y = \frac{\pi r^4}{8} )</td>
<td>( \frac{\pi r^2}{2} = \frac{\pi d^2}{8} )</td>
<td>( 0 )</td>
<td>( \frac{4r}{\sqrt{3}\pi} )</td>
</tr>
<tr>
<td><strong>Quarter circle</strong></td>
<td>( I_x = 0.0549r^4 ), ( I_y = 0.0549r^4 )</td>
<td>( \frac{\pi r^2}{4} = \frac{\pi d^2}{16} )</td>
<td>( \frac{4r}{\sqrt{3}\pi} )</td>
<td>( \frac{4r}{\sqrt{3}\pi} )</td>
</tr>
<tr>
<td><strong>Ellipse</strong></td>
<td>( I_x = \frac{1}{4}\pi ab^3 ), ( I_y = \frac{1}{4}\pi a'b )</td>
<td>( \pi ab )</td>
<td>( 0 )</td>
<td>( 0 )</td>
</tr>
<tr>
<td><strong>Parabolic area</strong></td>
<td>( I_x = 16ah^3/175 ), ( I_y = 4a^3h/15 )</td>
<td>( 4ah/3 )</td>
<td>( 0 )</td>
<td>( 3h/5 )</td>
</tr>
<tr>
<td><strong>Parabolic spandrel</strong></td>
<td>( I_x = 37ah^3/2100 ), ( I_y = a^3h/80 )</td>
<td>( ah/3 )</td>
<td>( 3a/4 )</td>
<td>( 3h/10 )</td>
</tr>
</tbody>
</table>
Example 1

Determine the centroidal $x$ and $y$ distances for the composite area shown. Use the lower left corner of the trapezoid as the reference origin.

<table>
<thead>
<tr>
<th>Component</th>
<th>$Area (A) \text{ (in.}^2\text{)}$</th>
<th>$\bar{x} \text{ (in.)}$</th>
<th>$\bar{x}A \text{ (in.}^3\text{)}$</th>
<th>$\bar{y} \text{ (in.)}$</th>
<th>$\bar{y}A \text{ (in.}^3\text{)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(\frac{9'(3')}{2} = 13.5 \text{ in.}^2)</td>
<td>6'</td>
<td>81 \text{ in.}^3</td>
<td>4'</td>
<td>54 \text{ in.}^3</td>
</tr>
<tr>
<td>(b)</td>
<td>(9' \text{ (3')} = 27 \text{ in.}^2)</td>
<td>4.5'</td>
<td>121.5 \text{ in.}^3</td>
<td>1.5'</td>
<td>40.5 \text{ in.}^3</td>
</tr>
<tr>
<td></td>
<td>(\sum A = 40.5 \text{ in.}^2)</td>
<td>(\sum \bar{x}A = 202.5 \text{ in.}^3)</td>
<td>(\sum \bar{y}A = 94.5 \text{ in.}^3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[\bar{x} = \frac{202.5 \text{ in.}^3}{40.5 \text{ in}^2} = 5 \text{ in}\]
\[\bar{y} = \frac{94.5 \text{ in.}^3}{40.5 \text{ in}^2} = 2.33 \text{ in}\]

Example 2

An alternate method that can be employed in solving this problem is referred to as the negative area method.

A 6" thick concrete wall panel is precast to the dimensions as shown. Using the lower left corner as the reference origin, determine the center of gravity (centroid) of the panel.
Example 3

Find the moments of inertia ($\hat{x} = 3.05''$, $\hat{y} = 1.05''$).

<table>
<thead>
<tr>
<th>Component</th>
<th>$I_{xc}$ (in.⁴)</th>
<th>$d_y$ (in.)</th>
<th>$Ad_{y}^2$ (in.⁴)</th>
<th>$I_{yc}$ (in.⁴)</th>
<th>$d_x$ (in.)</th>
<th>$Ad_{x}^2$ (in.⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$yc_1$</td>
<td>$\frac{(1)(4)^3}{12} = 5.33$</td>
<td>0.95</td>
<td>3.61</td>
<td>$\frac{(4)(1)^3}{12} = 0.33$</td>
<td>2.55</td>
<td>26.01</td>
</tr>
<tr>
<td>$yc_2$</td>
<td>$\frac{(7)(1)^3}{12} = 0.58 $</td>
<td>0.55</td>
<td>2.12</td>
<td>$\frac{(1)(7)^3}{12} = 28.58$</td>
<td>1.45</td>
<td>14.72</td>
</tr>
<tr>
<td>$\sum l_{xc}$</td>
<td>$5.91$</td>
<td></td>
<td>$\sum Ad_{y}^2 = 5.73$</td>
<td>$\sum I_{yc}$</td>
<td>$28.91$</td>
<td>$\sum Ad_{x} = 40.73$</td>
</tr>
</tbody>
</table>

Example 4

Determine the $l$ about the centroidal $x$-axis.
Example 5
Determine the moments of inertia about the centroid of the shape.

Solution:

There is no reference origin suggested in figure (a), so the bottom left corner is good.

In figure (b) area A will be a complete rectangle, while areas C and A are "holes" with negative area and negative moment of inertias.

Area A = 200 mm x 100 mm = 20000 mm\(^2\)
\[ I_x = (200\, \text{mm})(100\, \text{mm})^3/12 = 16.667 \times 10^6 \, \text{mm}^4 \]
\[ I_y = (200\, \text{mm})(100\, \text{mm})^3/12 = 66.667 \times 10^6 \, \text{mm}^4 \]

Area B = -\pi(30 \, \text{mm})^2 = -2827.4 \, \text{mm}^2
\[ I_x = I_y = -\pi(30\, \text{mm})^4/4 = -0.636 \times 10^6 \, \text{mm}^4 \]

Area C = -1/2\pi(50 \, \text{mm})^2 = 3927.0 \, \text{mm}^2
\[ I_x = -\pi(50\, \text{mm})^4/8 = -2.454 \times 10^6 \, \text{mm}^4 \]
\[ I_y = -0.1098(50\, \text{mm})^4 = -0.686 \times 10^6 \, \text{mm}^4 \]

Area D = 100 mm x 200 mm x 1/2 = 10000 mm\(^2\)
\[ I_x = (200\, \text{mm})(100\, \text{mm})^3/36 = 5.556 \times 10^6 \, \text{mm}^4 \]
\[ I_y = (200\, \text{mm})(1\, \text{mm})^3/36 = 22.222 \times 10^6 \, \text{mm}^4 \]

<table>
<thead>
<tr>
<th>shape</th>
<th>A (mm(^2))</th>
<th>(\bar{x}) (mm)</th>
<th>(\bar{A}) (mm(^3))</th>
<th>(\bar{y}) (mm)</th>
<th>(\bar{A}) (mm(^3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20000</td>
<td>100</td>
<td>20000000</td>
<td>50</td>
<td>10000000</td>
</tr>
<tr>
<td>B</td>
<td>-2827.43</td>
<td>150</td>
<td>-424115</td>
<td>50</td>
<td>-141372</td>
</tr>
<tr>
<td>C</td>
<td>-3926.99</td>
<td>21.22066</td>
<td>-833333.3</td>
<td>50</td>
<td>-196350</td>
</tr>
<tr>
<td>D</td>
<td>10000</td>
<td>66.66667</td>
<td>666666.7</td>
<td>133.3333</td>
<td>1333333</td>
</tr>
</tbody>
</table>

\[ \bar{x} = \frac{2159218 \, \text{mm}^3}{23245.58 \, \text{mm}^2} = 92.9 \, \text{mm} \]
\[ \dot{y} = \frac{1995612 \, \text{mm}^3}{23245.58 \, \text{mm}^2} = 85.8 \, \text{mm} \]

<table>
<thead>
<tr>
<th>shape</th>
<th>(I_x) (mm(^4))</th>
<th>(d_y) (mm)</th>
<th>(A_{dx}^2) (mm(^4))</th>
<th>(I_y) (mm(^4))</th>
<th>(d_x) (mm)</th>
<th>(A_{dy}^2) (mm(^4))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>16666667</td>
<td>35.8</td>
<td>25632800</td>
<td>66666667</td>
<td>-7.1</td>
<td>1008200</td>
</tr>
<tr>
<td>B</td>
<td>-636173</td>
<td>35.8</td>
<td>-3623751.73</td>
<td>-636173</td>
<td>-57.1</td>
<td>-9218592.093</td>
</tr>
<tr>
<td>C</td>
<td>-2454369</td>
<td>35.8</td>
<td>-5032988.51</td>
<td>-686250</td>
<td>71.67934</td>
<td>-20176595.22</td>
</tr>
<tr>
<td>D</td>
<td>5555556</td>
<td>-47.5333</td>
<td>22594177.8</td>
<td>22222222</td>
<td>26.23333</td>
<td>6881876.029</td>
</tr>
</tbody>
</table>

\[ \bar{y} = 39570237.5 \, \text{mm}^3 = 58701918 = 58.7 \times 10^6 \, \text{mm}^4 \]

So,
\[ I_x = 19131680 + 39570237.5 + 58701918 = 58.7 \times 10^6 \, \text{mm}^4 \]

\[ I_x = 87566466 + 21505111.3 = 66.1 \times 10^6 \, \text{mm}^4 \]
Example 6
Locate the centroidal $x$ and $y$ axes for the cross-section shown. Use the reference origin indicated and assume that the steel plate is centered over the flange of the wide-flange section. Compute the $I_x$ and $I_y$ about the major centroidal axes.
Beam Stresses – Bending and Shear

Notation:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>name for area</td>
</tr>
<tr>
<td>$A_{\text{web}}$</td>
<td>area of the web of a wide flange section</td>
</tr>
<tr>
<td>b</td>
<td>width of a rectangle</td>
</tr>
<tr>
<td>c</td>
<td>largest distance from the neutral axis to the top or bottom edge of a beam</td>
</tr>
<tr>
<td>d</td>
<td>calculus symbol for differentiation</td>
</tr>
<tr>
<td>$d_y$</td>
<td>difference in the y direction between an area centroid ($\bar{y}$) and the centroid of the composite shape ($\hat{y}$)</td>
</tr>
<tr>
<td>E</td>
<td>modulus of elasticity or Young’s modulus</td>
</tr>
<tr>
<td>$f_b$</td>
<td>bending stress</td>
</tr>
<tr>
<td>$f_c$</td>
<td>compressive stress</td>
</tr>
<tr>
<td>$f_{\text{max}}$</td>
<td>maximum stress</td>
</tr>
<tr>
<td>$f_t$</td>
<td>tensile stress</td>
</tr>
<tr>
<td>$f_v$</td>
<td>shear stress</td>
</tr>
<tr>
<td>$F_{\text{b}}$</td>
<td>allowable bending stress</td>
</tr>
<tr>
<td>$F_{\text{connector}}$</td>
<td>shear force capacity per connector</td>
</tr>
<tr>
<td>h</td>
<td>height of a rectangle</td>
</tr>
<tr>
<td>I</td>
<td>moment of inertia with respect to neutral axis bending</td>
</tr>
<tr>
<td>$I_x$</td>
<td>moment of inertia with respect to an x-axis</td>
</tr>
<tr>
<td>L</td>
<td>name for length</td>
</tr>
<tr>
<td>M</td>
<td>internal bending moment</td>
</tr>
<tr>
<td>n</td>
<td>number of connectors across a joint</td>
</tr>
<tr>
<td>n.a.</td>
<td>shorthand for neutral axis (N.A.)</td>
</tr>
<tr>
<td>O</td>
<td>name for reference origin</td>
</tr>
<tr>
<td>p</td>
<td>pitch of connector spacing</td>
</tr>
<tr>
<td>P</td>
<td>name for a force vector</td>
</tr>
<tr>
<td>q</td>
<td>shear per length (shear flow)</td>
</tr>
<tr>
<td>Q</td>
<td>first moment area about a neutral axis</td>
</tr>
<tr>
<td>$Q_{\text{connected}}$</td>
<td>first moment area about a neutral axis for the connected part</td>
</tr>
<tr>
<td>R</td>
<td>radius of curvature of a deformed beam</td>
</tr>
<tr>
<td>S</td>
<td>section modulus</td>
</tr>
<tr>
<td>$S_{\text{req’d}}$</td>
<td>section modulus required at allowable stress</td>
</tr>
<tr>
<td>$t_w$</td>
<td>thickness of web of wide flange</td>
</tr>
<tr>
<td>V</td>
<td>internal shear force</td>
</tr>
<tr>
<td>$V_{\text{longitudinal}}$</td>
<td>longitudinal shear force</td>
</tr>
<tr>
<td>$V_T$</td>
<td>transverse shear force</td>
</tr>
<tr>
<td>w</td>
<td>name for distributed load</td>
</tr>
<tr>
<td>x</td>
<td>horizontal distance</td>
</tr>
<tr>
<td>y</td>
<td>vertical distance</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>the distance in the y direction from a reference axis (n.a) to the centroid of a shape</td>
</tr>
<tr>
<td>$\hat{y}$</td>
<td>the distance in the y direction from a reference axis to the centroid of a composite shape</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>calculus symbol for small quantity</td>
</tr>
<tr>
<td>$\delta$</td>
<td>elongation or length change</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>strain</td>
</tr>
<tr>
<td>$\theta$</td>
<td>arc angle</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>summation symbol</td>
</tr>
</tbody>
</table>

Pure Bending in Beams

With bending moments along the axis of the member only, a beam is said to be in pure bending.
Normal stresses due to bending can be found for homogeneous materials having a plane of symmetry in the y axis that follow Hooke’s law.

**Maximum Moment and Stress Distribution**

In a member of constant cross section, the maximum bending moment will govern the design of the section size when we know what kind of normal stress is caused by it.

For internal equilibrium to be maintained, the bending moment will be equal to the $\sum M$ from the normal stresses $\times$ the areas $\times$ the moment arms. Geometric fit helps solve this statically indeterminate problem:

1. The normal planes remain normal for pure bending.
2. There is no net internal axial force.
3. Stress varies linearly over cross section.
4. Zero stress exists at the centroid and the line of centroids is the *neutral axis* (n. a)

![Figure 8.5(a) Beam elevation before loading.](image)

![Figure 8.5(b) Beam bending under load.](image)

![Beam cross section.](image)
Relations for Beam Geometry and Stress

Pure bending results in a circular arc deflection. $R$ is the distance to the center of the arc; $\theta$ is the angle of the arc (radians); $c$ is the distance from the n.a. to the extreme fiber; $f_{\text{max}}$ is the maximum normal stress at the extreme fiber; $y$ is a distance in $y$ from the n.a.; $M$ is the bending moment; $I$ is the moment of in\(\delta\) zertia; $S$ is the section modulus.

\[ L = R \theta \quad \varepsilon = \frac{\delta}{L} = R \quad f = E \varepsilon = \frac{y}{c} f_{\text{max}} \]

\[ M = \Sigma f_i A_i \quad M = \frac{f_{\text{max}}}{c} \Sigma y_i^2 A_i \quad I = \Sigma y^2 A \quad S = \frac{I}{c} \quad f_{\text{max}} = \frac{M c}{I} = \frac{M}{S} \]

Now: \[ f_b = \frac{M y}{I} \quad \text{for a rectangle of height } h \text{ and width } b: \quad S = \frac{b h^3}{12 h/2} = \frac{b h^2}{6} \]

RELATIONS:

\[ \frac{1}{R} = \frac{M}{E I} \quad f_b = \frac{M y^*}{I} \quad S = \frac{I}{c} \]

\[ f_{b,\text{max}} = \frac{M c}{I} = \frac{M}{S} \quad S_{\text{required}} \geq \frac{M}{F_b} \]

*Note: $y$ positive goes DOWN. With a positive $M$ and $y$ to the bottom fiber as positive, it results in a TENSION stress (we’ve called positive).

Transverse Loading in Beams

We are aware that transverse beam loadings result in internal shear and bending moments.

We designed sections based on bending stresses, since this stress dominates beam behavior.

There can be shear stresses \textit{horizontally} within a beam member.

It can be shown that $f_{\text{horizontal}} = f_{\text{vertical}}$
Equilibrium and Derivation

In order for equilibrium for any element CDD’C’, there needs to be a horizontal force $\Delta H$.

$V = f_D dA - f_C dA$

$Q$ is a moment area with respect to the neutral axis of the area above or below the horizontal where the $\Delta H$ occurs.

$Q$ is a maximum when $y = 0$ (at the neutral axis).

$q$ is a horizontal shear per unit length $\rightarrow$ shear flow

Shearing Stresses

$f_{v-ave} = 0$ on the beam’s surface. Even if $Q$ is a maximum at $y = 0$, we don’t know that the thickness is a minimum there.

\[
f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x} \quad \Rightarrow \quad f_{v-ave} = \frac{VQ}{lb}
\]

Rectangular Sections

$f_{v-max}$ occurs at the neutral axis:

\[
I = \frac{bh^3}{12} \quad Q = A\bar{y} = bh^2 \cdot \frac{h}{2} \cdot \frac{h}{2} = bh^2 / 8
\]

then:

\[
f_v = \frac{VQ}{lb} = \frac{V}{\sqrt{2}bh^3b} = \frac{3V}{2bh} \quad \Rightarrow \quad f_v = \frac{3V}{2A}
\]
Webs of Beams

In steel W or S sections the thickness varies from the flange to the web.

We neglect the shear stress in the flanges and consider the shear stress in the web to be constant:

\[ f_{v_{\text{max}}} = \frac{3V}{2A} \approx \frac{V}{A_{\text{web}}} \]

\[ f_{v_{\text{max}}} = \frac{V}{t_{\text{web}}d} \]

Webs of I beams can fail in tension shear across a panel with stiffeners or the web can buckle.

Shear Flow

Even if the cut we make to find \( Q \) is not horizontal, but arbitrary, we can still find the shear flow, \( q \), as long as the loads on thin-walled sections are applied in a plane of symmetry, and the cut is made perpendicular to the surface of the member.

\[ q = \frac{VQ}{l} \]

The shear flow magnitudes can be sketched by knowing \( Q \).
Connectors to Resist Horizontal Shear in Composite Members

Typical connections needing to resist shear are plates with nails or rivets or bolts in composite sections or splices.

The pitch (spacing) can be determined by the capacity in shear of the connector(s) to the shear flow over the spacing interval, \( p \).

\[
\frac{V_{\text{longitudinal}}}{p} = \frac{VQ}{I}
\]

where

\[ nF_{\text{connector}} \geq \frac{VQ_{\text{connected area}}}{I} \cdot p \]

\( p \) = pitch length

\( n \) = number of connectors connecting the connected area to the rest of the cross section

\( F \) = force capacity in one connector

\( Q_{\text{connected area}} = A_{\text{connected area}} \times y_{\text{connected area}} \)

\( y_{\text{connected area}} \) = distance from the centroid of the connected area to the neutral axis

Connectors to Resist Horizontal Shear in Composite Members

Even vertical connectors have shear flow across them.

The spacing can be determined by the capacity in shear of the connector(s) to the shear flow over the spacing interval, \( p \).

\[ p \leq \frac{nF_{\text{connector}}}{VQ_{\text{connected area}}} \]

Unsymmetrical Sections or Shear

If the section is not symmetric, or has a shear not in that plane, the member can bend and twist.

If the load is applied at the shear center there will not be twisting. This is the location where the moment caused by shear flow = the moment of the shear force about the shear center.
Example 1

Calculate the maximum bending and shear stress for the beam shown.

ALSO: Determine the minimum nail spacing required (pitch) if the shear capacity of a nail (F) is 250 lb.

<table>
<thead>
<tr>
<th>Component</th>
<th>$A$ (in.$^2$)</th>
<th>$\overline{y}$ (in.)</th>
<th>$\overline{y}A$ (in.$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
<td>7</td>
<td>84</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>3</td>
<td>36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Component</th>
<th>$I_y$ (in.$^4$)</th>
<th>$A$ (in.$^2$)</th>
<th>$d_y$ (in.)</th>
<th>$Ad_y^2$ (in.$^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>12</td>
<td>2</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>36</td>
<td>12</td>
<td>2</td>
<td>48</td>
</tr>
</tbody>
</table>
Example 2

8.11 A built-up plywood box beam with $2 \times 4$ S4S top and bottom flanges is held together by nails. Determine the pitch (spacing) of the nails if the beam supports a uniform load of 200 #/ft. along the 26-foot span. Assume the nails have a shear capacity of 80#/each.

Solution:

Construct the shear ($V$) diagram to obtain the critical shear condition and its location.

Note that the condition of shear is critical at the support, and the shear intensity decreases as you approach the center line of the beam. This would indicate that the nail spacing $P$ varies from the support to midspan. Nails are closely spaced at the support, but increasing spacing occurs toward midspan, following the shear diagram.

\[
f_v = \frac{VQ}{lb}
\]

\[
I_s = \frac{(4.5")(18")^3}{12} = 1,202.6 \text{ in.}^4
\]

\[
Q = \sum A\bar{y} = (9")(\frac{1}{2}")(4.5")+(9")(\frac{1}{2}")(4.5")+(1.5")(3.5")(8.25") = 83.8 \text{ in}^3
\]

\[
f_{v_{\text{max}}} = \frac{(2,600\#)(83.3\text{in.}^3)}{(1,202.6\text{in.}^4)(\frac{1}{2}" + \frac{1}{2}")} = 180.2 \text{ psi}
\]

\[
Q = A\bar{y} = (5.25 \text{ in.}^2)(8.25") = 43.3 \text{ in}^3
\]

Shear force = $f_v \times A_v$, where:

$A_v$ = shear area

Assume:

$F$ = Capacity of two nails (one each side) at the flange; representing two shear surfaces

\[
(n)F \geq f_v \times b \times p = \frac{VQ}{lb} \times bp
\]

\[
\therefore (n)F \geq p \times \frac{VQ}{I}; \quad p \leq \frac{(n)FI}{VQ}
\]

At the maximum shear location (support) where $V = 2,600#$

\[
p \leq \frac{(2 \text{ nails} \times 80 \#/\text{nail})(1,202.6 \text{ in.}^4)}{(2,600\#)(43.3 \text{ in}^3)} = 1.71"
\]
Advanced Beam Analysis

Notation:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I )</td>
<td>moment of inertia with respect to neutral axis bending</td>
</tr>
<tr>
<td>( L )</td>
<td>beam span length</td>
</tr>
<tr>
<td>( M )</td>
<td>internal bending moment</td>
</tr>
<tr>
<td>( n )</td>
<td>relative location of the load on a span</td>
</tr>
<tr>
<td>( P )</td>
<td>name for a force vector</td>
</tr>
<tr>
<td>( R )</td>
<td>name for reaction force vector</td>
</tr>
<tr>
<td>( w )</td>
<td>name for distributed load</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>summation symbol</td>
</tr>
</tbody>
</table>

*Statically indeterminate* beams have more unknowns than equations provided by statics. But by adding more restraints, the deflections are significantly impacted.

This means that the maximum moment, if the beam was statically determinate with simple supports, can be said to be **redistributed** between positive and negative moments (which means the absolute value of the moments sum).

Approximate Analysis Methods

There are analysis methods based on the way the structure deforms which assume where the inflection points (having zero moment) may be. These inflection points are treated as *hinges*.

For example, the following beam has supports that aren’t entirely rigid, so the inflection points can be assumed to be closer to the supports than a rigidly supported beam (see *Beam Diagrams and Formulas*). Statics is used to isolate the center span, find the support forces reactions that end up as loads on the remaining bodies.
Analysis Methods

There are two general methods for analysis of statically indeterminate structures; the force or flexibility method, and the stiffness or displacement method.

- **Force Method** – The method obtains additional equations from writing equations that satisfy compatibility (consistent displacements) and force-displacement requirements. The **Theorem of Three Moments** is a force method.

- **Displacement Method** – The method is based on writing force-displacement relations for the members and then satisfying the equilibrium requirements for the structures. The unknowns are the displacements. Matrix methods use this format, as do most computer programs (like Multiframe3D).

**Theorem of Three Moments**

The general three-moment equation applies to continuous beams of constant section, the supports either being unyielding or settling* by known amounts. It gives a relationship between the moments at three adjacent supports, in terms of the loading on the two associated spans.

**General relation for fixed supports:** (*The settling equation has more terms.)

\[
M_1 \frac{L_1}{I_1} + 2M_2 \left( \frac{L_1}{I_1} + \frac{L_2}{I_2} \right) + M_3 \frac{L_2}{I_2} = -\sum \frac{P_1 I_1^2}{I_1} (n_1 - n_1^3) - \sum \frac{P_2 I_2^2}{I_2} (n_2 - n_2^3) - \frac{w_1 L_1^3}{4 I_1} - \frac{w_2 L_2^3}{4 I_2}
\]

where:
- \( M_1 \) is the bending moment at the left support of the two spans
- \( M_2 \) is the bending moment at the center support of the two spans
- \( M_3 \) is the bending moment at the right support of the two spans
- \( L_1 \) is the length of the left span
- \( L_2 \) is the length of the right span
- \( I_1 \) is the moment of inertia of the left span
- \( I_2 \) is the moment of inertia of the right span
- \( P_1 \) is the concentrated load on the left span
- \( P_2 \) is the concentrated load on the right span
- \( n_1 \) is the relative location of the concentrated load on the left span with respect to the span length
- \( n_1 \) is the relative location of the concentrated load on the left span with respect to the span length

For spans with only distributed loading AND constant moment of inertia \((I_1 = I_2)\), the general equation becomes:

\[
M_1 L_1 + 2M_2 (L_1 + L_2) + M_3 L_2 = -\frac{w_1 L_1^3}{4} - \frac{w_2 L_2^3}{4}
\]
For spans with only concentrated loading AND constant moment of inertia \((I_1 = I_2)\), the general equation becomes:

\[
M_1L_1 + 2M_2(L_1 + L_2) + M_3L_2 = -\sum P_1L_1^2(n_i - n_i^3) - \sum P_2L_2^2(n_2 - n_2^3)
\]

Continuous Beams with Two Spans and Symmetrical Loading

With symmetrical loading, the center support of a two equal-span continuous beam acts like a fixed support preventing any rotation and displacement. We can treat one span like a beam fixed at one end, supported at the other and use beam formulas and diagrams.
Example 1 (pg 128)

*Example 18.* Construct the shear and moment diagrams for the beam in Figure 3.28.

![Beam diagram with load and reactions](attachment:beam_diagram.png)

- $R_1 = 5.607\, k = 25.23\, kN$
- $R_2 = 15.343\, k = 69.04\, kN$
- $R_3 = 3.05\, k = 13.73\, kN$

Shear diagram:

- $V = 5.607\, k$ at $x = 0$
- $V = 6.95\, k$ at $x = 14\, ft$
- $V = 3.05\, k$ at $x = 10\, ft$

Moment diagram:

- $M_1 = 15.7\, k\cdot ft = 21.22\, kN\cdot m$
- $M_2 = 19.5\, k\cdot ft = 26.32\, kN\cdot m$
- $M_3 = 4.65\, k\cdot ft = 6.28\, kN\cdot m$
Frame & Pinned Systems

Notation:
- $F$ = name for force vectors
- $F_x$ = force component in the x direction
- $F_y$ = force component in the y direction
- $FBD$ = free body diagram
- $M$ = name for reaction moment, as is $M_R$
- $\Sigma$ = summation symbol
- $R$ = name for reaction force vector
- $w$ = name for distributed load
- $W$ = name for total force due to distributed load

Frame Systems

- A FRAME is made up of members where at least one member has more than 3 forces on it
  - Usually stationary and fully constrained

- A PINNED FRAME has member connected by pins
  - Considered non-rigid if it would collapse when the supports are removed
  - Considered rigid if it retains its original shape when the supports are removed

- A RIGID FRAME is all one member with no internal pins
  - Typically statically indeterminate
  - Portal frames look like door frames
  - Gable frames have a peak.

- INTERNAL PIN CONNECTIONS:
  - Pin connection forces are equal and opposite between the bodies they connect.
  - There are 2 unknown forces at a pin, but if we know a body is a two-force body the direction of the resultant force is known.
AN ARCH is a structural shape that can span large distances and sees compression along its slope. It may have no hinges (or pins), two hinges at the supports, or two hinges at the supports with a hinge at the apex. The three-hinged arch types are statically determinate with 2 bodies and 6 unknown forces.

CONTINUOUS BEAMS WITH PINS:
- If pins within the span of a beam over multiple supports result in static determinacy (the right number of unknowns for the number of equilibrium equations), the internal forces at the pins are applied as reactions to the adjacent span.

The location of the internal pins can be chosen to increase or decrease the moments in order to make the section economical for both positive bending and negative bending (similar values for the moments).

Solution Procedure

1. Solve for the support forces on the entire frame (FBD) if possible.
2. Draw a FBD of each member:
   - Consider all two-force bodies first.
   - Pins are integral with members
   - Pins with applied forces should belong to members with greater than two forces [Same if pins connect 3 or more members]
   - Draw forces on either side of a pin equal and opposite with arbitrary direction chosen for the first side
   - Consider all multi-force bodies
   - Represent connection forces not known by x & y components
   - There are still three equilibrium equations available, but the moment equations may be more helpful when the number of unknowns is greater than two.
Example 1

**Example 23.** Find the components of the reactions for the structure shown in Figure 3.44a.
Example 2

Example 4.13 (Three-Hinged Arch)

An industrial building is framed using tapered steel sections (haunches) and connected with three hinges (Figure 4.70). Assuming that the loads shown are from gravity loads and wind, determine the support reactions at $A$ and $D$ and the pin reactions at $B$.

Solution:

Construct a FBD (Figure 4.71) of the entire three-hinged arch and determine as many of the support reactions as possible using the three available equations of equilibrium.

Since there are four support reactions and only three equations of equilibrium, only $A_y$ and $D_y$ can be solved at this time.

$$
\sum M_A = +10\text{kN}(7\text{ m}) + 22\text{kN}(7\text{ m}) + 25\text{kN}(14\text{ m})
+ 12\text{kN}(21\text{ m}) - D_y(21\text{ m}) = 0
\therefore D_y = 39.3\text{kN}\ (\uparrow)
$$

$$
\sum F_y = +A_y - 10\text{kN} - 22\text{kN} - 25\text{kN}
- 12\text{kN} + 39.3\text{kN} = 0
\therefore A_y = +29.7\text{kN}\ (\uparrow)
$$

To determine $A_y$ and $D_y$, and the pin reactions at $B$, additional free-body diagrams are needed.

Separate the three-hinged arch into its two main components and draw the FBD of each.

Using Figure 4.72a, note that the 25-kN load at $B$ was assigned to member $AB$ (an assumption).

$$
\sum M_B = -22\text{kN}(7\text{ m}) - 10\text{kN}(14\text{ m})
- 10\text{kN}(3.5\text{ m}) + 29.7\text{kN}(14\text{ m})
- A_y(10.5\text{ m}) = 0
\therefore A_y = 8.3\text{kN}\ (\rightarrow)
$$

$$
\sum F_x = +8.3\text{kN} + 10\text{kN} - B_x = 0
\therefore B_x = +18.3\text{kN}
$$

$$
\sum F_y = +29.7\text{kN} - 10\text{kN} - 22\text{kN}
- 25\text{kN} + B_y = 0
\therefore B_y = +27.3\text{kN}
$$

The remaining unknown $D_x$ can be solved using free-body diagrams Figure 4.71 or Figure 4.72b.

Using Figure 4.72b:

$$
\sum F_x = +18.3\text{kN} - D_x = 0
\therefore D_x = +18.3\text{kN}\ (\leftarrow)
$$

As a check, substitute the answer for $D_x$ into an equation for the horizontal condition of equilibrium using Figure 4.71.
Example 3

Example 25. Investigate the beam shown in Figure 3.47.
Example 4

Example 24. Investigate the beam shown in Figure 3.46a. Find the reactions, draw the shear and moment diagrams, and sketch the deflected shape.

Solution: Because of the internal pin, the first 12 ft of the left-hand span acts as a simple beam. Its two reactions are therefore equal, being one half the total load, and its shear, moment, and deflected shape diagrams are those for a simple beam with a uniformly distributed load. (See Case 2, Figure 3.25). As shown in b and c in Figure 3.46, the simple beam reaction at the right end of the 12-ft portion of the left span becomes a 6-kip concentrated load at the left end of the remainder of the beam. This beam (Figure 3.46c) is then investigated as a beam with one overhanging end, carrying a single concentrated load at the cantilevered end and the total distributed load of 20 kips. (Note that on the diagram the total uniformly distributed load is indicated in the form of a single force, representing its resultant.) The second portion of the beam is statically determinate, and its reactions can now be determined by statics equations.

For left beam, the reaction forces are equal because it is symmetrical:

\[ R = W/2 = 12\text{k}/2 = 6\text{k} \]

With the equal and opposite for to the reaction of the 6 k at the pin, the right hand beam reactions can be found:

\[ \Sigma F_x: R_{1x} = 0 \]
\[ \Sigma F_y: -6k + R_{1y} -20k + R_2 = 0 \]
\[ \Sigma M_1: -6k(4\text{ft}) + 20k(6\text{ft}) - R_2(16\text{ft}) = 0 \]

So, \( R_2 = 6\text{k} \) and \( R_{1y} = 20\text{k} \)

The loaded area from the first support to the second support is \(-10(16\text{ft})(1\text{k/ft}) = -16\text{k} \), while the area from second to third supports is \(-16\text{k} \).

The shear starts at 0, adds 6k, subtracts 16k at the middle support (−10), adds 20k (10), subtracts 16k (−6), and adds 6k.

The triangle base is:

\[ x = 6\text{k}/1\text{k/ft} = 6\text{ft} \]

The first area is \( 6\text{k}(6\text{ft})/2 = 18\text{k-ft} \)

The second area is \( -10\text{k}(10\text{ft})/2 = -50\text{k-ft} \).
(Mirrored negatively on the right half.)

The moment starts at 0, adds 18k-ft, subtracts 50k-ft (−32k-ft), adds 50k-ft (18k-ft) and subtracts 18k-ft to close.

Figure 3.46 Reference for Example 24.
Rigid Frames -
Compression & Buckling

Notation:

- $A$ = name for area
- $d$ = name for depth
- $E$ = modulus of elasticity or Young’s modulus
- $f_a$ = axial stress
- $f_b$ = bending stress
- $f_z$ = stress in the x direction
- $F_a$ = allowable axial stress
- $F_b$ = allowable bending stress
- $F_x$ = force component in the x direction
- $F_y$ = force component in the y direction
- $FBD$ = free body diagram
- $G$ = relative stiffness of columns to beams in a rigid connection, as is $\Psi$
- $I$ = moment of inertia with respect to neutral axis bending
- $k$ = effective length factor for columns
- $\ell_b$ = length of beam in rigid joint
- $\ell_c$ = length of column in rigid joint
- $L$ = name for length
- $L_e$ = effective length that can buckle for column design, as is $\ell_e$
- $M$ = internal bending moment
- $P$ = name for a moment vector
- $P_{\text{crit}}$ = critical buckling load in column calculations, as is $P_{\text{critical}}, P_{cr}$
- $r$ = radius of gyration
- $V$ = internal shear force
- $y$ = vertical distance
- $\Delta$ = displacement due to bending
- $\pi$ = pi ($180^\circ$)
- $\Sigma$ = summation symbol
- $\Psi$ = relative stiffness of columns to beams in a rigid connection, as is $G$

**Rigid Frames**

Rigid frames are identified by the lack of pinned joints within the frame. The joints are rigid and resist rotation. They may be supported by pins or fixed supports. They are typically statically indeterminate.

Frames are useful to resist lateral loads.

Frame members will see
- shear
- bending
- axial forces

and behave like beam-columns.
Behavior

The relation between the joints has to be maintained, but the whole joint can rotate. The amount of rotation and distribution of moment depends on the stiffness \((EI/L)\) of the members in the joint.

End restraints on columns reduce the effective length, allowing columns to be more slender. Because of the rigid joints, deflections and moments in beams are reduced as well.

Frames are sensitive to settlement because it induces strains and changes the stress distribution.

Types

*Gabled* – has a peak

*Portal* – resembles a door. Multi-story, multiple bay portal frames are commonly used for commercial and industrial construction. The floor behavior is similar to that of continuous beams.

*Staggered Truss* – Full story trusses are staggered through the frame bays, allowing larger clear stories.

Connections

*Steel* – Flanges of members are fully attached to the flanges of the other member. This can be done with welding, or bolted plates.

*Reinforced Concrete* – Joints are monolithic with continuous reinforcement for bending. Shear is resisted with stirrups and ties.

Braced Frames

Braced frames have beams and columns that are “pin” connected with bracing to resist lateral loads.
Types of Bracing

- knee-bracing
- diagonal (including eccentric)
- X
- K or chevron
- shear walls – which resist lateral forces in the plane of the wall

Compression Members - Columns

Including strength (stresses) and servicability (including deflections), another requirement is that the structure or structural member be stable.

Stability is the ability of the structure to support a specified load without undergoing unacceptable (or sudden) deformations.

A column loaded centrically can experience unstable equilibrium, called buckling, because of how tall and slender they are. This instability is sudden and not good.

Buckling can occur in sheets (like my “memory metal” cookie sheet), pressure vessels or slender (narrow) beams not braced laterally.

Buckling can be thought of with the loads and motion of a column having a stiff spring at mid-height. There exists a load where the spring can’t resist the moment in it any longer.

Short (stubby) columns will experience crushing before buckling.
Critical Buckling Load

The critical axial load to cause buckling is related to the deflected shape we could get (or determine from bending moment of \( P \cdot \Delta \)).

The buckled shape will be in the form of a sine wave.

Euler Formula

Swiss mathematician Euler determined the relationship between the critical buckling load, the material, section and effective length (as long as the material stays in the elastic range):

\[
P_{\text{critical}} = \frac{\pi^2 EI}{L^2} \quad \text{or} \quad P_{\text{cr}} = \frac{\pi^2 EI}{(L_e)^2} = \frac{\pi^2 E A}{(L_e/r)^2}
\]

and the critical stress (if less than the normal stress) is:

\[
f_{\text{critical}} = \frac{P_{\text{critical}}}{A} = \frac{\pi^2 EA r^2}{A(L_e)^2} = \frac{\pi^2 E}{(L_e/r)^2}
\]

where \( I = Ar^2 \) and \( L_e/r \) is called the slenderness ratio. The smallest I of the section will govern, if the effective length is the same for box axes.

Yield Stress and Buckling Stress

The two design criteria for columns are that they do not buckle and the strength is not exceeded. Depending on slenderness, one will control over the other.

But, because in the real world, things are rarely perfect – and columns will not actually be loaded concentrically, but will see eccentricity – Euler’s formula is used only if the critical stress is less than half of the yield point stress, in the elastic buckling region. A transition formula is used for inelastic buckling.
Effective Length and Bracing

Depending on the end support conditions for a column, the effective length can be found from the deflected shape (elastic equations). If a very long column is braced intermittently along its length, the column length that will buckle can be determined. The effective length can be found by multiplying the column length by an effective length factor, $K$. 

$$L_e = K \cdot L$$
### Bending in Columns

Bending can occur in column like members when there are transverse loads such as wind and seismic loads, when the column is in a frame, or when the column load does not go through the axes. This situation is referred to as *eccentric loading* and the moment is of size $P \times e$.

![Diagram of column bending](image)

#### Eccentric Loading

<table>
<thead>
<tr>
<th>Theoretical $K$ value</th>
<th>0.5</th>
<th>0.7</th>
<th>1.0</th>
<th>1.0</th>
<th>2.0</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recommended design values when ideal conditions are approximated</td>
<td>0.65</td>
<td>0.80</td>
<td>1.0</td>
<td>1.2</td>
<td>2.10</td>
<td>2.0</td>
</tr>
</tbody>
</table>

- **Rotation fixed, Translation fixed**
- **Rotation free, Translation fixed**
- **Rotation fixed, Translation free**
- **Rotation free, Translation free**

---

**Note:**

*Wind Load Left*

*Seismic Load Left*

*Eccentric Load Left*

*Wind Load Left Braced at Top*
P-Δ (delta) Effect

The bending moment on a column will produce a lateral deflection. Because there is an axial load \( P \) on the column, there will be an addition moment produced of the size \( P \times \Delta \), which in turn will cause more deflection, increasing the moment, etc.. This non-linear increase in moment is called the \( P-\Delta \) effect. Design methods usually take this into account with magnification factors.

Combined Stresses

Within the elastic range (linear stresses) we can superposition or add up the normal and bending stresses (where \( M \) can be from \( P_e \) or calculated):

\[
f_x = f_a + f_b = \frac{P}{A} + \frac{My}{I}
\]

The resulting stress distribution is still linear. And the n.a. can move (if there is one)!

Interaction Design

Because there are combined stresses, we can’t just compare the axial stress to a limit axial stress or a bending stress to a limit bending stress. We use a limit called the interaction diagram. The diagram can be simplified as a straight line from the ratio of axial stress to allowable stress= 1 (no bending) to the ratio of bending stress to allowable stress = 1 (no axial load).

The interaction diagram can be more sophisticated (represented by a curve instead of a straight line). These types of diagrams take the effect of the bending moment increasing because the beam deflects. This is called the \( P-\Delta \) (P-delta) effect.
Limit Criteria Methods

1) \[ \frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \] interaction formula (bending in one direction)

2) \[ \frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \] interaction formula (biaxial bending)

3) \[ \frac{f_a}{F_a} + \frac{f_b \times (Magnification \ factor)}{F_{bx}} \leq 1.0 \] interaction formula (P-Δ effect)

Rigid Frame Analysis

Structural analysis methods such as the portal method (approximate), the method of virtual work, Castigliano’s theorem, the force method, the slope-displacement method, the stiffness method, and matrix analysis, can be used to solve for internal forces and moments and support reactions.

Shear and bending moment diagrams can be drawn for frame members by isolating the member from a joint and drawing a free body diagram. The internal forces at the end will be equal and opposite, just like for connections in pinned frames. Direction of the “beam-like” member is usually drawn by looking from the “inside” of the frame.

Frame Columns

Because joints can rotate in frames, the effective length of the column in a frame is harder to determine. The stiffness (EI/L) of each member in a joint determines how rigid or flexible it is. To find k, the relative stiffness, G or Ψ, must be found for both ends, plotted on the alignment charts, and connected by a line for braced and unbraced frames.
where

\[ E = \text{modulus of elasticity for a member} \]
\[ I = \text{moment of inertia of for a member} \]
\[ l_c = \text{length of the column from center to center} \]
\[ l_b = \text{length of the beam from center to center} \]

- For pinned connections we typically use a value of 10 for \( \Psi \).
- For fixed connections we typically use a value of 1 for \( \Psi \).
Lateral Buckling in Beams

With compression stresses in the top of a beam, a sudden “popping” or buckling can happen even at low stresses. In order to prevent it, we need to brace it along the top, or laterally brace it, or provide a bigger I_y.

Torsional buckling can result with simultaneous twisting and bending, which can be a problem with thin walled, non-symmetric sections.

Example 1 (pg 152)

Example 23. Find the components of the reactions and draw the shear and moment diagrams and the deformed shape of the frame in Figure 3.58a.
Example 2 (pg 154)

Example 24. Investigate the frame shown in Figure 3.60 for the reactions and internal conditions. Note that the right-hand support allows for an upward vertical reaction only, whereas the left-hand support allows for both vertical and horizontal components. Neither support provides moment resistance.
Example 3

Find the column effective lengths for a steel frame with 12 ft columns, a 15 ft beam when the support connections are pins for a) when it is braced and b) when it is allowed to sway. The relative stiffness of the beam is twice that of the columns (2I).
Frame Analysis Using Multiframe

1. The software is on the computers in the College of Architecture in Programs under the Windows Start menu. Multiframe is under the Bentley Engineering menu. It is also available at the Open Access Labs (OAL) and the Virtual OAL.

2. There are tutorials available on line at http://www.daystarsoftware.com/support/mftutorials that list the tasks and order in greater detail. The first task is to define the unit system:
   - Choose Units… from the View menu. Unit sets are available, but specific units can also be selected by double clicking on a unit or format and making a selection from the menu.

3. To see the scale of the geometry, a grid option is available:
   - Choose Grid… from the View menu

4. To create the geometry, you must be in the Frame window (default). The symbol is the frame in the window toolbar:

The Member toolbar shows ways to create members:

The Generate toolbar has convenient tools to create typical structural shapes.

- To create a frame, use the multi-bay frame button:
• Enter the number of bays (horizontally), number of stories (vertically) and the corresponding spacings:

![Generate Frame](image)

• If the frame does not have regular bays, use the add connected members button to create segments:

![Add Connected Members](image)

• Select a starting point and ending point with the cursor. The location of the cursor and the segment length is displayed at the bottom of the geometry window. The ESC button will end the segmented drawing.

• The geometry can be set precisely by selecting the joint (drag), and bringing up the joint properties menu (right click) to set the coordinates.

![Joint Properties Menu](image)

• The support types can be set by selecting the joint (drag) and using the Joint Toolbar (fixed shown), or the Frame / Joint Restraint ... menu (right click).

![Joint Toolbar](image)

NOTE: If the support appears at both ends of the member, you had the member selected rather than the joint. Select the joint to change support for and right click to select the joint restraints menu or select the correct support on the joint toolbar.

![Restraints Menu](image)

The support forces will be determined in the analysis.
5. All members must have sections assigned (see section 6.) in order to calculate reactions and deflections. To use a standard steel section proceed to step 6. For custom sections the section information must be entered. To define a section:

- Choose Edit Sections / Add Section... from the Edit menu
- Type a name for your new section
- Choose group Frame from the group names provided so that the section will remain with the file data
- Choose a shape. The Flat Bar shape is a rectangular section.
- Enter the cross section data.

Table values 1-9 must have values for a Flat Bar, but not all are used for every analysis. A recommendation is to put the value of 1 for those properties you don’t know or care about. Properties like \( t_f \), \( t_w \), etc. refer to wide flange sections.

- Answer any query. If the message says there is an error, the section will not be created until the error is corrected.

6. The standard sections library loaded is for the United States. If another section library is needed, use the Open Sections Library... command under the file menu, choose the library folder, and select the SectionsLibrary.slb file.

Select the members (drag to make bold) and assign sections with the Section button on the Member toolbar:

- Choose the group name and section name:
7. If there is an area that has a uniformly distributed load, load panels may be defined in the Frame window. Because the loaded area may not be visible in the current view, choose the View button at the lower left of the Frame window. The options for view are shown. (See 3D Frames, last page.)

- Choose the panel type (rectangular, 4-node, or 3-node) from the menu and select the corners. If the area is rectangular, only the opposite corners need to be selected.

- Select the panel and from the pop-up menu, or the Frame menu, specify the load panel supports. The default supports are on all sides. If the panel is one way, chose the corresponding picture.

8. The frame geometry is complete, and in order to define the load conditions you must be in the Load window represented by the green arrow:

9. The Load toolbar allows a joint to be loaded with a force or a moment in global coordinates, shown by the first two buttons after the display numbers button. It allows a member to be loaded with a distributed load, concentrated load or moment (next three buttons) in global coordinates, as well as loading with distributed or single force or moment in the local coordinate system (next three buttons). It allows a load panel to be loaded with a distributed load in global or local coordinates (last two buttons).

- Choose the member to be loaded (drag) and select the load type (here shown for global distributed loading):

- Choose the distribution type and direction. Note that the arrow shown is the direction of the loading. There is no need to put in negative values for downward loading.

- Enter the values of the load and distances (if any). Distances can be entered as a function of the length, i.e. L/2, L/4...

- Area load units may have to be changed in the View Units dialogue.

**NOTE:** Do not put support reactions as applied loads. The analysis will determine the reaction values.
Multiframe will automatically generate a grouping called a Load Case named **Load Case 1** when a load is created. All additional loads will be added to this load case unless a new load case is defined (Add case under the Case menu).

10. In order to run the analysis after the geometry, member properties and loading has been defined:
   - Choose Linear from the Analyze menu

11. If the analysis is successful, you can view the results in the Plot window represented by the red moment diagram:

12. The Plot toolbar allows the numerical values to be shown (1.0 button), the reaction arrows to be shown (brown up arrow) and reaction moments to be shown (brown curved arrow):
   - To show the moment diagram, Choose the red Moment button
   - To show the shear diagram, Choose the green Shear button
   - To show the axial force diagram, Choose the purple Axial Force button
   - To show the deflection diagram, Choose the blue Deflection button
   - To animate the deflection diagram, Choose Animate... from the Display menu. You can also save the animation to a .avi file by checking the box.
   - To see exact values of shear, moment and deflection, double click on the member and move the vertical cross hair with the mouse. The ESC key will return you to the window.
13. The Data window (D) allows you to view all data “entered” for the geometry, sections and loading. These values can be edited.

14. The Results window (R) allows you to view all results of the analysis including displacements, reactions, member forces (actions) and stresses. These values can be cut and pasted into other Windows programs such as Word or Excel.

NOTE: \( P_x' \) refers to the axial load (P) in the local axis \( x \) direction (\( x' \)). \( V_y' \) refers to the shear perpendicular to the local x axis, and \( M_z' \) refers to the bending moment.

15. To save the file Choose Save from the File menu.

16. To load an existing file Choose Open... from the File menu.

17. To print a plot Choose Print Window... from the File menu. As an alternative, you may copy the plot (Ctrl+c) and paste it in a word processing document (Ctrl+v).

Example of Combined Stresses:

for member 3: \( M_{\text{max}} = 19.6 \text{ k-ft}, P = 1.76 \text{ k} \)

knowing \( A = 21.46 \text{ in}^2, I = 796.0 \text{ in}^4, c = 7.08 \text{ in} \)

\[
f_{\text{max}} = \frac{1.76}{21.46} + \frac{19.6}{796} \cdot \frac{7.08}{2.092} = 2.174 \text{ksi}
\]

Results window:

<table>
<thead>
<tr>
<th>Member</th>
<th>Label</th>
<th>Joint</th>
<th>( S_{b2}' ) top</th>
<th>( S_{b2}' ) bot</th>
<th>( S_y' )</th>
<th>( S_x' )</th>
<th>( S_x' + S_{b2}' ) top</th>
<th>( S_x' + S_{b2}' ) bot</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Column 1</td>
<td>1</td>
<td>1.036</td>
<td>-1.036</td>
<td>-1152.461</td>
<td>0.286</td>
<td>1.235</td>
<td>-0.753</td>
</tr>
<tr>
<td>2</td>
<td>Column 3</td>
<td>1</td>
<td>2.062</td>
<td>2.062</td>
<td>-1152.461</td>
<td>0.286</td>
<td>2.092</td>
<td>2.378</td>
</tr>
<tr>
<td>3</td>
<td>Column 2</td>
<td>2</td>
<td>-1.036</td>
<td>1.036</td>
<td>1152.461</td>
<td>0.286</td>
<td>-0.753</td>
<td>1.235</td>
</tr>
<tr>
<td>4</td>
<td>Column 4</td>
<td>2</td>
<td>2.062</td>
<td>2.062</td>
<td>1152.461</td>
<td>0.286</td>
<td>2.092</td>
<td>2.378</td>
</tr>
<tr>
<td>5</td>
<td>X Prima 3</td>
<td>3</td>
<td>-2.062</td>
<td>2.062</td>
<td>4032.245</td>
<td>0.002</td>
<td>-2.011</td>
<td>2.174</td>
</tr>
<tr>
<td>6</td>
<td>X Prima 4</td>
<td>3</td>
<td>-2.062</td>
<td>2.062</td>
<td>-4032.245</td>
<td>0.002</td>
<td>-2.011</td>
<td>2.174</td>
</tr>
</tbody>
</table>

where \( S_x' \) refers to the axial stress, \( S_y' \) refers to the bending stress around the local vertical axis and \( S_z' \) refers to the bending stress around the local horizontal axis.
For 3D Frames:

- There are tutorials available online at [http://www.formsys.com/mflearning](http://www.formsys.com/mflearning) that list the tasks and order in greater detail. It expects that you have been through the 2D tutorial to build on the steps already mastered.

- There are standard 3D frame shapes on the frame toolbar.

- It is very useful to change the view to isometric with the View Button.

- If you wish to have additional beams supported by the beams of your frame, choose the beam and use the Subdivide Member menu under Geometry. This will make additional joints, but keep the segments together.

- In order to model a beam end as simply supported, you must release the restraint preventing rotation about the x-x axis of the beam. The pinned ends menu is useful for segments or subdivided members.
Or, by selecting a segment and right clicking for a menu, you can use Member Releases (also under the Frame menu) to release the Major Bending ($M'_Z$) for one end or both.

- It is necessary to understand the local member axes to assign the correct load direction. Choosing the *local* loading types will show the member orientation with respect to the load direction.
## Design Loads and Methodology

### Notation:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>name for area</td>
</tr>
<tr>
<td>$ASCE$</td>
<td>American Society of Civil Engineers</td>
</tr>
<tr>
<td>$ASD$</td>
<td>allowable stress design</td>
</tr>
<tr>
<td>$D$</td>
<td>dead load symbol</td>
</tr>
<tr>
<td>$E$</td>
<td>earthquake load symbol</td>
</tr>
<tr>
<td>$F$</td>
<td>hydraulic loads from fluids symbol</td>
</tr>
<tr>
<td>$H$</td>
<td>hydraulic loads from soil symbol</td>
</tr>
<tr>
<td>$L$</td>
<td>live load symbol</td>
</tr>
<tr>
<td>$L_r$</td>
<td>live roof load symbol</td>
</tr>
<tr>
<td>$LRFD$</td>
<td>load and resistance factor design</td>
</tr>
<tr>
<td>$R$</td>
<td>rainwater load or ice water load symbol</td>
</tr>
<tr>
<td>$S$</td>
<td>snow load symbol</td>
</tr>
<tr>
<td>$T$</td>
<td>effect of material &amp; temperature symbol</td>
</tr>
<tr>
<td>$t$</td>
<td>name for thickness</td>
</tr>
<tr>
<td>$W$</td>
<td>wind load symbol</td>
</tr>
<tr>
<td>$V$</td>
<td>name for volume</td>
</tr>
<tr>
<td>$w$</td>
<td>name for distributed load</td>
</tr>
<tr>
<td>$V$</td>
<td>name for total force due to distributed load</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>density or unit weight</td>
</tr>
</tbody>
</table>

### Design Codes in General

Design codes are issued by a professional organization interested in insuring safety and standards. They are legally backed by the engineering profession. Different design methods are used, but they typically defined the *load cases or combination*, stress or strength limits, and deflection limits.

### Load Types

Loads used in design load equations are given letters by type:

- $D$ = dead load
- $L$ = live load
- $L_r$ = live roof load
- $W$ = wind load
- $S$ = snow load
- $E$ = earthquake load
- $R$ = rainwater load or ice water load
- $T$ = effect of material & temperature
- $H$ = hydraulic loads from soil
- $F$ = hydraulic loads from fluids

### Determining Dead Load from Material Weights

Material density is a measure of how much mass in a unit volume causes a force due to gravity. The common symbol for density is $\gamma$. When volume, $V$, is multiplied by density, a force value results:

$$W = \gamma \cdot V$$

Materials “weight” can also be presented as a weight per unit area. This takes into account that the volume is a constant thickness times an area: $V = t \cdot A$; so the calculation becomes:

$$W = (\text{weight/unit area}) \cdot A$$
Allowable Stress Design (ASD)

Combinations of service (also referred to as working) loads are evaluated for maximum stresses and compared to allowable stresses. The allowed stresses are some fraction of limit stresses.

ASCE-7 (2010) combinations of loads:

1. $D$
2. $D + L$
3. $D + 0.75(L_r \text{ or } S \text{ or } R)$
4. $D + 0.75L + 0.75(L_r \text{ or } S \text{ or } R)$
5. $D + (0.6W \text{ or } 0.7E)$
6a. $D + 0.75L + 0.75(0.6W) + 0.75(L_r \text{ or } S \text{ or } R)$
6b. $D + 0.75L + 0.75(0.7E) + 0.75S$
7. $0.6D + 0.6W$
8. $0.6D + 0.7E$

When $F$ loads are present, they shall be included with the same load factor as dead load $D$ in 1 through 6 and 8.

When $H$ loads are present, they shall have a load factor of 1.0 when adding to load effect, or 0.6 when resisting the load when permanent.

Load and Resistance Factor Design – LRFD

Combinations of loads that have been factored are evaluated for maximum loads, moments or stresses. These factors take into consideration how likely the load is to happen and how often. This “imaginary” worse case load, moment or stress is compared to a limit value that has been modified by a resistance factor. The resistance factor is a function of how “comfortable” the design community is with the type of limit, ie. yielding or rupture...

ASCE-7 (2010) combinations of factored nominal loads:

1. $1.4D$
2. $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
3. $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W)$
4. $1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R)$
5. $1.2D + 1.0E + L + 0.2S$
6. $0.9D + 1.0W$
7. $0.9D + 1.0E$

When $F$ loads are present, they shall be included with the same load factor as dead load $D$ in 1 through 5 and 7.

When $H$ loads are present, they shall have a load factor of 1.6 when adding to load effect, or 0.9 when resisting the load when permanent.

Load Tracing

- LOAD TRACING is the term used to describe how the loads on and in the structure are transferred through the members (load paths) to the foundation, and ultimately supported by the ground.

- It is a sequence of actions, NOT reactions. Reactions in statically determinate members (using FBD’s) can be solved for to determine the actions on the next member in the hierarchy.
• The tributary area is a loaded area that contributes to the load on the member supporting that area, ex. the area from the center between two beams to the center of the next two beams for the full span is the load on the center beam. It can also be called the load periphery.

• The tributary load on the member is found by concentrating (or consolidating) the load into the center.

\[ w = \left( \frac{\text{load}}{\text{area}} \right) \times (\text{tributary width}) \]

where:

\[ w = \text{distributed load in units of load/length} \]

Distribution of Loads with Irregular Configurations

When a bay (defined by the area bounded by vertical supports) is not rectangular, it is commonly constructed with parallel or non-parallel spanning members of non-uniform lengths. With parallel spanning members, the tributary width is uniform. With non-parallel members, the tributary width at each end is different, but still defined as half the distance (each side) to the next member. The resulting distribution will be linear (and not uniform).

The most efficient one-way systems have regular, rectangular bays. Two way systems are most efficient when they are square. With irregular bays, attempts are made to get as many parallel members as possible with similar lengths, resulting in an economy of scale.
Distribution of Loads on Edge Supported Slabs

Distributed loads on two-way slabs (i.e. not one-way like beams) do not have obvious tributary “widths”. The distribution is modeled using a 45 degree tributary “boundary” in addition to the tributary boundary that is half way between supporting elements, in this case, edge beams.

![Figure 2-16: Supporting beams' contributing areas for reinforced concrete floor system.](image)

The tributary distribution from the area loads result in a trapezoidal distribution. Self weight will be a uniform distributed load, and will also have to be included for design of beam AB.

![Figure 2-17: Trapezoidal distributed load for Beam AB of Fig. 2-16.](image)

Openings in Floor/Roof Plans

Openings in a horizontal system usually are framed on all sides. This provides for stiffness and limiting the deflection. The edge beams may not be supporting the flooring, however, so care needs to be taken to determine if an opening edge beam must support tributary area, or just itself.

- Any edge beam supporting a load has load on only one side to the next supporting element.

Beams Supported by Other Beams

Joists are commonly supported by beams with beam hangers. The reaction at the support is transferred to the beam as a single force. A beam, in turn, can be supported by a larger beam or girder, and the reaction from this beam having a uniform distributed self weight, and the forces, will be an action on the girder.
Framing Plans

Framing plans are diagrams representing the placement and organization of structural members. Until the final architecture has been determined, framing plans are often drawn freehand with respect to the floor plans, and quite often use the formal conventions for structural construction drawings.

Parts of the building are identified by letter symbols:

- **B** – Beams
- **C** – Columns
- **D** – Dowels
- **F** – Footings
- **G** – Girders
- **J** – Joists
- **L** – Lintels
- **S** – Slabs
- **T** – Ties
- **U** – Stirrups
- **W** – Walls
- **D** – Dowels

Other parts are represented with lines (beams and joists), dots, squares, rectangles or wide-flange shapes for columns. Column and footing locations in structural drawings are referred to by letters and numbers, with vertical lines at column centers given letters – A, B, C, etc., and horizontal lines at columns given numbers – 1, 2, 3, etc. The designation *do* may be used to show like members (like *ditto*).
Breaks in the lines are commonly used to indicate the *end* of a beam that is supported by another member, such as a girder or column. Beams can span over a support (as a continuous beam) and therefore, there is no break shown at the column.

Joists can span over a supporting beam, and the lines will cross. (Looking for the ends of the crossing members give information about which is below and which is above.)

Concrete systems often have slabs, ribs or drop panels or strips, which aren’t easily represented by centerlines, so hidden lines represent the edges. Commonly isolated “patches” of repeated geometry are used for brevity.
Example 1 (pg 50)
Identify the tributary area for beams A-C, and columns 1-4 for the plan shown (twice).
Example 2
In the single-bay, post-and-beam deck illustrated, planks typically are available in nominal widths of 4" or 6", but for the purposes of analysis it is permissible to assume a unit width equal to one foot. Determine the plank, beam, and column reactions.
The loads are: 60 lb/ft² live load, 8 lb/ft² dead load, 10 lb/ft self weight of 12' beams, and 100 lb self weight of columns.
Example 3

Assume that the average dead plus live load on the structure shown in Figure 3.15 is 60 lbs/ft$^2$. Determine the reactions for Beam D. This is the same structure as shown in Figure 3.1.

Assuming all beams are weightless!

Solution:

Note carefully the directions of the decking span. Beam D carries floor loads from the decking to the left (see the contributory area and load strip), but not to the right, since the center decking runs parallel to Beam D and is not carried by it. Beam D also picks up the end of Beam G and thus also "carries" the reactive force from Beam G. It is therefore necessary to analyze Beam G first to determine the magnitude of this force. The analysis appears in Figure 3.15. The reactive force from Beam G of 2160 lbs is then treated as a downward force acting on Beam D. The load model for Beam D thus consists of distributed forces from the decking plus the 2160-lb force. It is then analyzed by means of the equations of statics to obtain reactive forces of 4896 lbs and 4464 lbs at its ends.

Beam A:

\[
\begin{align*}
R_{CA} &= 4896 \text{ lb} \\
R_{EA} &= 4896 \text{ lb}
\end{align*}
\]

By symmetry: \( R_{CC1} = R_{CC3} = (4896 \text{ lb} + 4896 \text{ lb})/2 = 4896 \text{ lb} \)

Beam B:

\[
\begin{align*}
R_{DB} &= 4464 \text{ lb} \\
R_{EB} &= 4464 \text{ lb}
\end{align*}
\]

By symmetry: \( R_{CC2} = R_{CC4} = (4464 \text{ lb} + 4464 \text{ lb})/2 = 4464 \text{ lb} \)

Additional loads are transferred to the column from the reactions on Beams C and F:

\[
\begin{align*}
R_{C1} &= R_{C2} = R_{F1} = R_{F2} = wL/2 = (6 \text{ ft})(60 \text{ lb/ft})^2(20 \text{ ft})/2 = 3600 \text{ lb} \\
R_{D1} &= 4464 \text{ lb} = R_{E1}
\end{align*}
\]

C1 = 4896 lb + 3600 lb = 8,496 lb
C2 = 6624 lb + 3600 lb = 10,224 lb
C3 = 4896 lb + 3600 lb = 8,496 lb
C4 = 6624 lb + 3600 lb = 10,224 lb
Example 4

A steel-framed floor for an office building, as shown in Figures 5.54 to 5.56, was designed to support a load condition as follows:

**Loads:**

- **Live load** = 50 psf
- **Dead loads:**
  - Concrete = 150 lb/ft.³
  - Steel decking = 5 psf
  - Mechanical equipment = 10 psf
  - Suspended ceiling = 5 psf
  - Steel beams = 25 lb/ft.
  - Steel girders = 35 lb/ft.

Using appropriate FBDs, determine the reaction forces for beams B-1, B-2, and B-3, and girder G-1.

**Solution:**

**Loads:**

- Slab load = \( \left( \frac{4 \text{ in.}}{12 \text{ in./ft}} \right) \times (150 \text{ lb/ft.}^3) = 50 \text{ lb/ft.}^2 \)
- Dead loads:
  - 50 psf (slab)
  - 5 psf (decking)
  - 10 psf (mech. equip.)
  - 5 psf (ceiling)

**Total DL** = 70 psf

Dead load + Live load = 70 psf + 50 psf = 120 psf

**Beam B-1** (Figures 5.57 and 5.58):

(Tributary width of load is 6')

\[
\omega_1 = \left( 120 \text{ lb/ft.}^2 \right) \times (6 \text{ ft.}) + \left( 25 \text{ lb./ft.} \right) \left( \frac{\text{beam wt.}}{\text{beam wt.}} \right) = 745 \text{ lb./ft.}
\]

Figure 5.54  (a) Isometric view of partial steel framing arrangement. (b) Partial floor framing—office structure.

Figure 5.55  Section A at girder G-1.

Figure 5.56  Section B at beam B-2.

Figure 5.57  Tributary width for beam B-1.

Figure 5.58  FBD of beam B-1.
Example 4 (continued)

Beam B-2 (Figures 5.59 and 5.60):
(tributary width of load is 6' + 6' = 12')

\[ \omega_2 = \left(120 \text{ lb/ft}^2 \right) \times (12 \text{ ft.}) + 25 \text{ lb/ft} = 1465 \text{ lb/ft.} \]

Beam B-2

\[ \omega_2 = 1465 \text{ lb/ft.} \]

L = 20 ft.
(\(\omega_{\text{beam}} = 25 \text{ lb/ft.}\))

R
(14,850 lb.)
(14,650 lb.)

Figure 5.59 Tributary width for beam B-2. Figure 5.60 FBD of beam B-2.

Beam B-3 (Figures 5.61 to 5.62): This beam has two different load conditions due to the changing tributary width created by the opening.

For 12' of span:

\[ \omega_3 = \left(120 \text{ lb/ft}^2 \right) \times (12 \text{ ft.}) + 25 \text{ lb/ft} = 1465 \text{ lb/ft.} \]

\[ \sum M_a = 0 \]
\[ - (745 \text{ lb/ft}) (8 \text{ ft.}) (4 \text{ ft.}) - (1465 \text{ lb/ft}) (12 \text{ ft.}) (4 \text{ ft.}) + B_y (20 \text{ ft.}) = 0 \]
\[ \therefore B_y = 13,498 \text{ lb.} \]
\[ \sum F_y = 0 \]
\[ - (745 \text{ lb/ft}) (8 \text{ ft.}) - (1465 \text{ lb/ft}) (12 \text{ ft.}) + 13,498 \text{ lb.} + A_y = 0 \]
\[ \therefore A_y = 10,042 \text{ lb.} \]

Beam B-3

\[ \omega_3 = 1465 \text{ lb/ft.} \]

Figure 5.61 Tributary widths for beam B-3.

Figure 5.62 FBD of beam B-3.

Girder G-1 (Figures 5.63 and 5.64): Girder G-1 supports reactions from beams B-2 and B-3. Beam B-1 sends its reaction directly to the column and causes no load to appear in girder G-1.

\[ \left[ \sum M_a = 0 \right] \]
\[ - (14,650 \text{ lb.}) (12 \text{ ft.}) - (13,498 \text{ lb.}) (24 \text{ ft.}) \]
\[ - (35 \text{ lb/ft}) (36 \text{ ft.}) (18 \text{ ft.}) + B_y (36 \text{ ft.}) = 0 \]
\[ \therefore B_y = 14,512 \text{ lb.} \]

\[ \left[ \sum F_y = 0 \right] - 14,650 \text{ lb.} - 13,498 \text{ lb.} \]
\[ + 14,512 \text{ lb.} + A_y = 0 \]
\[ \therefore A_y = 14,896 \text{ lb.} \]

Girder G-1 (partial framing).

Figure 5.63 Girder G-1 (partial framing).
### Common Design Loads in Building Codes
adapted from SEI/ASCE 7-10: Minimum Design Loads for Buildings and Other Structures

#### Minimum Concentrated Loads

<table>
<thead>
<tr>
<th>Location</th>
<th>Concentrated load lb (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catwalks for maintenance access</td>
<td>300 (1.33)</td>
</tr>
<tr>
<td>Elevator machine room grating (on area of 2 in. by 2 in. (50 mm by 50 mm))</td>
<td>300 (1.33)</td>
</tr>
<tr>
<td>Finish light floor plate construction (on area of 1 in. by 1 in. (25 mm by 25 mm))</td>
<td>200 (0.89)</td>
</tr>
<tr>
<td>Hospital floors</td>
<td>1,000 (4.45)</td>
</tr>
<tr>
<td>Library floors</td>
<td>1,000 (4.45)</td>
</tr>
<tr>
<td>Manufacturing</td>
<td></td>
</tr>
<tr>
<td>Light</td>
<td>2,000 (8.90)</td>
</tr>
<tr>
<td>Heavy</td>
<td>3,000 (13.40)</td>
</tr>
<tr>
<td>Office floors</td>
<td>2,000 (8.90)</td>
</tr>
<tr>
<td>Awnings and canopies</td>
<td></td>
</tr>
<tr>
<td>Screen enclosure support frame</td>
<td>200 (0.89)</td>
</tr>
<tr>
<td>Roofs – primary members and subject to maintenance workers</td>
<td>300 (1.33)</td>
</tr>
<tr>
<td>School floors</td>
<td>1,000 (4.45)</td>
</tr>
<tr>
<td>Sidewalks, vehicular driveways, and yards subject to trucking (over wheel area of 4.5 in. by 4.5 in. (114 mm x 114 mm))</td>
<td>8,000 (35.60)</td>
</tr>
<tr>
<td>Stairs and exit ways on area of 2 in. by 2 in. (50 mm by 50 mm) non-concurrent with uniform load</td>
<td>300 (1.33)</td>
</tr>
<tr>
<td>Store floors</td>
<td>1,000 (4.45)</td>
</tr>
</tbody>
</table>

#### Minimum Uniformly Distributed Live Loads

<table>
<thead>
<tr>
<th>Location</th>
<th>Uniform load psf (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apartments (see Residential)</td>
<td></td>
</tr>
<tr>
<td>Access floor systems</td>
<td></td>
</tr>
<tr>
<td>Office use</td>
<td>50 (2.4)</td>
</tr>
<tr>
<td>Computer use</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Armories and drill rooms</td>
<td>150 (7.18)</td>
</tr>
<tr>
<td>Assembly areas</td>
<td></td>
</tr>
<tr>
<td>Fixed seats (fastened to floor)</td>
<td>60 (2.87)</td>
</tr>
<tr>
<td>Lobbies</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Movable seats</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Platforms (assembly)</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Stage floors</td>
<td>150 (7.18)</td>
</tr>
<tr>
<td>Assembly areas (other)</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Balconies and decks</td>
<td></td>
</tr>
<tr>
<td>Catwalks for maintenance access</td>
<td>40 (1.92)</td>
</tr>
<tr>
<td>Corridors</td>
<td></td>
</tr>
<tr>
<td>First floor</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Other floors</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.5 times the live load for the area served. Not required to exceed 100 psf (4.79 kN/m²)</td>
</tr>
<tr>
<td></td>
<td>same as occupancy served except as indicated</td>
</tr>
<tr>
<td>Location</td>
<td>Uniform load psf (kN/m²)</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>Dining rooms and restaurants</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Dwellings (see Residential)</td>
<td></td>
</tr>
<tr>
<td>Elevator machine room grating (on area of 2 in. by 2 in. (50 mm by 50 mm))</td>
<td>300 (1.33)</td>
</tr>
<tr>
<td>Finish light floor plate construction (on area of 1 in. by 1 in. (25 mm by 25 mm))</td>
<td>200 (0.89)</td>
</tr>
<tr>
<td>Fire escapes</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Garages</td>
<td></td>
</tr>
<tr>
<td>Passenger vehicles only</td>
<td>40 (1.92)</td>
</tr>
<tr>
<td>Helipads</td>
<td>60 (2.87)</td>
</tr>
<tr>
<td>Hospitals</td>
<td></td>
</tr>
<tr>
<td>Operating rooms, laboratories</td>
<td>60 (2.87)</td>
</tr>
<tr>
<td>Patient rooms</td>
<td>40 (1.92)</td>
</tr>
<tr>
<td>Corridors above first floor</td>
<td>80 (3.83)</td>
</tr>
<tr>
<td>Hotels (see Residential)</td>
<td></td>
</tr>
<tr>
<td>Libraries</td>
<td></td>
</tr>
<tr>
<td>Reading rooms</td>
<td>60 (2.87)</td>
</tr>
<tr>
<td>Stack rooms</td>
<td>150 (7.18)</td>
</tr>
<tr>
<td>Corridors above first floor</td>
<td>80 (3.83)</td>
</tr>
<tr>
<td>Manufacturing</td>
<td></td>
</tr>
<tr>
<td>Light</td>
<td>125 (6.00)</td>
</tr>
<tr>
<td>Heavy</td>
<td>250 (11.97)</td>
</tr>
<tr>
<td>Office buildings</td>
<td></td>
</tr>
<tr>
<td>File and computer rooms shall be designed for heavier loads based on anticipated occupancy</td>
<td></td>
</tr>
<tr>
<td>Lobbies and first floor corridors</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Offices</td>
<td>50 (2.40)</td>
</tr>
<tr>
<td>Corridors above first floor</td>
<td>80 (3.83)</td>
</tr>
<tr>
<td>Penal institutions</td>
<td></td>
</tr>
<tr>
<td>Cell blocks</td>
<td>40 (1.92)</td>
</tr>
<tr>
<td>Corridors</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Recreational uses</td>
<td></td>
</tr>
<tr>
<td>Bowling alleys, poolrooms, and similar uses</td>
<td>75 (3.59)</td>
</tr>
<tr>
<td>Dance halls and ballrooms</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Gymnasiums</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Reviewing stands, grandstands, and bleachers</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Stadiums and arenas with fixed seats (fastened to the floor)</td>
<td>60 (2.87)</td>
</tr>
<tr>
<td>Residential</td>
<td></td>
</tr>
<tr>
<td>One- and two-family dwellings</td>
<td></td>
</tr>
<tr>
<td>Uninhabitable attics without storage</td>
<td>10 (0.48)</td>
</tr>
<tr>
<td>Uninhabitable attics with storage</td>
<td>20 (0.96)</td>
</tr>
<tr>
<td>Habitable attics and sleeping areas</td>
<td>30 (1.44)</td>
</tr>
<tr>
<td>All other areas except stairs</td>
<td>40 (1.92)</td>
</tr>
<tr>
<td>All other residential occupancies</td>
<td></td>
</tr>
<tr>
<td>Private rooms and corridors serving them</td>
<td>40 (1.92)</td>
</tr>
<tr>
<td>Public rooms and corridors serving them</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Roofs</td>
<td></td>
</tr>
<tr>
<td>Ordinary flat, pitched, and curved roofs</td>
<td>20 (0.96n)</td>
</tr>
<tr>
<td>Roofs used for roof gardens</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Roofs used for assembly purposes</td>
<td>Same as occupancy served</td>
</tr>
<tr>
<td>Roofs used for other occupancies</td>
<td>As approved by authority</td>
</tr>
<tr>
<td>Awnings and canopies</td>
<td></td>
</tr>
<tr>
<td>Fabric construction supported by a skeleton structure</td>
<td>5 (0.24) nonreducible</td>
</tr>
<tr>
<td>Location</td>
<td>Uniform load psf (kN/m²)</td>
</tr>
<tr>
<td>--------------------------------------------------------------</td>
<td>--------------------------------------------------------------</td>
</tr>
<tr>
<td>Screen enclosure support frame</td>
<td>5 (0.24) nonreducible and based on the tributary area of the roof supported by the frame</td>
</tr>
<tr>
<td>All other construction</td>
<td>20 (0.96)</td>
</tr>
<tr>
<td>Schools</td>
<td></td>
</tr>
<tr>
<td>Classrooms</td>
<td>40 (1.92)</td>
</tr>
<tr>
<td>Corridors above first floor</td>
<td>80 (3.83)</td>
</tr>
<tr>
<td>First-floor corridors</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Scuttles, skylight ribs, and accessible ceilings</td>
<td>200 (0.89)</td>
</tr>
<tr>
<td>Sidewalks, vehicular driveways, and yards subject to trucking</td>
<td>250 (11.97)</td>
</tr>
<tr>
<td>Stairs and exit ways</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>One- and two-family dwellings only</td>
<td>40 (1.92)</td>
</tr>
<tr>
<td>Storage areas above ceilings</td>
<td>20 (0.96)</td>
</tr>
<tr>
<td>Storage warehouses (shall be designed for heavier loads if required for anticipated storage)</td>
<td></td>
</tr>
<tr>
<td>Light</td>
<td>125 (6.00)</td>
</tr>
<tr>
<td>Heavy</td>
<td>250 (11.97)</td>
</tr>
<tr>
<td>Stores</td>
<td></td>
</tr>
<tr>
<td>Retail</td>
<td></td>
</tr>
<tr>
<td>First floor</td>
<td>100 (4.79)</td>
</tr>
<tr>
<td>Upper floors</td>
<td>75 (3.59)</td>
</tr>
<tr>
<td>Wholesale, all floors</td>
<td>125 (6.00)</td>
</tr>
<tr>
<td>Walkways and elevated platforms (other than exit ways)</td>
<td>60 (2.87)</td>
</tr>
<tr>
<td>Yards and terraces, pedestrian</td>
<td>100 (4.79)</td>
</tr>
</tbody>
</table>

Live load reductions are not permitted for specific types (see code).
Some occupancies must be designed for appropriate loads as approved by the authority having jurisdiction.
Library stack room floors have specified limitations (see code)
AASHTO lane loads should also be considered where appropriate.
<table>
<thead>
<tr>
<th>Substance</th>
<th>Weight lb per cu ft</th>
<th>Specific Gravity</th>
<th>Substance</th>
<th>Weight lb per cu ft</th>
<th>Specific Gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASHLEAI, MASONRY</td>
<td>165</td>
<td>2.3-3.0</td>
<td>Aiken</td>
<td>153</td>
<td>2.1-2.8</td>
</tr>
<tr>
<td>Granite, syenite, gneiss</td>
<td>160</td>
<td>2.3-2.8</td>
<td>Barytes</td>
<td>281</td>
<td>2.8-3.2</td>
</tr>
<tr>
<td>Lime, mastic</td>
<td>160</td>
<td>2.3-2.8</td>
<td>Basalt</td>
<td>281</td>
<td>2.8-3.2</td>
</tr>
<tr>
<td>Sandstone, bluestone</td>
<td>130</td>
<td>2.0-2.2</td>
<td>Barite</td>
<td>281</td>
<td>2.8-3.2</td>
</tr>
<tr>
<td>MORROR RUBBLE</td>
<td>155</td>
<td>2.2-2.8</td>
<td>Bentonite</td>
<td>139</td>
<td>2.4-2.8</td>
</tr>
<tr>
<td>MASONRY</td>
<td>150</td>
<td>2.2-2.8</td>
<td>Clay marl</td>
<td>137</td>
<td>2.4-2.8</td>
</tr>
<tr>
<td>Lime, mastic</td>
<td>150</td>
<td>2.2-2.8</td>
<td>Dolomite</td>
<td>201</td>
<td>2.4-2.8</td>
</tr>
<tr>
<td>Sandstone, bluestone</td>
<td>130</td>
<td>2.0-2.2</td>
<td>Feldspar, orthoclase</td>
<td>159</td>
<td>2.4-2.8</td>
</tr>
<tr>
<td>DRY RUBBLE MASONRY</td>
<td>130</td>
<td>1.9-2.3</td>
<td>Greenstone, trap</td>
<td>175</td>
<td>2.4-2.8</td>
</tr>
<tr>
<td>Granite, syenite, gneiss</td>
<td>125</td>
<td>1.9-2.1</td>
<td>Gypsum, plaster</td>
<td>175</td>
<td>2.4-2.8</td>
</tr>
<tr>
<td>Lime, mastic</td>
<td>125</td>
<td>1.9-2.1</td>
<td>Hornblende</td>
<td>159</td>
<td>2.4-2.8</td>
</tr>
<tr>
<td>Brick</td>
<td>110</td>
<td>1.8-1.9</td>
<td>Lime, mastic</td>
<td>165</td>
<td>2.5-2.8</td>
</tr>
<tr>
<td>Pressed brick</td>
<td>140</td>
<td>2.2-2.3</td>
<td>Magnesite</td>
<td>187</td>
<td>2.5-2.8</td>
</tr>
<tr>
<td>Common brick</td>
<td>130</td>
<td>1.8-2.0</td>
<td>Phosphate rock, apatite</td>
<td>200</td>
<td>2.5-2.8</td>
</tr>
<tr>
<td>Soft brick</td>
<td>100</td>
<td>1.5-1.7</td>
<td>Prophyll</td>
<td>172</td>
<td>2.5-2.8</td>
</tr>
<tr>
<td>CONCRETE MASONRY</td>
<td>144</td>
<td>2.2-2.4</td>
<td>Pumice, natural</td>
<td>40</td>
<td>0.7-0.9</td>
</tr>
<tr>
<td>Cement, concrete, sand</td>
<td>130</td>
<td>1.9-2.3</td>
<td>Quartz, fire</td>
<td>165</td>
<td>2.5-2.8</td>
</tr>
<tr>
<td>Cement, cinder, etc</td>
<td>100</td>
<td>1.5-1.7</td>
<td>Sandstone, bluestone</td>
<td>147</td>
<td>2.5-2.8</td>
</tr>
<tr>
<td>SAND</td>
<td>67-72</td>
<td>-</td>
<td>Slate, shale</td>
<td>175</td>
<td>2.7-2.9</td>
</tr>
<tr>
<td>Sands, slag</td>
<td>70-148</td>
<td>-</td>
<td>Soapstone, talc</td>
<td>165</td>
<td>2.6-2.8</td>
</tr>
<tr>
<td>STONE QUARRIED, PILED</td>
<td>-</td>
<td>-</td>
<td>STONE QUARRIED, PILED</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Basalt, granite, gneiss</td>
<td>90</td>
<td>-</td>
<td>STONE QUARRIED, PILED</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Lime, mastic, quartz</td>
<td>95</td>
<td>-</td>
<td>STONE QUARRIED, PILED</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Sandstone</td>
<td>82</td>
<td>-</td>
<td>STONE QUARRIED, PILED</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Slate</td>
<td>92</td>
<td>-</td>
<td>STONE QUARRIED, PILED</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Greenstone, hornblende</td>
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<td>-</td>
<td>STONE QUARRIED, PILED</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>BITUMINOUS SUBSTANCES</td>
<td>81</td>
<td>1.1-1.5</td>
<td>Asphalt</td>
<td>97</td>
<td>1.4-1.7</td>
</tr>
<tr>
<td>Coke</td>
<td>163</td>
<td>1.2-1.5</td>
<td>Coal, bituminous</td>
<td>84</td>
<td>1.2-1.5</td>
</tr>
<tr>
<td>Coal, lignite</td>
<td>78</td>
<td>1.1-1.2</td>
<td>Coal, peat, turf, dry</td>
<td>47</td>
<td>0.6-0.8</td>
</tr>
<tr>
<td>Coal, charcoal, pine</td>
<td>53</td>
<td>0.5-0.6</td>
<td>Coal, charcoal, oak</td>
<td>33</td>
<td>0.5-0.7</td>
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<tr>
<td>Coal, coke</td>
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<td>Coal, coke</td>
<td>75</td>
<td>1.0-1.4</td>
</tr>
<tr>
<td>Graphite</td>
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<td>1.0-2.3</td>
<td>Graphite</td>
<td>75</td>
<td>1.0-1.4</td>
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<tr>
<td>Petroleum, refined</td>
<td>50</td>
<td>0.7-0.8</td>
<td>Petroleum, benzene</td>
<td>46</td>
<td>0.7-0.8</td>
</tr>
<tr>
<td>Petroleum, gasoline</td>
<td>42</td>
<td>0.6-0.8</td>
<td>Petroleum, gasoline</td>
<td>42</td>
<td>0.6-0.8</td>
</tr>
<tr>
<td>Pitch</td>
<td>69</td>
<td>1.07-1.15</td>
<td>Pitch</td>
<td>69</td>
<td>1.07-1.15</td>
</tr>
<tr>
<td>TAR, bituminous</td>
<td>75</td>
<td>1.20</td>
<td>TAR, bituminous</td>
<td>75</td>
<td>1.20</td>
</tr>
<tr>
<td>COAL AND COKE, PILLED</td>
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<td>COAL AND COKE, PILLED</td>
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<td>Coal, anthracite</td>
<td>47-58</td>
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<td>Coal, bituminous, lignite</td>
<td>40-54</td>
<td>-</td>
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<tr>
<td>Coal, peat, turf</td>
<td>40-54</td>
<td>-</td>
<td>Coal, peat, turf</td>
<td>40-54</td>
<td>-</td>
</tr>
<tr>
<td>Coal, charcoal</td>
<td>16-14</td>
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<td>Coal, charcoal</td>
<td>16-14</td>
<td>-</td>
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<tr>
<td>Coal coke</td>
<td>23-32</td>
<td>-</td>
<td>Coal coke</td>
<td>23-32</td>
<td>-</td>
</tr>
</tbody>
</table>

The specific gravities of solids and liquids refer to water at 4°C, those of gasses to air at 0°C and 760 mm pressure. The weights per cubic foot are derived from average specific gravities, except where stated that weights are for bulk, heaped, or loose material, etc.
### Table 17-13. Weights of Building Materials

<table>
<thead>
<tr>
<th>Materials</th>
<th>Weight lb per sq ft</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CEILINGS</strong></td>
<td></td>
</tr>
<tr>
<td>Channel suspended system</td>
<td>1</td>
</tr>
<tr>
<td>Lathing and plastering</td>
<td>See Table</td>
</tr>
<tr>
<td>Acoustical fiber</td>
<td>1</td>
</tr>
<tr>
<td><strong>FLOORS</strong></td>
<td></td>
</tr>
<tr>
<td>Steel deck</td>
<td>See Manufacturer</td>
</tr>
<tr>
<td>Concrete-Reinforced 1 in.</td>
<td>12 1/2</td>
</tr>
<tr>
<td>Stone</td>
<td>12 1/2</td>
</tr>
<tr>
<td>Slat</td>
<td>11 1/2</td>
</tr>
<tr>
<td>Light weight</td>
<td>6 to 10</td>
</tr>
<tr>
<td><strong>CONCRETE-PLAN 1 in.</strong></td>
<td></td>
</tr>
<tr>
<td>Stone</td>
<td>12</td>
</tr>
<tr>
<td>Slat</td>
<td>11</td>
</tr>
<tr>
<td>Light weight</td>
<td>3 to 9</td>
</tr>
<tr>
<td><strong>FINS 1 inch</strong></td>
<td></td>
</tr>
<tr>
<td>Gypsum</td>
<td>6</td>
</tr>
<tr>
<td>Sand</td>
<td>8</td>
</tr>
<tr>
<td>Cinder</td>
<td>4</td>
</tr>
<tr>
<td><strong>FINISHES</strong></td>
<td></td>
</tr>
<tr>
<td>Terrazzo 1 in.</td>
<td>13</td>
</tr>
<tr>
<td>Ceramic or Quarry Tile 12 1/2 in.</td>
<td>10</td>
</tr>
<tr>
<td>Linoleum 1/4 in.</td>
<td>1</td>
</tr>
<tr>
<td>Masonry stone</td>
<td>4</td>
</tr>
<tr>
<td>Handwood 1/4 in.</td>
<td>4</td>
</tr>
<tr>
<td>Softwood 2 1/4 in.</td>
<td>12 1/2</td>
</tr>
<tr>
<td><strong>WALLS</strong></td>
<td></td>
</tr>
<tr>
<td>Brick</td>
<td>4</td>
</tr>
<tr>
<td>4 in.</td>
<td>40</td>
</tr>
<tr>
<td>8 in.</td>
<td>80</td>
</tr>
<tr>
<td>Hollow concrete block</td>
<td>Heavy aggregate</td>
</tr>
<tr>
<td><strong>ROofs</strong></td>
<td></td>
</tr>
<tr>
<td>Copper or In.</td>
<td>1</td>
</tr>
<tr>
<td>Corrugated steel</td>
<td>See Manufacturer</td>
</tr>
<tr>
<td>3-ply ready roofing</td>
<td>1</td>
</tr>
<tr>
<td>3-ply felt and gravel</td>
<td>5 1/2</td>
</tr>
<tr>
<td>5-ply felt and gravel</td>
<td>6</td>
</tr>
<tr>
<td>Shingles</td>
<td></td>
</tr>
<tr>
<td>Wood</td>
<td>2</td>
</tr>
<tr>
<td>Asphalt</td>
<td>3</td>
</tr>
<tr>
<td>Clay tile</td>
<td>9 to 14</td>
</tr>
<tr>
<td>Slate 1 1/4 in.</td>
<td>10</td>
</tr>
<tr>
<td><strong>Sheathing</strong></td>
<td></td>
</tr>
<tr>
<td>Wood 3 1/4 in.</td>
<td>3</td>
</tr>
<tr>
<td>Gypsum 1 in.</td>
<td>4</td>
</tr>
<tr>
<td><strong>Insulation 1 in.</strong></td>
<td></td>
</tr>
<tr>
<td>Loose</td>
<td>1/2</td>
</tr>
<tr>
<td>Poured</td>
<td>2</td>
</tr>
<tr>
<td>Rigid</td>
<td>1 1/2</td>
</tr>
</tbody>
</table>

For weights of other materials used in building construction, see Table 17-12.

### Table 17-14. Weights and Measures

<table>
<thead>
<tr>
<th>United States System</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th><strong>LINEAR MEASURE</strong></th>
<th>Inches</th>
<th>Feet</th>
<th>Yards</th>
<th>Rods</th>
<th>Furlongs</th>
<th>Miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>.08333</td>
<td>.02778</td>
<td>.002978</td>
<td>.000246</td>
<td>.0000196</td>
<td>.0000196</td>
</tr>
<tr>
<td>12.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>198.0</td>
<td>16.5</td>
<td>5.5</td>
<td>1.0</td>
<td>1.0</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>7,500.0</td>
<td>666.0</td>
<td>222.0</td>
<td>75.0</td>
<td>15.0</td>
<td>1.5</td>
<td>1.5</td>
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<tr>
<td>63,360.0</td>
<td>5,280.0</td>
<td>1,760.0</td>
<td>586.0</td>
<td>117.0</td>
<td>11.7</td>
<td>11.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>SQUARE AND LAND MEASURE</strong></th>
<th>Sq. Inches</th>
<th>Square Feet</th>
<th>Square Yards</th>
<th>Square Rods</th>
<th>Acres</th>
<th>Sq. Miles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>.006944</td>
<td>.000772</td>
<td>.000026</td>
<td>.00000133</td>
<td>.000000066</td>
<td>.000000066</td>
</tr>
<tr>
<td>144.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>1,296.0</td>
<td>9.0</td>
<td>9.0</td>
<td>9.0</td>
<td>9.0</td>
<td>0.3306</td>
<td>0.00207</td>
</tr>
<tr>
<td>39,204.0</td>
<td>272.25</td>
<td>272.25</td>
<td>272.25</td>
<td>272.25</td>
<td>1.0</td>
<td>0.000009</td>
</tr>
<tr>
<td>43,560.0</td>
<td>2,840.0</td>
<td>2,840.0</td>
<td>2,840.0</td>
<td>2,840.0</td>
<td>1.0</td>
<td>0.000026</td>
</tr>
<tr>
<td>3,097,600.0</td>
<td>212,000.0</td>
<td>212,000.0</td>
<td>212,000.0</td>
<td>212,000.0</td>
<td>1.0</td>
<td>0.067</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>AVOIRDUPOIS WEIGHTS</strong></th>
<th>Grains</th>
<th>Drains</th>
<th>Quarters</th>
<th>Pounds</th>
<th>Tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.0657</td>
<td>0.00266</td>
<td>0.00133</td>
<td>0.0000714</td>
<td></td>
</tr>
<tr>
<td>27.34375</td>
<td>1.0</td>
<td>0.0657</td>
<td>0.00133</td>
<td>0.0000714</td>
<td></td>
</tr>
<tr>
<td>437.5</td>
<td>15.0</td>
<td>1.0</td>
<td>0.058</td>
<td>0.003158</td>
<td></td>
</tr>
<tr>
<td>7,000.0</td>
<td>256.0</td>
<td>16.0</td>
<td>1.0</td>
<td>0.058</td>
<td></td>
</tr>
<tr>
<td>14,000,000.0</td>
<td>512,000.0</td>
<td>32,000.0</td>
<td>2,000.0</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>DRIY MEASURE</strong></th>
<th>Pints</th>
<th>Quarts</th>
<th>Gallons</th>
<th>Cubic Feet</th>
<th>Bushels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.08333</td>
<td>0.02778</td>
<td>0.002978</td>
<td>0.000246</td>
<td>0.0000196</td>
</tr>
<tr>
<td>2.0</td>
<td>0.16667</td>
<td>0.05556</td>
<td>0.005956</td>
<td>0.000492</td>
<td>0.0000392</td>
</tr>
<tr>
<td>16.0</td>
<td>1.0</td>
<td>0.06667</td>
<td>0.041667</td>
<td>0.003125</td>
<td>0.00025</td>
</tr>
<tr>
<td>51.0</td>
<td>2.5</td>
<td>1.25</td>
<td>0.075757</td>
<td>0.05625</td>
<td>0.0045</td>
</tr>
<tr>
<td>64.0</td>
<td>3.2</td>
<td>1.6</td>
<td>0.10</td>
<td>0.125</td>
<td>0.009765</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>LIQUID MEASURE</strong></th>
<th>U.S. Gallons</th>
<th>Cubic Feet</th>
<th>Bushels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>.08333</td>
<td>0.002978</td>
<td>0.000246</td>
</tr>
<tr>
<td>5.0</td>
<td>0.41667</td>
<td>0.024137</td>
<td>0.001225</td>
</tr>
<tr>
<td>25.0</td>
<td>2.0</td>
<td>0.125</td>
<td>0.009765</td>
</tr>
<tr>
<td>125.0</td>
<td>10.0</td>
<td>0.625</td>
<td>0.05625</td>
</tr>
<tr>
<td>1,000.0</td>
<td>80.0</td>
<td>4.0</td>
<td>0.375</td>
</tr>
<tr>
<td>7,480.52</td>
<td>598.3833</td>
<td>30.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

**American Institute of Steel Construction**
### TABLE 1607.1—continued
MINIMUM UNIFORMLY DISTRIBUTED LIVE LOADS, \( L_{pl} \), AND MINIMUM CONCENTRATED LIVE LOADS

<table>
<thead>
<tr>
<th>OCCUPANCY OR USE</th>
<th>UNIFORM ( (\text{psf}) )</th>
<th>CONCENTRATED ( (\text{lbs}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>23. Plant institutions</td>
<td>Cell blocks</td>
<td>40</td>
</tr>
<tr>
<td>Corridors</td>
<td>100</td>
<td>—</td>
</tr>
<tr>
<td>24. Recreational uses:</td>
<td>Bowling alleys, pool rooms and similar uses</td>
<td>72.5</td>
</tr>
<tr>
<td>Dance halls and ballrooms</td>
<td>100</td>
<td>—</td>
</tr>
<tr>
<td>Gymnasiums</td>
<td>100</td>
<td>—</td>
</tr>
<tr>
<td>Reviewing stands, grandstands and stands and areas with fixed seats (flooded to floor)</td>
<td>60</td>
<td>—</td>
</tr>
<tr>
<td>25. Residential</td>
<td>One- and two-family dwellings</td>
<td></td>
</tr>
<tr>
<td>Uninhabitable attics without storage</td>
<td>30</td>
<td>—</td>
</tr>
<tr>
<td>Uninhabitable attics with storage</td>
<td>30</td>
<td>—</td>
</tr>
<tr>
<td>Habitable attics and sleeping areas</td>
<td>40</td>
<td>—</td>
</tr>
<tr>
<td>All other areas</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hotels and multifamily dwellings</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private rooms and corridors serving them</td>
<td>40</td>
<td>—</td>
</tr>
<tr>
<td>Public rooms and corridors serving them</td>
<td>100</td>
<td>—</td>
</tr>
<tr>
<td>26. Roofs</td>
<td></td>
<td>300</td>
</tr>
<tr>
<td>All roof surfaces subject to maintenance workers</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vents and capstays</td>
<td>5</td>
<td>—</td>
</tr>
<tr>
<td>Fabric construction supported by a skeleton structure</td>
<td>20</td>
<td>nonreducible</td>
</tr>
<tr>
<td>Ordinary flat, pitched, and curved roofs (that are not occupiable)</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Where primary roof members are exposed to a work floor, at single panel point of lower chord of roof trusses or any point along primary structural members supporting roofs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Over manufacturing, storage warehouses, and repair garages</td>
<td>2,000</td>
<td></td>
</tr>
<tr>
<td>All other primary roof members</td>
<td></td>
<td>300</td>
</tr>
<tr>
<td>Occupiable roofs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Roof gardens</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>Assembly areas</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>All other similar areas</td>
<td>Note 1</td>
<td>Note 1</td>
</tr>
<tr>
<td>27. Schools</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Classrooms</td>
<td>40</td>
<td>1,000</td>
</tr>
<tr>
<td>Corridors above first floor</td>
<td>80</td>
<td>1,000</td>
</tr>
<tr>
<td>First-floor corridors</td>
<td>100</td>
<td>1,000</td>
</tr>
<tr>
<td>28. Stables, skyscraper ribbons and accessible ceilings</td>
<td></td>
<td>—</td>
</tr>
<tr>
<td>Sidewalks, vehicular drive ways and yards, subject to trucking</td>
<td>250</td>
<td>8,000</td>
</tr>
</tbody>
</table>

---

Note Set 12.3

- a. Floors in garages or portions of buildings used for the storage of motor vehicles shall be designed for the uniformly distributed live load of Table 1607.1 or the following concentrated loads: (1) for garages restricted to passenger vehicles accommodating not more than nine passengers, 3,000 pounds acting on an area of 4.5 inches by 4.5 inches, (2) for mechanical parking structures without slab or deck that are used for storing passenger vehicles only, 2,250 pounds per wheel.
- b. The loading applies to stack room floors that support nonmobile, double-faced library book stacks, subject to the following limitations:
  1. The nominal stack unit height shall not exceed 90 inches;
  2. The nominal shelf depth shall not exceed 12 inches for each face; and
  3. Parallel rows of double-faced book stacks shall be separated by aisles not less than 36 inches wide.
- c. Design in accordance with ICC 300.
- d. Other uniform loads in accordance with an approved method containing provisions for truck loadings shall also be considered where appropriate.
- e. The concentrated wheel load shall be applied on an area of 4.5 inches by 4.5 inches.
- f. The minimum concentrated load on stair treads shall be applied on an area of 2 inches by 2 inches. This load need not be assumed to act concurrently with the uniform load.
- g. Where snow loads occur that are in excess of the design conditions, the structure shall be designed to support the loads due to the increased loads caused by drift build-up or a greater snow design determined by the building official (see Section 1608).
- h. See Section 1608.8.5 for decks attached to exterior walls.
- i. Uninhabitable attics without storage are those where the maximum clear height between the joists and rafters is less than 42 inches, or where there are not two or more adjacent trusses with web configurations capable of accommodating an assumed rectangle 42 inches in height by 24 inches in width, or greater, within the plane of the trusses. This live load need not be assumed to act concurrently with any other live load requirements.
1607.10 Reduction in uniform live loads. Except for uniform live loads at roofs, all other minimum uniformly distributed live loads, \( L_o \), in Table 1607.1 are permitted to be reduced in accordance with Section 1607.10.1 or 1607.10.2. Uniform live loads at roofs are permitted to be reduced in accordance with Section 1607.12.2.

1607.10.1 Basic uniform live load reduction. Subject to the limitations of Sections 1607.10.1.1 through 1607.10.1.3 and Table 1607.1, members for which a value of \( K_{UL} \), is 400 square feet (37.16 m²) or more are permitted to be designed for a reduced uniformly distributed live load, \( L \), in accordance with the following equation:

\[
L = L_o \left(0.25 + \frac{15}{K_{UL} A_T} \right)
\]

(Equation 16-23)

For SI: \( L = L_o \left(0.25 + \frac{4.57}{K_{UL} A_T} \right) \)

where:

- \( L \) = Reduced design live load per square foot (m²) of area supported by the member.
- \( L_o \) = Unreduced design live load per square foot (m²) of area supported by the member (see Table 1607.1).
- \( K_{UL} \) = Live load element factor (see Table 1607.10.1).
- \( A_T \) = Tributary area, in square feet (m²).

\( L \) shall not be less than 0.50\( L_o \) for members supporting one floor and \( L \) shall not be less than 0.40\( L_o \) for members supporting two or more floors.

### TABLE 1607.10.1

<table>
<thead>
<tr>
<th>ELEMENT</th>
<th>( K_{UL} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior columns</td>
<td>4</td>
</tr>
<tr>
<td>Interior columns without cantilever slabs</td>
<td>4</td>
</tr>
<tr>
<td>Edge columns with cantilever slabs</td>
<td>3</td>
</tr>
<tr>
<td>Center columns with cantilever slabs</td>
<td>2</td>
</tr>
<tr>
<td>Edge beams without cantilever slabs</td>
<td>2</td>
</tr>
<tr>
<td>Interior beams</td>
<td>2</td>
</tr>
<tr>
<td>All other members not identified above including:</td>
<td>1</td>
</tr>
<tr>
<td>Edge beams with cantilever slabs</td>
<td></td>
</tr>
<tr>
<td>Cantilever beams</td>
<td></td>
</tr>
<tr>
<td>One-way slabs</td>
<td></td>
</tr>
<tr>
<td>Two-way slabs</td>
<td></td>
</tr>
<tr>
<td>Members without provisions for continuous shear transfer normal to their span</td>
<td></td>
</tr>
</tbody>
</table>

1607.10.1.1 One-way slabs. The tributary area, \( A_o \), for use in Equation 16-23 for one-way slabs shall not exceed an area defined by the slab span times a width normal to the span of 1.5 times the slab span.

1607.10.1.2 Heavy live loads. Live loads that exceed 100 psf (4.79 kN/m²) shall not be reduced.

Exceptions:

1. The live loads for members supporting two or more floors are permitted to be reduced by a maximum of 20 percent, but the live load shall not be less than \( L \) as calculated in Section 1607.10.1.
2. For uses other than storage, where approved, additional live load reductions shall be permitted where shown by the registered design professional that a rational approach has been used and that such reductions are warranted.

1607.10.1.3 Passenger vehicle garages. The live loads shall not be reduced in passenger vehicle garages.

Exception: The live loads for members supporting two or more floors are permitted to be reduced by a maximum of 20 percent, but the live load shall not be less than \( L \) as calculated in Section 1607.10.1.

1607.10.2 Alternative uniform live load reduction. As an alternative to Section 1607.10.1 and subject to the limitations of Table 1607.1, uniformly distributed live loads are permitted to be reduced in accordance with the following provisions. Such reductions shall apply to slab systems, beams, girders, columns, piers, walls and foundations.

1. A reduction shall not be permitted where the live load exceeds 100 psf (4.79 kN/m²) except that the design live load for members supporting two or more floors is permitted to be reduced by a maximum of 20 percent.

Exception: For uses other than storage, where approved, additional live load reductions shall be permitted where shown by the registered design professional that a rational approach has been used and that such reductions are warranted.

2. A reduction shall not be permitted in passenger vehicle parking garages except that the live loads for members supporting two or more floors are permitted to be reduced by a maximum of 20 percent.

3. For live loads not exceeding 100 psf (4.79 kN/m²), the design live load for any structural member supporting 150 square feet (13.94 m²) or more is permitted to be reduced in accordance with Equation 16-24.

4. For one-way slabs, the area, \( A \), for use in Equation 16-24 shall not exceed the product of the slab span and a width normal to the span of 0.5 times the slab span.

\[
R = 0.08(A - 150)
\]

(Equation 16-24)

For SI: \( R = 0.861(A - 13.94) \)

Such reduction shall not exceed the smallest of:

1. 0.40 percent for horizontal members;
2. 0.60 percent for vertical members; or
3. \( R \) as determined by the following equation.

\[
R = 23.1(1 + D/L_o)
\]

(Equation 16-25)

where:

- \( A \) = Area of floor supported by the member, square feet (m²).
- \( D \) = Dead load per square foot (m²) of area supported.
- \( L_o \) = Unreduced live load per square foot (m²) of area supported.
- \( R \) = Reduction in percent.

1607.11 Distribution of floor loads. Where uniform floor live loads are involved in the design of structural members arranged so as to create continuity, the minimum applied loads shall be the full dead loads on all spans in combination with the floor live loads on spans selected to produce the greatest load effect at each location under consideration. Floor live loads are permitted to be reduced in accordance with Section 1607.10.
Minimum Roof Loads

1607.12 Roof loads. The structural supports of roofs and marquees shall be designed to resist wind and, where applicable, snow and earthquake loads, in addition to the dead load of construction and the appropriate live loads as prescribed in this section, or as set forth in Table 1607.1. The live loads acting on a sloping surface shall be assumed to act vertically on the horizontal projection of that surface.

1607.12.1 Distribution of roof loads. Where uniform roof live loads are reduced to less than 20 psf (0.96 kN/m²) in accordance with Section 1607.12.2.1 and are applied to the design of structural members arranged so as to create continuity, the reduced roof live load shall be applied to adjacent spans or to alternate spans, whichever produces the most unfavorable load effect. See Section 1607.12.2 for reductions in minimum roof live loads and Section 7.5 of ASCE 7 for partial snow loading.

1607.12.2 General. The minimum uniformly distributed live loads of roofs and marquees, \( L_u \), in Table 1607.1 are permitted to be reduced in accordance with Section 1607.12.2.1.

1607.12.2.1 Ordinary roofs, awnings and canopies. Ordinary flat, pitched and curved roofs, and awnings and canopies other than of fabric construction supported by a skeleton structure, are permitted to be designed for a reduced uniformly distributed roof live load, \( L_u \), as specified in the following equations or other controlling combinations of loads as specified in Section 1605, whichever produces the greater load effect.

In structures such as greenhouses, where special scaffolding is used as a work surface for workers and materials during maintenance and repair operations, a lower roof load than specified in the following equations shall not be used unless approved by the building official. Such structures shall be designed for a minimum roof live load of 12 psf (0.58 kN/m²).

\[
L_u = L_{un} R_1 R_2 \quad \text{(Equation 16-26)}
\]

where: \( 12 \leq L_u \leq 20 \)

For SI: \( L_u = L_{un} R_1 R_2 \)

where: \( 0.58 \leq L_u \leq 0.96 \)

\( L_{un} = \) Unreduced roof live load per square foot (m²) of horizontal projection supported by the member (see Table 1607.1).

\( L_u = \) Reduced roof live load per square foot (m²) of horizontal projection supported by the member.

The reduction factors \( R_1 \) and \( R_2 \) shall be determined as follows:

\[
R_1 = 1 \text{ for } A_i \leq 200 \text{ square feet (18.58 m²)}
\]

(Equation 16-27)

\[
R_2 = 1.2 \times 0.001 A_i \text{, for 200 square feet}
\]

\(< A_i < 600 \text{ square feet} \quad \text{(Equation 16-28)}\)

For SI: \( 1.2 \times 0.011 A_i \), for 18.58 square meters \(< A_i < 55.74 \text{ square meters} \)

\[
R_1 = 0.6 \text{ for } A_i \geq 600 \text{ square feet (55.74 m²)}
\]

(Equation 16-29)

where:

\( A_i = \) Tributary area (span length multiplied by effective width) in square feet (m²) supported by the member, and

\[
R_1 = 1 \text{ for } F \leq 4 \quad \text{(Equation 16-30)}
\]

\[
R_2 = 1.2 \times 0.05 F \text{ for } 4 < F < 12 \quad \text{(Equation 16-31)}
\]

\[
R_3 = 0.6 \text{ for } F \geq 12 \quad \text{(Equation 16-32)}
\]

where:

\( F = \) For a sloped roof, the number of inches of rise per foot (for SI: \( F = 0.12 \times \text{slope} \), with slope expressed as a percentage), or for an arch or dome, the rise-to-span ratio multiplied by 32.

1607.12.3 Occupiable roofs. Areas of roofs that are occupiable, such as roof gardens, or for assembly or other similar purposes, and marquees are permitted to have their uniformly distributed live loads reduced in accordance with Section 1607.10.

1607.12.3.1 Landscaped roofs. The uniform design live load in unoccupied landscaped areas on roofs shall be 20 psf (0.958 kN/m²). The weight of all landscaping materials shall be considered as dead load and shall be computed on the basis of saturation of the soil.

1607.12.4 Awnings and canopies. Awnings and canopies shall be designed for uniform live loads as required in Table 1607.1 as well as for snow loads and wind loads as specified in Sections 1608 and 1609.
Minimum Snow Loads

In CS areas, site-specific case studies are required to establish snow load at these areas. The map below shows the general map of snow loads in these areas.

Numbers in parentheses represent the upper elevation limits in feet for the ground snow load values presented below. Site-specific case studies are required to establish ground snow loads at elevations not covered.

To convert lbs/ft² to kN/m², multiply by 0.0472.

To convert feet to meters, multiply by 0.3048.

FIGURE 1008.2—continued
GROUND SNOW LOADS, Pₛ, FOR THE UNITED STATES (psi)
SECTION 1603
CONSTRUCTION DOCUMENTS

1603.1 General. Construction documents shall show the size, section and relative locations of structural members with floor levels, column centers and offsets dimensioned. The design loads and other information pertinent to the structural design required by Sections 1603.1.1 through 1603.1.9 shall be indicated on the construction documents.

Exception: Construction documents for buildings constructed in accordance with the conventional light-frame construction provisions of Section 2308 shall indicate the following structural design information:

1. Floor and roof live loads.
2. Ground snow load, $P_e$.
3. Ultimate design wind speed, $V_{wu}$, (3-second gust), miles per hour (mph) (km/hr) and nominal design wind speed, $V_{wu}$, as determined in accordance with Section 1609.3.1 and wind exposure.
4. Seismic design category and site class.
5. Flood design data, if located in flood hazard areas established in Section 1612.3.
6. Design load-bearing values of soils.

1603.1.1 Floor live load. The uniformly distributed, concentrated and impact floor load used in the design shall be indicated for floor areas. Use of live load reduction in accordance with Section 1607.10 shall be indicated for each type of live load used in the design.

1603.1.2 Roof live load. The roof live load used in the design shall be indicated for roof areas (Section 1607.12).

1603.1.3 Roof snow load data. The ground snow load, $P_e$, shall be indicated. In areas where the ground snow load, $P_e$, exceeds 10 pounds per square foot (psf) (0.479 kN/m²), the following additional information shall also be provided, regardless of whether snow loads govern the design of the roof:

1. Flat-roof snow load, $P_f$.
2. Snow exposure factor, $C_e$.
4. Thermal factor, $C_r$.

1603.1.4 Wind design data. The following information related to wind loads shall be shown, regardless of whether wind loads govern the design of the lateral force-resisting system of the structure:

1. Ultimate design wind speed, $V_{wu}$, (3-second gust), miles per hour (km/hr) and nominal design wind speed, $V_{wu}$, as determined in accordance with Section 1609.3.1.
2. Risk category.
3. Wind exposure. Where more than one wind exposure is utilized, the wind exposure and applicable wind direction shall be indicated.
4. The applicable internal pressure coefficient.
5. Components and cladding. The design wind pressures in terms of psf (kN/m²) to be used for the design of exterior component and cladding materials not specifically designed by the registered design professional.

1603.1.5 Earthquake design data. The following information related to seismic loads shall be shown, regardless of whether seismic loads govern the design of the lateral force-resisting system of the structure:

1. Risk category.
2. Seismic importance factor, $I_s$.
4. Site class.
5. Design spectral response acceleration parameters, $S_{0x}$ and $S_{0y}$.
6. Seismic design category.
7. Basic seismic force-resisting system(s).
8. Design base shear(s).
9. Seismic response coefficient(s), $C_v$.
10. Response modification coefficient(s), $R$.
11. Analysis procedure used.

1603.1.6 Geotechnical information. The design load-bearing values of soils shall be shown on the construction documents.

1603.1.7 Flood design data. For buildings located in whole or in part in flood hazard areas as established in Section 1612.3, the documentation pertaining to design, if required in Section 1612.5, shall be included and the following information, referenced to the datum on the community's Flood Insurance Rate Map (FIRM), shall be shown, regardless of whether flood loads govern the design of the building:

1. In flood hazard areas not subject to high-velocity wave action, the elevation of the proposed lowest floor, including the basement.
2. In flood hazard areas not subject to high-velocity wave action, the elevation to which any nonresidential building will be dry flood proofed.
3. In flood hazard areas subject to high-velocity wave action, the proposed elevation of the bottom of the lowest horizontal structural member of the lowest floor, including the basement.

1603.1.8 Special loads. Special loads that are applicable to the design of the building, structure or portions thereof shall be indicated along with the specified section of this code that addresses the special loading condition.

1603.1.9 Systems and components requiring special inspections for seismic resistance. Construction documents or specifications shall be prepared for those systems and components requiring special inspection for seismic resistance as specified in Section 1705.11 by the registered design professional responsible for their design and shall be submitted for approval in accordance with Section 107.1. Reference to seismic standards in lieu of detailed drawings is acceptable.
1604.3 Serviceability. Structural systems and members thereof shall be designed to have adequate stiffness to limit deflections and lateral drift. See Section 12.12 of ASCE 7 for drift limits applicable to earthquake loading.

1604.3.1 Deflections. The deflections of structural members shall not exceed the more restrictive of the limitations of Sections 1604.3.2 through 1604.3.5 or that permitted by Table 1604.3.

1604.3.2 Reinforced concrete. The deflection of reinforced concrete structural members shall not exceed that permitted by ACI 318.

1604.3.3 Steel. The deflection of steel structural members shall not exceed that permitted by AISC 360, AISI S100, ASCE 8, SJI CJ-1.0, SJI JG-1.1, SJI K-1.1 or SJI LH/DLH-1.1, as applicable.

1604.3.4 Masonry. The deflection of masonry structural members shall not exceed that permitted by TMS 402/ACI 530/ASCE 5.

1604.3.5 Aluminum. The deflection of aluminum structural members shall not exceed that permitted by AA ADM1.

1604.3.6 Limits. The deflection limits of Section 1604.3.1 shall be used unless more restrictive deflection limits are required by a referenced standard for the element or finish material.

### TABLE 1604.3

<table>
<thead>
<tr>
<th>CONSTRUCTION</th>
<th>L</th>
<th>S or W</th>
<th>D + L³/8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof members:</td>
<td>L/360</td>
<td>L/360</td>
<td>L/240</td>
</tr>
<tr>
<td>Supporting plaster or stucco ceiling</td>
<td>L/240</td>
<td>L/240</td>
<td>L/180</td>
</tr>
<tr>
<td>Supporting nonplaster ceiling</td>
<td>L/180</td>
<td>L/180</td>
<td>L/120</td>
</tr>
<tr>
<td>Not supporting ceiling</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Floor members</td>
<td>L/360</td>
<td>—</td>
<td>L/240</td>
</tr>
<tr>
<td>Exterior walls and interior partitions:</td>
<td>—</td>
<td>L/360</td>
<td>—</td>
</tr>
<tr>
<td>With plaster or stucco finishes</td>
<td>—</td>
<td>L/240</td>
<td>—</td>
</tr>
<tr>
<td>With other brittle finishes</td>
<td>—</td>
<td>L/120</td>
<td>—</td>
</tr>
<tr>
<td>With flexible finishes</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Farm buildings</td>
<td>—</td>
<td>—</td>
<td>L/180</td>
</tr>
<tr>
<td>Greenhouses</td>
<td>—</td>
<td>—</td>
<td>L/120</td>
</tr>
</tbody>
</table>

For SI: 1 foot = 304.8 mm.

a. For structural roofing and siding made of formed metal sheets, the total load deflection shall not exceed 1/900. For secondary roof structural members supporting formed metal roofing, the live load deflection shall not exceed 1/150. For secondary wall members supporting formed metal siding, the design wind load deflection shall not exceed 1/900. For roofs, this exception only applies when the metal sheets have no roof covering.

b. Interior partitions not exceeding 6 feet in height and flexible, folding and portable partitions are not governed by the provisions of this section. The deflection criterion for interior partitions is based on the horizontal load defined in Section 1607.14.

c. See Section 2403 for glass supports.

d. For wood structural members having a moisture content of less than 16 percent at time of installation and used under dry conditions, the deflection resulting from \( L + 0.5D \) is permitted to be subtracted for the deflection resulting from \( L + D \).

e. The above deflections do not ensure against ponding. Roofs that do not have sufficient slope or camber to assure adequate drainage shall be investigated for ponding. See Section 1611 for rain and ponding requirements and Section 1503.4 for roof drainage requirements.

f. The wind load is permitted to be taken as 0.42 times the “component and cladding” loads for the purpose of determining deflection limits herein.

\( D + L\text{/}8 \) for steel structural members, the dead load shall be taken as zero.

h. For aluminum structural members or aluminum panels used in skylights and sloped glazing framing, roofs or walls of sunroom additions or patio covers, not supporting edge of glass or aluminum sandwich panels, the total load deflection shall not exceed 1/900. For continuous aluminum structural members supporting edge of glass, the total load deflection shall not exceed 1/175 for each glass lute or 1/600 for the entire length of the member, whichever is more stringent. For aluminum sandwich panels used in roofs or walls of sunroom additions or patio covers, the total load deflection shall not exceed 1/120.

\( D + L\text{/}8 \) for cantilever members, \( f \) shall be taken as twice the length of the cantilever.
Simplified Design Wind Pressures
SEI/ASCE 7-10:

Main Wind Force Resisting System – Method 2

Figure 28.6-1

Design Wind Pressures

Walls & Roofs

Enclosed Buildings

Case A

Case B

Notes:
1. Pressures shown are applied to the horizontal and vertical projections, for exposure B, at h=30 ft (9.1m). Adjust to other exposures and heights with adjustment factor λ.
2. The load patterns shown shall be applied to each corner of the building in turn as the reference corner. (See Figure 28.4-1)
3. For Case B use θ = 0°.
4. Load cases 1 and 2 must be checked for 25° < θ ≤ 45°. Load case 2 at 25° is provided only for interpolation between 25° and 30°.
5. Plus and minus signs signify pressures acting toward and away from the projected surfaces, respectively.
6. For roof slopes other than those shown, linear interpolation is permitted.
7. The total horizontal load shall not be less than that determined by assuming pE - q in zones B & D.
8. Where zone E or G falls on a roof overhang on the windward side of the building, use Eow and Gow for the pressure on the horizontal projection of the overhang. Overhangs on the leeward and side edges shall have the basic zone pressure applied.
9. Notation:
   a: 10 percent of least horizontal dimension or 0.4h, whichever is smaller, but not less than either 4% of least horizontal dimension or 5 ft (0.9 m).
   h: Mean roof height, in feet (meters), except that eave height shall be used for roof angles <10°.
   θ: Angle of plane of roof from horizontal, in degrees.
<table>
<thead>
<tr>
<th>Basic Wind Speed (mph)</th>
<th>Roof Angle (degrees)</th>
<th>Load Case</th>
<th>Horizontal Pressures</th>
<th>Vertical Pressures</th>
<th>Overhangs</th>
<th>Zones</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>0 to 5°</td>
<td>10°</td>
<td>1</td>
<td>-19.2</td>
<td>-10.0</td>
<td>12.7</td>
<td>-5.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>-21.7</td>
<td>-9.9</td>
<td>14.4</td>
<td>-6.2</td>
</tr>
<tr>
<td>10°</td>
<td></td>
<td>1</td>
<td>26.1</td>
<td>-7.0</td>
<td>17.7</td>
<td>-3.9</td>
</tr>
<tr>
<td>15°</td>
<td></td>
<td>1</td>
<td>24.4</td>
<td>-6.0</td>
<td>16.0</td>
<td>-4.6</td>
</tr>
<tr>
<td>20°</td>
<td></td>
<td>1</td>
<td>26.6</td>
<td>-7.0</td>
<td>17.7</td>
<td>-3.9</td>
</tr>
<tr>
<td>25°</td>
<td></td>
<td>1</td>
<td>24.1</td>
<td>-6.0</td>
<td>16.0</td>
<td>-4.6</td>
</tr>
<tr>
<td>30 to 45°</td>
<td></td>
<td>1</td>
<td>21.6</td>
<td>14.8</td>
<td>17.2</td>
<td>11.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>21.0</td>
<td>14.8</td>
<td>17.2</td>
<td>11.8</td>
</tr>
</tbody>
</table>

Unit Conversions – 1.0 ft = 0.3048 m; 1.0 psf = 0.0479 kN/m²

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**Main Wind Force Resisting System – Method 2**

Figure 28.6-1 (cont’d) Design Wind Pressures

**Enclosed Buildings**

**Walls & Roofs**

**Simplified Design Wind Pressure, p_{50} (psf)** (Exposure B at h = 30 ft, with l = 1.0)
## Simplified Design Wind Pressure, $p_{330}$ (psf) (Exposure B at $h = 30$ ft.)

<table>
<thead>
<tr>
<th>Basic Wind Speed (mph)</th>
<th>Roof Angle (degrees)</th>
<th>Zones</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Horizontal Pressures</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A</td>
</tr>
<tr>
<td>0 to 5°</td>
<td>1</td>
<td>40.6</td>
</tr>
<tr>
<td>10°</td>
<td>1</td>
<td>45.8</td>
</tr>
<tr>
<td>15°</td>
<td>1</td>
<td>51.0</td>
</tr>
<tr>
<td>20°</td>
<td>1</td>
<td>56.2</td>
</tr>
<tr>
<td>25°</td>
<td>1</td>
<td>60.9</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>30 to 45°</td>
<td>1</td>
<td>45.7</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>45.7</td>
</tr>
<tr>
<td>0 to 5°</td>
<td>1</td>
<td>51.4</td>
</tr>
<tr>
<td>10°</td>
<td>1</td>
<td>58.0</td>
</tr>
<tr>
<td>15°</td>
<td>1</td>
<td>64.5</td>
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<td>71.1</td>
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<td>25°</td>
<td>1</td>
<td>84.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>30 to 45°</td>
<td>1</td>
<td>57.8</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>57.8</td>
</tr>
<tr>
<td>0 to 5°</td>
<td>1</td>
<td>83.4</td>
</tr>
<tr>
<td>10°</td>
<td>1</td>
<td>71.5</td>
</tr>
<tr>
<td>15°</td>
<td>1</td>
<td>79.7</td>
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<td>25°</td>
<td>1</td>
<td>93.6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>30 to 45°</td>
<td>1</td>
<td>71.3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>71.3</td>
</tr>
</tbody>
</table>

### Adjustment Factor

**for Building Height and Exposure, $\lambda$**

<table>
<thead>
<tr>
<th>Mean roof height (ft)</th>
<th>Exposure</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td></td>
<td>1.00</td>
<td>1.21</td>
<td>1.47</td>
</tr>
<tr>
<td>20</td>
<td></td>
<td>1.00</td>
<td>1.29</td>
<td>1.55</td>
</tr>
<tr>
<td>25</td>
<td></td>
<td>1.00</td>
<td>1.35</td>
<td>1.61</td>
</tr>
<tr>
<td>30</td>
<td></td>
<td>1.00</td>
<td>1.40</td>
<td>1.66</td>
</tr>
<tr>
<td>35</td>
<td></td>
<td>1.05</td>
<td>1.45</td>
<td>1.70</td>
</tr>
<tr>
<td>40</td>
<td></td>
<td>1.09</td>
<td>1.49</td>
<td>1.74</td>
</tr>
<tr>
<td>45</td>
<td></td>
<td>1.12</td>
<td>1.53</td>
<td>1.76</td>
</tr>
<tr>
<td>50</td>
<td></td>
<td>1.16</td>
<td>1.56</td>
<td>1.81</td>
</tr>
<tr>
<td>55</td>
<td></td>
<td>1.19</td>
<td>1.59</td>
<td>1.84</td>
</tr>
<tr>
<td>60</td>
<td></td>
<td>1.22</td>
<td>1.62</td>
<td>1.87</td>
</tr>
</tbody>
</table>

**Unit Conversions**

- 1.0 ft = 0.3048 m; 1.0 psf = 0.0479 kN/m²
Table 1.5-1  Risk Category of Buildings and Other Structures for Flood, Wind, Snow, Earthquake, and Ice Loads

<table>
<thead>
<tr>
<th>Use or Occupancy of Buildings and Structures</th>
<th>Risk Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buildings and other structures that represent a low risk to human life in the event of failure</td>
<td>I</td>
</tr>
<tr>
<td>All buildings and other structures except those listed in Risk Categories I, III, and IV</td>
<td>II</td>
</tr>
<tr>
<td>Buildings and other structures, the failure of which could pose a substantial risk to human life.</td>
<td>III</td>
</tr>
<tr>
<td>Buildings and other structures, not included in Risk Category IV, with potential to cause a substantial economic impact and/or mass disruption of day-to-day civilian life in the event of failure.</td>
<td></td>
</tr>
<tr>
<td>Buildings and other structures not included in Risk Category IV (including, but not limited to, facilities that manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous chemicals, hazardous waste, or explosives) containing toxic or explosive substances where their quantity exceeds a threshold quantity established by the authority having jurisdiction and is sufficient to pose a threat to the public if released.</td>
<td></td>
</tr>
<tr>
<td>Buildings and other structures designated as essential facilities.</td>
<td>IV</td>
</tr>
<tr>
<td>Buildings and other structures, the failure of which could pose a substantial hazard to the community.</td>
<td></td>
</tr>
<tr>
<td>Buildings and other structures (including, but not limited to, facilities that manufacture, process, handle, store, use, or dispose of such substances as hazardous fuels, hazardous chemicals, or hazardous waste) containing sufficient quantities of highly toxic substances where the quantity exceeds a threshold quantity established by the authority having jurisdiction to be dangerous to the public if released and is sufficient to pose a threat to the public if released.*</td>
<td></td>
</tr>
<tr>
<td>Buildings and other structures required to maintain the functionality of other Risk Category IV structures.</td>
<td></td>
</tr>
</tbody>
</table>

*Buildings and other structures containing toxic, highly toxic, or explosive substances shall be eligible for classification to a lower Risk Category if it can be demonstrated to the satisfaction of the authority having jurisdiction by a hazard assessment as described in Section 1.5.2 that a release of the substances is commensurate with the risk associated with that Risk Category.
Figure 26.5.1-A Basic Wind Speeds for Occupancy Category II Buildings and Other Structures.

Notes:
1. Values are nominal design 3-second gust wind speeds in miles per hour (mph) at 33 ft (10m) above ground for Exposure C category.
2. Linear interpolation between contours is permitted.
3. Islands and coastal areas outside the last contour shall use the last wind speed contour of the coastal area.
4. Mountainsous terrain, gorges, ocean pronomories, and special wind regions shall be examined for unusual wind conditions.
5. Wind speeds correspond to approximately a 1% probability of exceedance in 50 years (Annual Exceedance Probability = 0.00145; MREI = 700 Years).
Earthquake Ground Motion, 0.2 Second Spectral Response

International Building Code 2012:
Spectral Response Maps

Following earthquakes larger than magnitude 5.5, spectral response maps are made. Response spectra portray the response of a damped, single-degree-of-freedom oscillator to the recorded ground motions. This data representation is useful for engineers determining how a structure will react to ground motions. The response is calculated for a range of periods. Within that range, the Uniform Building Code (UBC) refers to particular reference periods that help define the shape of the "design spectra" that reflects the building code.
Properties of Structural Materials

5.1 INTRODUCTION

It is important for the structural designer to realize that different engineering materials have different characteristics and will exhibit different behaviors under load. A knowledge of such characteristics or properties will help to ensure proper use of these materials, both architecturally and structurally.

It is assumed that the reader will have already been exposed to the study of materials through courses in building construction or materials science. This chapter will only highlight a few selected structural materials in the interest of emphasizing the range of structural characteristics and their diversity. Tables of properties of selected structural materials are given in Appendix E.

5.2 NATURE OF WOOD

Wood is a natural material and has a broad range of physical properties because of the different characteristics of its many species. Softwoods such as fir, pine, and hemlock are most often used for structural applications, because they are more plentiful (grow fast and tall) and are easier to fabricate. These woods are generally strong in tension and compression in a direction parallel to the grain and weak when stressed perpendicularly to the grain. Wood is also weak in shear because of its tendency to split along the natural grain laminations. The allowable stresses for three selected species are given in Appendix H.

Wood is light and soft compared to most other structural materials and is easily shaped and fastened together. A minimum of materials-handling equipment is needed to erect wood structures because of their weight. Wood is also very versatile in terms of its adaptability to the making of geometric shapes and even nonlinear forms.

Most softwoods are fairly ductile and will not fail suddenly when overloaded. Because of their lack of homogeneity or uniformity, the allowable stresses are quite low compared to failure stresses. Consequently, when wood structures are properly engineered, a statistically high margin of safety is present. Wood is often known as the “forgiving” material because of its apparent ability to sustain loads not accounted for when the structure was designed.

Wood, on the other hand, is not very stiff. It is subject to excessive deflection and creep deformation if not designed with these characteristics in mind (see Section 5.6). It is prone to damage by fire and to deterioration by moisture and insects. It expands and contracts with variations in humidity, and markedly so in the direction perpendicular to the grain. Timber structures that are to be exposed to the elements must be carefully treated or highly maintained to preserve their integrity.

The American Forest and Paper Association publishes the “National Design Specification,” which is the primary reference guide for timber designers. The Association also publishes a number of useful bulletins and manuals, as do other groups, such as the Western Wood Products Association and the Southern Forest Products Association.

5.3 CONCRETE AND REINFORCED CONCRETE

Concrete is a man-made conglomerate stone composed of essentially four ingredients: portland cement, water, sand, and coarse aggregate. The cement and water combine to make a paste that binds the sand and stones together. Ideally, the aggregates are graded so that the volume of paste is at a minimum, merely surrounding every piece with a thin layer. Most structural concrete is stone concrete, but structural lightweight concrete (roughly two-thirds the density of stone concrete) is becoming increasingly popular.

Concrete is essentially a compressive material having almost no tensile strength. As explained in Chapter 8, shearing stresses are always accompanied by tension, so concrete’s weakness in tension also causes it to be weak in shear. These deficiencies are overcome by using steel bars for reinforcement at the places where tensile and shearing stresses are generated. Under load, reinforced concrete beams actually have numerous minute cracks, which run at right angles to the direction of major tensile stresses. The tensile forces at such locations are being taken completely by the steel “re-bars.”

The compressive strength of a given concrete is a function of the quality and proportions of its constituents and the manner in which the fresh concrete is cured. (Curing is the provision of an appropriate environment surrounding freshly placed concrete while it gains its initial strength. During this time (7 to 14 days in duration), the concrete should be kept at a reasonable temperature and must be prevented from “drying out,” because the presence of water is necessary for the chemical action to progress.)

Coarse aggregate that is hard and well graded is particularly essential for quality concrete.

The most important factor governing the strength, however, is the percentage of water used in the mix. A minimum amount of water is needed for proper hydration of the cement. Additional water is needed for handling and placing the concrete, but excess amounts cause the strength to drop markedly.

These and other topics are fully covered in the booklet “Design and Control of Concrete Mixtures,” published by the Portland Cement Association. This is an excellent reference, treating both concrete mix design and proper construction practices. The American
Concrete Institute publishes a widely adopted code specifying the structural requirements for reinforced concrete.

Concrete is known as the "formable" or "moldable" structural material. Compared to other materials, it is easy to make curvilinear members and surfaces with concrete. It has no inherent texture but absorbs the texture of the forming material, so it can range widely in surface appearance. It is relatively inexpensive to make, both in terms of raw materials and labor, and the basic ingredients of Portland cement are available the world over. (It should be noted, however, that the necessary reinforcing bars for concrete may not be readily available in less-developed countries.)

The best structural use of reinforced concrete, in terms of the characteristics of the material, is in those structures requiring continuity and/or rigidity. It has a monolithic quality that automatically makes fixed or continuous connections. These moment-resistant joints are such that many low-rise concrete buildings do not require a secondary bracing system for lateral loads. In essence, a concrete beam joins a concrete column very differently from the way steel and wood pieces join, and the sensitive designer will not ignore this difference. (These remarks do not apply to precast structural elements, which are usually not joined in a continuous manner.)

Concrete is naturally fireproof and needs no separate protection system. Because of its mass, it can also serve as an effective barrier to sound transmission.

In viewing the negative aspects, concrete is unfortunately quite heavy, and it is often noted that a concrete structure expends a large portion of its capacity merely carrying itself. Attempts to make concrete less dense, while maintaining high-quality levels, have generally resulted in increased costs. Nevertheless, use of lightweight concrete can sometimes result in overall economies.

Concrete requires more quality control than most other building materials. Modern transit-mixed concrete suppliers are available to all U.S. urban areas, and the mix is usually of a uniformly high quality. Field- or job-mixed concrete requires knowledgeable supervision, however. In any type of concrete work, missing or mislocated reinforcing bars can result in elements with reduced load capacities. Poor handling and/or curing conditions can seriously weaken any concrete. For these and other reasons, most building codes require independent field inspections at various stages of construction.

Proper concrete placement is also somewhat dependent on the ambient weather conditions. Extremely high temperatures and, more important, those below (or near) freezing can make concrete work very difficult.

5.4 STRUCTURAL STEEL

Steel is the strongest and stiffest building material in common use today. Relative to wood and concrete, it is a high-technology material made by highly refined and controlled processes. Structural steel has a uniformly high strength in tension and compression and is also very good in shear. It comes in a range of yield strengths made by adjusting the chemistry of the material in its molten state. It is the most consistent of all structural materials and is, for practical purposes, homogeneous and isotropic, meaning it has like characteristics in all directions. (By contrast, wood is anisotropic.)

The greatest asset of steel is its strength and "plastic reserve," as shown in Figure 4-7. It is highly ductile and deforms greatly before failing if overloaded. Because of steel's strength, the individual members of a frame are usually small in cross-section and have very little visual mass. Steel is a linear material and can be economically made into a visual curve only by using a segmented geometry. It is most appropriately used in rectilinear structures where bolted or welded connections are easy to make. The structural shapes (i.e., pipes, tubes, channels, angles, and wide-flange sections) are manufactured to uniform dimensions having low tolerances. They are fully prepared (cut, trimmed or milled, drilled or punched, etc.) in a fabrication shop, remote from the site, and then delivered ready for erection. Such structures go up rapidly with a minimum of on-site labor. The most popular form of construction used today is referred to as "shop-welded, field-bolted." In this method, the various clip angles, beam seats, and so on are welded to the members in a shop, and then the members are bolted together in the field.

A major disadvantage of structural steel is its need to be fire-protected in most applications. It loses its strength at around 1100°F (600°C) and will then yield rapidly under low loads. A few municipalities require that all structural steel be fire-protected, and most codes will not permit any exposed elements to be within approximately 12 ft (4 m) of a combustible fire source.

The making of steel requires large physical plants and a high capital outlay; therefore, relatively few countries of the world have extensive mill facilities. The cost of manufacturing, coupled with the cost of transportation, can make steel a relatively expensive material. Just the same, in most urban areas, concrete and steel are quite competitive with one another in terms of in-place construction costs.

Continuity in the connections is much harder to achieve in steel than in concrete, and most buildings are constructed with simple connections or ones that are only partially moment-resistant. Some type of lateral load-bracing system is almost always required in a steel-framed building and must be considered early in the design process.

Rolled steel is manufactured in a wide range of strengths. The standard low-carbon mild steel in use today has a yield strength of 36 ksi (250 MPa). However, recent changes in industry practices have made 50-ksi (345-MPa) steel as economical to produce as the 36 (250) grade steel, and the stronger material is increasing in popularity. Steel plate can be obtained with an Fy value of 100 ksi (700 MPa), and most standard shapes can be rolled in steel as strong as 65 ksi (450 MPa), although this can be expensive. Examples and problems in this text are limited to shapes of Fy = 36 ksi (250 MPa) and Fy = 50 ksi (345 MPa).

Information about the various kinds of steel available can be obtained directly from manufacturers and fabricators. The reader is also advised to purchase the latest edition of the Manual of Steel Construction, published by the American Institute of Steel Construction. (With reference to Section 4.8, it is published in two versions: ASD, and LRFD.) It is an indispensable reference work for the design professional.

5.5 MASONRY AND REINFORCED MASONRY

Like concrete, brick and concrete masonry units are strong in compression and weak in tension. These materials have traditionally been used in walls, both bearing and non-bearing.
CHAPTER 5

Usually, wall thicknesses required by code specifications to prevent lateral instability are such that the actual compressive stresses are low. Crushing is seldom an important design constraint.

Masonry walls are more permanent than wood walls and provide effective barriers to both fire and noise. They are less expensive and often more attractive than formed concrete walls. Brick generally has more variation of pattern and texture than does concrete block, but it is also more expensive.

It is becoming increasingly common to use reinforced concrete block for retaining walls and structural pilasters. In this construction, individual reinforcing bars are grouted in some of the vertically aligned cells of the concrete units and serve as tensile reinforcement. This greatly increases the lateral load capacity of the block. Reinforcing can also be placed in special channel-shaped blocks to serve as lintels and tie beams. Brick can be reinforced by using two wythes to create a cavity for grout and reinforcing bars. The brick not only serves as formwork but also carries compressive forces under load.

5.6 CREEP

Section 4.3 explained how structural elements change their size and shape on application of load. This is called elastic strain, and provided that we do not stress the material too greatly, such deformation will disappear on removal of the load. Most materials, if left under load for a long time, will exhibit an additional strain referred to as creep. In most cases, these strains will remain after removal of the load.

The amount of creep, which takes place under long-term load, seems to vary directly with the stress level present and the ambient temperature and inversely with the material stiffness. Many plastics exhibit considerable creep within a short period of time. Steel exhibits very little creep, except at elevated temperatures. Concrete and wood both exhibit creep appreciably if stressed highly for long periods of time.

Members that must support constantly applied loads such as dead weight should be "overdesigned" so that the stresses will be low. For example, the increased deflection (over a couple of years) of a reinforced concrete beam carrying a heavy masonry wall can be double the initial elastic deflection. Many cantilevered portions of wood structures develop an unsightly sag with time that could have been prevented or minimized through the proper consideration of creep.
Wood Design

Notation:

\(\alpha\) = name for width dimension
\(A\) = name for area
\(A_{\text{req'd}}\) = area required at allowable stress when shear is adjusted to include self weight
\(b\) = width of a rectangle
\(c\) = largest distance from the neutral axis to the top or bottom edge of a beam
\(c_1\) = coefficient for shear stress for a rectangular bar in torsion
\(C_D\) = load duration factor
\(C_{fu}\) = flat use factor for other than decks
\(C_F\) = size factor
\(C_H\) = shear stress factor
\(C_i\) = incising factor
\(C_L\) = beam stability factor
\(C_M\) = wet service factor
\(C_p\) = column stability factor for wood design
\(C_r\) = repetitive member factor for wood design
\(C_V\) = volume factor for glue laminated timber design
\(C_t\) = temperature factor for wood design
\(d\) = name for depth
\(d_{\text{min}}\) = dimension of timber critical for buckling
\(D\) = shorthand for dead load
\(DL\) = shorthand for dead load
\(E\) = modulus of elasticity
\(E_{\text{min}}\) = reference modulus of elasticity for stability
\(E_{\text{min n}}\) = reference nominal modulus of elasticity for stability (LRFD)
\(E'_{\text{min}}\) = adjusted modulus of elasticity for stability
\(E'_{\text{min n}}\) = adjusted nominal modulus of elasticity for stability (LRFD)
\(f\) = stress (strength is a stress limit)
\(f_b\) = bending stress
\(f_c\) = compressive stress
\(f_{\text{from table}}\) = tabular strength (from table)
\(f_p\) = bearing stress
\(f_v\) = shear stress
\(f_{v-max}\) = maximum shear stress
\(F_{\text{allow}}\) = allowable stress
\(F_b\) = tabular bending strength
\(F_{b'}\) = allowable bending stress (adjusted)
\(F_{\text{bn}}\) = nominal bending stress (adjusted)
\(F_c\) = tabular compression strength parallel to the grain
\(F_{c'}\) = allowable compressive stress (adjusted)
\(F_{cn}\) = nominal compressive stress (adjusted)
\(F_{cE}\) = intermediate compressive stress dependent on load duration
\(F_{cE}\) = theoretical allowed buckling stress
\(F_{c,E}\) = tabular compression strength perpendicular to the grain
\(F_{\text{connector}}\) = shear force capacity per connector
\(F_{p}\) = tabular bearing strength parallel to the grain
\(F_{p'}\) = allowable bearing stress
\(F_t\) = tabular tensile strength
\(F_u\) = ultimate strength
\(F_v\) = tabular bending strength
\(F_{v'}\) = allowable shear stress
\(F_{vn'}\) = nominal shear stress
\(h\) = height of a rectangle
\(I\) = moment of inertia with respect to neutral axis bending
\(I_{\text{trial}}\) = moment of inertia of trial section
\(I_{\text{req'd}}\) = moment of inertia required at limiting deflection
\(I_y\) = moment of inertia about the y axis
\(J\) = polar moment of inertia
\(K\) = effective length factor for columns
\(K_F\) = format conversion factor for timber LRFD design
\(K_{cE}\) = material factor for wood column design
\(L\) = name for length or span length
\( L_e \) = effective length that can buckle for column design, as is \( \ell_e \)

\( LL \) = shorthand for live load

\( LRFD \) = load and resistance factor design

\( M \) = internal bending moment

\( M_{\text{max}} \) = maximum internal bending moment

\( M_{\text{max}-\text{adj}} \) = maximum bending moment adjusted to include self weight

\( M_n \) = nominal flexure strength for LRFD beam design

\( M_d \) = maximum moment from factored loads for LRFD beam design

\( n \) = number of connectors across a joint, as is \( N \)

\( p \) = pitch of connector spacing

= safe connector load parallel to the grain

\( P \) = name for axial force vector

\( P_{\text{allowable}} \) = allowable axial force

\( P_n \) = nominal column load capacity in LRFD design

\( q \) = safe connector load perpendicular to the grain

\( Q_{\text{connected}} \) = first moment area about a neutral axis for the connected part

\( r \) = radius of gyration

\( R \) = radius of curvature of a deformed beam

\( S \) = section modulus

\( S_{\text{req'd}} \) = section modulus required at allowable stress

\( S_{\text{req'd}}-\text{adj} \) = section modulus required at allowable stress when moment is adjusted to include self weight

\( T \) = torque (axial moment)

\( V \) = internal shear force

\( V_{\text{max}} \) = maximum internal shear force

\( V_{\text{max}-\text{adj}} \) = maximum internal shear force adjusted to include self weight

\( V_n \) = nominal shear strength capacity for LRFD beam design

\( w \) = name for distributed load

\( w_{\text{equivalent}} \) = equivalent distributed load to produce the maximum moment

\( w_{\text{self wt}} \) = name for distributed load from self weight of member

\( x \) = horizontal distance

\( y \) = vertical distance

\( Z \) = force capacity of a connector

\( \Delta_{\text{actual}} \) = actual beam deflection

\( \Delta_{\text{allowable}} \) = allowable beam deflection

\( \Delta_{\text{limit}} \) = allowable beam deflection limit

\( \Delta_{\text{max}} \) = maximum beam deflection

\( \phi \) = resistance factor in LRFD

\( \lambda \) = time effect factor in LRFD design

\( \gamma \) = density or unit weight

= load factor in LRFD design

\( \theta \) = slope of the beam deflection curve

\( \rho \) = radial distance

\( \int \) = symbol for integration

\( \Sigma \) = summation symbol

---

**Wood or Timber Design**

Structural design standards for wood are established by the *National Design Specification (NDS)* published by the American Wood Council. There is a combined specification (from 2005) for **Allowable Stress Design** and limit state design (LRFD).

Tabulated (or *reference*) wood strength values are used as the base allowable strength and modified by appropriate **adjustment** factors:

\[
 f = C_D C_M C_F \ldots \times f_{\text{from table}}
\]

**Size and Use Categories**

- **Boards:** 1 to 1½ in. thick 2 in. and wider
- **Dimension lumber:** 2 to 4 in. thick 2 in. and wider
- **Timbers:** 5 in. and thicker 5 in. and wider
Adjustment Factors

(partial list)

- **C\(_D\)**: load duration factor
- **C\(_M\)**: wet service factor (1.0 dry \(\leq 19\%\) moisture content sawn, \(\leq 16\%\) moisture content glu-lam)
- **C\(_F\)**: size factor for visually graded sawn lumber and round timber > 12” depth
  \[ C_F = \left( \frac{12}{d} \right)^{\frac{1}{6}} \leq 1.0 \]
- **C\(_{fu}\)**: flat use factor (excluding decking)
- **C\(_i\)**: incising factor (from increasing the depth of pressure treatment)
- **C\(_t\)**: temperature factor (at high temperatures strength decreases)
- **C\(_r\)**: repetitive member factor
- **C\(_H\)**: shear stress factor (amount of splitting)
- **C\(_V\)**: volume factor for glued laminated timber (similar to C\(_F\))
- **C\(_L\)**: beam stability factor (for beams without full lateral support)

Tabular (or Reference) Design Values

- **F\(_b\)**: bending stress
- **F\(_t\)**: tensile stress
- **F\(_v\)**: horizontal shear stress
- **F\(_c\)\(_\perp\)**: compression stress (perpendicular to grain)
- **F\(_c\)\(_\parallel\)**: compression stress (parallel to grain)
- **E**: modulus of elasticity (E\(_\text{min}\) used for column design)
- **F\(_p\)**: bearing stress (parallel to grain)

Wood is significantly weakest in shear and strongest along the direction of the grain (tension and compression).

Load Combinations and Deflection

The critical load combination (ASD) is determined by the largest of either:

\[
\frac{\text{dead load}}{0.9} \text{ or } \left( \frac{\text{dead load} + \text{any combination of live load}}{C_D} \right)
\]

The deflection limits may be increased for less stiffness with total load: LL + 0.5(DL)

Criteria for Design of Beams

Allowable normal stress or normal stress from LRFD should not be exceeded:

\[ F_b' \text{ or } \phi F_u \geq f_b = \frac{M_c}{I} \]

Knowing M and F\(_b\)'\(_b\), the minimum section modulus fitting the limit is:

\[ S_{\text{req}} \geq \frac{M}{F_b'} \]
Besides strength, we also need to be concerned about serviceability. This involves things like limiting deflections & cracking, controlling noise and vibrations, preventing excessive settlements of foundations and durability. When we know about a beam section and its material, we can determine beam deformations.

**Determining Maximum Bending Moment**

Drawing V and M diagrams will show us the maximum values for design. Remember:

\[
V = \Sigma(-w)dx \\
M = \Sigma(V)dx \\
\frac{dV}{dx} = -w \\
\frac{dM}{dx} = V
\]

**Determining Maximum Bending Stress**

For a prismatic member (constant cross section), the maximum normal stress will occur at the maximum moment.

For a non-prismatic member, the stress varies with the cross section AND the moment.

**Deflections**

If the bending moment changes, M(x) across a beam of constant material and cross section then the curvature will change:

The slope of the n.a. of a beam, \( \theta \), will be tangent to the radius of curvature, \( R \):

\[ \theta = \text{slope} = \frac{1}{EI} \int M(x)dx \]

The equation for deflection, y, along a beam is:

\[ y = \frac{1}{EI} \int \theta dx = \frac{1}{EI} \int M(x)dx \]

Elastic curve equations can be found in handbooks, textbooks, design manuals, etc...Computer programs can be used as well (like Multiframe).

Elastic curve equations can be **superpositioned** ONLY if the stresses are in the elastic range. The deflected shape is roughly the same shape flipped as the bending moment diagram but is constrained by supports and geometry.
Boundary Conditions

The boundary conditions are geometrical values that we know – slope or deflection – which may be restrained by supports or symmetry.

At Pins, Rollers, Fixed Supports: \( y = 0 \)

At Fixed Supports: \( \theta = 0 \)

At Inflection Points From Symmetry: \( \theta = 0 \)

The Slope Is Zero At The Maximum Deflection \( y_{\text{max}} \):

\[
\frac{\partial y}{\partial x} = \text{slope} = 0
\]

Allowable Deflection Limits

All building codes and design codes limit deflection for beam types and damage that could happen based on service condition and severity.

\[
y_{\text{max}} (x) = \Delta_{\text{actual}} \leq \Delta_{\text{allowable}} = \frac{L}{\text{value}}
\]

<table>
<thead>
<tr>
<th>Use</th>
<th>LL only</th>
<th>DL+LL</th>
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<tbody>
<tr>
<td>Roof beams:</td>
<td></td>
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</tr>
<tr>
<td>Industrial</td>
<td>L/180</td>
<td>L/120</td>
</tr>
<tr>
<td>Commercial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>plaster ceiling</td>
<td>L/240</td>
<td>L/180</td>
</tr>
<tr>
<td>no plaster</td>
<td>L/360</td>
<td>L/240</td>
</tr>
<tr>
<td>Floor beams:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ordinary Usage</td>
<td>L/360</td>
<td>L/240</td>
</tr>
<tr>
<td>Roof or floor (damageable elements)</td>
<td>L/480</td>
<td></td>
</tr>
</tbody>
</table>

Lateral Buckling

With compression stresses in the top of a beam, a sudden “popping” or buckling can happen even at low stresses. In order to prevent it, we need to brace it along the top, or laterally brace it, or provide a bigger \( I_y \).

Beam Loads & Load Tracing

In order to determine the loads on a beam (or girder, joist, column, frame, foundation...) we can start at the top of a structure and determine the \textit{tributary area} that a load acts over and the beam needs to support. Loads come from material weights, people, and the environment. This area is assumed to be from half the distance to the next beam over to halfway to the next beam.

The reactions must be supported by the next lower structural element \textit{ad infinitum}, to the ground.
Design Procedure

The intent is to find the most lightweight member satisfying the section modulus size.

1. Know $F_{all}$ ($F_b$) for the material or get adjustment factors and resistance factor for LRFD.
2. Draw V & M, finding $M_{max}$.
3. Calculate $S_{req'd}$. This step is equivalent to determining $f_b = \frac{M_{max}}{S} \leq F_b'$
4. For rectangular beams $S = \frac{bh^2}{6}$
   - For timber: use the section charts to find $S$ that will work and remember that the beam self weight will increase $S_{req'd}$. $w_{self\ wt} = \gamma A$

****Determine the “updated” $V_{max}$ and $M_{max}$ including the beam self weight, and verify that the updated $S_{req'd}$ has been met.******

5. Consider lateral stability.
6. Evaluate horizontal shear stresses using $V_{max}$ to determine if $f_v \leq F_v'$ or find $A_{req'd}$

   For rectangular beams $f_{v-max} = \frac{3V}{2A} = 1.5 \frac{V}{A}$ $\therefore A_{req'd} \leq \frac{3V}{2F_v'}$
7. Provide adequate bearing area at supports: $f_p = \frac{P}{A} \leq F_p'$ (from $F_c$ or $F_{c1}$)
8. Evaluate shear due to torsion $f_v = \frac{Tp}{J} or \frac{T}{c_iab^2} \leq F_v'$
9. Evaluate the deflection to determine if $\Delta_{maxLL} \leq \Delta_{LL-allowed}$ and/or $\Delta_{maxTotal} \leq \Delta_{Total-allowed}$

**** note: when $\Delta_{calculated} > \Delta_{limit}$, $I_{required}$ can be found with: $I_{req'd} \geq \frac{\Delta_{total-allowed}}{\Delta_{limit}} I_{trial}$

FOR ANY EVALUATION:

Redesign (with a new section) at any point that a stress or serviceability criteria is NOT satisfied and re-evaluate each condition until it is satisfactory.

Load Tables for Uniformly Loaded Joists & Rafters

Tables exist for the common loading situation for joists and rafters – that of uniformly distributed load. The tables either provide the safe distributed load based on bending and deflection limits, they give the allowable span for specific live and dead loads. If the load is not uniform, an equivalent distributed load can be calculated from the maximum moment equation.

If the deflection limit is less, the design live load to check against allowable must be increased, ex. $w_{adjusted} = \frac{\left(L/400\right)^{\text{wanted}}}{\left(L/360\right)^{\text{table limit}}}$

$M_{max} = \frac{w_{equivalent}L^2}{8}$
Decking

Flat panels or planks that span several joists or evenly spaced support behave as continuous beams. Design tables consider a “1 unit” wide strip across the supports and determine maximum bending moment and deflections in order to provide allowable loads depending on the depth of the material.

The other structural use of decking is to construct what is called a diaphragm, which is a horizontal or vertical (if the panels are used in a shear wall) unit tying the sheathing to the joists or studs that resists forces parallel to the surface of the diaphragm.

LRFD Introduction

With the publication of the 2005 edition of the National Design Specifications, the load and resistance factor design method was included in the same document as the ASD method using adjustment factors for format conversion (\(K_F\)) and a time effect factor (\(\lambda\)) rather than the use of the load duration factor (\(C_D\)).

The general format, with bending as an example, requires the factored design value (\(M_u\)) not exceed the maximum adjusted capacity (\(M_n\)) which is determined from determining the adjusted nominal strength (\(F'_{bu}\)): 
\[
M_u \leq M_n'
\]

where 
\[
M_n = F'_{bu} \times S
\]
\[
F'_{bu} = F_{bu} (\phi_b) (\lambda) (C_M)(C_i)(C_{tl})(C_f)(C_r)
\]
\[
F_{bu} = F_b \times K_F
\]

For shear the relationships for \(V_u \leq V'\) are:

where 
\[
V_u = \frac{2}{3} F'_{\text{sw}} \times A
\]
\[
F'_{\text{sw}} = F_{\text{sw}} (\phi_v) (\lambda) (C_M)(C_r)(C_i)
\]
\[
F_{\text{sw}} = F_v \times K_F
\]

For axial compression the relationships for \(P_u \leq P'\) are:

where 
\[
P_u = F'_{\text{con}} \times A
\]
\[
F'_{\text{con}} = F_{\text{con}} (\phi_c) (\lambda) (C_M)(C_r)(C_f)(C_p)(C_t)
\]
\[
F_{\text{con}} = F_c \times K_F
\]

and 
\[
F'_{\text{con},n} = F_{\text{con},n} (\phi_c) (C_M)(C_r)(C_f)(C_t)
\]

\[
F_{\text{con},n} = F_{\text{con},n} \times K_F
\]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_b)</td>
<td>Flexure (bending)</td>
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</tr>
<tr>
<td>(\phi_c)</td>
<td>Compression, bearing</td>
<td>0.90</td>
</tr>
<tr>
<td>(\phi_t)</td>
<td>Tension</td>
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</tr>
<tr>
<td>(\phi_v)</td>
<td>Shear</td>
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<tr>
<td>(\phi_s)</td>
<td>Stability, (E_{\text{min}})</td>
<td>0.85</td>
</tr>
<tr>
<td>(\phi_t)</td>
<td>Connections</td>
<td>0.65</td>
</tr>
</tbody>
</table>
Criteria for Design of Columns

If we know the loads, we can select a section that is adequate for strength & buckling.

If we know the length, we can find the limiting load satisfying strength & buckling.

Any slenderness ratio, $L/e/d \leq 50$:

$$f_c = \frac{P}{A} \leq F'_c$$

$$F'_c = F_c \left( C_D \right) \left( C_M \right) \left( C_t \right) \left( C_F \right) \left( C_p \right)$$

The allowable stress equation uses factors to replicate the combination crushing-buckling curve:

where:

- $F'_c$ = allowable compressive stress parallel to the grain
- $F_c$ = compressive strength parallel to the grain
- $c = 0.8$ for sawn lumber, 0.85 for poles, 0.9 for glulam timber
- $C_D$ = load duration factor
- $C_M$ = wet service factor (1.0 for dry)
- $C_t$ = temperature factor
- $C_F$ = size factor
- $C_p$ = column stability factor off chart or equation: $C_p = \frac{1+\left(\frac{F_{cL}}{F'_c}\right)}{2c} \sqrt{\frac{1+\left(\frac{F_{cL}}{F'_c}\right)^2}{2c}} - \frac{F_{cL}}{F'_c} \cdot \frac{1}{c}$

For preliminary column design:

$$F'_c = F_c^* C_p = \left( F_c C_D \right) C_p$$
Procedure for Analysis

1. Calculate \( L_e/d_{\min} \) (KL/d for each axis and chose largest)
2. Obtain \( F'_c \)
   
   \[
   F'_c = \frac{0.822E'_{\min}}{\left(\frac{L_e}{d}\right)^2}
   \]
   
   with \( E'_{\min} = E_{\min}(C_M)(C_i)(C_T)(C_j) \)
3. Compute \( F'_c \equiv F_c C_p \) with \( C_D = 1 \), normal, \( C_D = 1.25 \) for 7 day roof...
4. Calculate \( F'_{cE} / F'_c \) and get \( C_p \) from table or calculation
5. Calculate \( F'_c = F'_c C_p \)
6. Compute \( P_{\text{allowable}} = F'_c A \) or alternatively compute \( f_{\text{actual}} = P/A \)
7. Is the design satisfactory?
   
   Is \( P \leq P_{\text{allowable}} \)? \( \Rightarrow \) yes, it is; no, it is no good
   
   or Is \( f_{\text{actual}} \leq F'_c \)? \( \Rightarrow \) yes, it is; no, it is no good

Procedure for Design

1. Guess a size by picking a section
2. Calculate \( L_e/d_{\min} \) (KL/d for each axis and chose largest)
3. Obtain \( F'_c \)
   
   \[
   F'_c = \frac{0.822E'_{\min}}{\left(\frac{L_e}{d}\right)^2}
   \]
   
   with \( E'_{\min} = E_{\min}(C_M)(C_i)(C_T)(C_j) \)
4. Compute \( F'_c \equiv F_c C_p \) with \( C_D = 1 \), normal, \( C_D = 1.25 \) for 7 day roof...
5. Calculate \( F'_{cE} / F'_c \) and get \( C_p \) from table or calculation
6. Calculate \( F'_c = F'_c C_p \)
7. Compute \( P_{\text{allowable}} = F'_c A \) or alternatively compute \( f_{\text{actual}} = P/A \)
8. Is the design satisfactory?
   
   Is \( P \leq P_{\text{allowable}} \)? \( \Rightarrow \) yes, it is; no, pick a bigger section and go back to step 2.
   
   or Is \( f_{\text{actual}} \leq F'_c \)? \( \Rightarrow \) yes, it is; no, pick a bigger section and go back to step 2.

Columns with Bending (Beam-Columns)

The modification factors are included in the form:

\[
\left[ \frac{f_c}{F'_c} \right]^2 + \left( \frac{f_{bx}}{F'_{bx}} \right) \left[ 1 - \frac{f_c}{F'_{cEx}} \right] \leq 1.0
\]

where:

\[
1 - \frac{f_c}{F'_{cEx}} = \text{magnification factor accounting for P-\Delta}
\]

\( F'_{bx} = \text{allowable bending stress} \)

\( f_{bx} = \text{working stress from bending about x-x axis} \)
In order to design an adequate section for allowable stress, we have to start somewhere:

1. Make assumptions about the limiting stress from:
   - buckling
   - axial stress
   - combined stress
2. See if we can find values for $r$ or $A$ or $S$ ($=I/c_{max}$)
3. Pick a trial section based on if we think $r$ or $A$ is going to govern the section size.
4. Analyze the stresses and compare to allowable using the allowable stress method or interaction formula for eccentric columns.
5. Did the section pass the stress test?
   - If not, do you increase $r$ or $A$ or $S$?
   - If so, is the difference really big so that you could decrease $r$ or $A$ or $S$ to make it more efficient (economical)?
6. Change the section choice and go back to step 4. Repeat until the section meets the stress criteria.

Criteria for Design of Connections

Connections for wood are typically mechanical fasteners. Shear plates and split ring connectors are common in trusses. Bolts of metal bear on holes in wood, and nails rely on shear resistance transverse and parallel to the nail shaft. Timber rivets with steel side plates are allowed with glue laminated timber.

Connections must be able to transfer any axial force, shear, or moment from member to member or from beam to column.

Bolted Joints

Stress must be evaluated in the member being connected using the load being transferred and the reduced cross section area called net area. Bolt capacities are usually provided in tables and take into account the allowable shearing stress across the diameter for single and double shear, and the allowable bearing stress of the connected material based on the direction of the load with respect to the grain. Problems, such as ripping of the bolt hole at the end of the member, are avoided by following code guidelines on minimum edge distances and spacing.

Nailed Joints

Because nails rely on shear resistance, a common problem when nailing is splitting of the wood at the end of the member, which is a shear failure. Tables list the shear force capacity per unit length of embedment per nail. Jointed members used for beams will have shear stress across the connector, and the pitch spacing, $p$, can be determined from the shear stress equation when the capacity, $F$, is known:

$$nF_{\text{connector}} \geq \frac{VQ_{\text{connected area}}}{I} \cdot p$$
Example 1 (pg 204)

Example 2. A simple beam has a span of 16 ft (4.88 m) and supports a total uniformly distributed load, including its own weight, of 6500 lb (28.9 kN). Using Douglas fir-larch, select structural grade, determine the size of the beam with the least cross-sectional area on the basis of limiting bending stress. Density of Douglas fir-larch is 32 lb/ft$^3$.

Example 2 (pg 207)

Example 3. A 6 × 10 beam of Douglas fir-larch, No. 2 grade, has a total horizontally distributed load of 6000 lb (26.7 kN). Investigate for shear stress.

Example 3 (pg 209)

Example 6. A two-span 3 × 12 beam of Douglas fir-larch, No. 1 grade, bears on a 3 × 14 beam at its center support. If the reaction force is 4200 lb (18.7 kN), is this safe for bearing?
Example 4 (pg 212)

Example 7. An 8 × 12 wood beam with \( E = 1,600,000 \, \text{psi} \) is used to carry a total uniformly distributed load of 10 kips on a simple span of 16 ft. Find the maximum deflection of the beam.

\[ F_b = 1500 \, \text{psi}; \quad F_v = 110 \, \text{psi}; \quad F_c\perp = 440 \, \text{psi}; \quad E = 1.6 \times 10^6 \, \text{psi}; \quad \gamma = 36.3 \, \text{lb/ft}^3. \]

SOLUTION:

Because this beam appears to support other beams at the locations of the roof construction loads, we have to assume that this beam is not closely spaced to others and the repetitive use adjustment factor doesn’t apply. The load duration factor, \( C_D \), is 1.25 for roof construction loads. The other conditions (like temperature and moisture) must be assumed to be normal (and have values of 1.0). The allowable stresses can be determined from:

\[
F_b' = C_D F_b = (1.25)(1500) = 1875 \, \text{psi} \\
F_v' = C_D F_v = (1.25)(110) = 137.5 \, \text{psi} \\
F_c\perp' = C_D F_c\perp = (1.25)(440) = 550 \, \text{psi}
\]

Example 5 (pg 223)

Example 13. Using Table 5.10 select joists to carry a live load of 40 psf and a dead load of 10 psf on a span of 15 ft 6 in. if the spacing is 16 in. on center.

<table>
<thead>
<tr>
<th>Joint Size</th>
<th>2 × 6</th>
<th>2 × 8</th>
<th>2 × 10</th>
<th>2 × 12</th>
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<td>12</td>
<td>10-9</td>
<td>14-2</td>
<td>17-9</td>
<td>20-7</td>
</tr>
<tr>
<td>16</td>
<td>9-9</td>
<td>12-7</td>
<td>15-5</td>
<td>17-10</td>
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<tr>
<td>19.2</td>
<td>9-1</td>
<td>11-6</td>
<td>14-1</td>
<td>16-3</td>
</tr>
<tr>
<td>24</td>
<td>8-1</td>
<td>10-3</td>
<td>12-7</td>
<td>14-7</td>
</tr>
</tbody>
</table>

Source: Compiled from data in the International Building Code (Ref. 4), with permission of the publisher, International Code Council.

Joists are Douglas fir-larch, No. 2 grade. Assumed maximum available length of single piece is 24 ft.

Example 6

Design a Southern pine No. 1 beam to carry the loads shown (roof beam, no plaster). Assume the beam is supported at each end of by an 8” block wall. \( F_b = 1500 \, \text{psi}; \ F_v = 110 \, \text{psi}; \ F_{c\perp} = 440 \, \text{psi}; \ E = 1.6 \times 10^6 \, \text{psi}; \ \gamma = 36.3 \, \text{lb/ft}^3. \)

SOLUTION:

Because this beam appears to support other beams at the locations of the roof construction loads, we have to assume that this beam is not closely spaced to others and the repetitive use adjustment factor doesn’t apply. The load duration factor, \( C_D \), is 1.25 for roof construction loads. The other conditions (like temperature and moisture) must be assumed to be normal (and have values of 1.0). The allowable stresses can be determined from:

\[
F_b' = C_D F_b = (1.25)(1500) = 1875 \, \text{psi} \\
F_v' = C_D F_v = (1.25)(110) = 137.5 \, \text{psi} \\
F_{c\perp}' = C_D F_{c\perp} = (1.25)(440) = 550 \, \text{psi}
\]
Bending:

\[ S_{req'd} \geq \frac{M}{F_b} \geq \frac{12,813^{3/2} \cdot (12 \text{ in} / \text{ft})}{1875 \text{ psi}} = 82.0 \text{ in}^3 \]

Try a 3 x 16. This satisfies both requirements with the least amount of area. (See the 4 x 14, 6 x 10, and 8 x 10.)

\[ A_{req'd} \geq \frac{3V}{2F_c'} \geq \frac{3(2,750 \text{ lb})}{2(137.5 \text{ psi})} = 30.0 \text{ in}^2 \]

Shear:

\[ \text{NOTE: If the area or section that I have is not adequate, I need to choose one that is. This will have a } \]

\[ \text{smaller } S_{req'd} \text{ and } A_{req'd} \text{ bigger as well, and the new section properties must be evaluated with respect to these new values.} \]

Deflection:

The total deflection due to dead and live loads must not exceed a limit specified by the building code adopted (for example, the International Building Code) or recommended by construction manuals. For a commercial roof beam with no plaster, the usual limits are L/360 for live load only and L/240 for live and dead load.

\[ \Delta_{live-lim} = \frac{15 \text{ ft} \cdot (12 \text{ in} / \text{ft})}{360} = 0.5 \text{ in} \quad \Delta_{total-lim} = \frac{15 \text{ ft} \cdot (12 \text{ in} / \text{ft})}{240} = 0.75 \text{ in} \]

Superpositioning (combining or superimposing) of several load conditions can be performed, but care must be taken that the deflections calculated for the separate cases to obtain the maximum must be deflections at the same location in order to be added together:

\[ \text{two symmetrically placed equal point loads (live load): } (a \text{ is the distance from the supports to the loads) } \]

\[ \Delta_{max (at center)} = \frac{P_a}{2AE} \left( \frac{3a^2 - 4a^2}{2} \right) = 0.35 \text{ in} \]

\[ \text{distributed load (dead load) } \]

\[ \Delta_{max (at center)} = \frac{5W a^2}{384EI} = 0.11 \text{ in} \]

Bearing:

Determine if the bearing stress between the beam and the block wall support less than the allowable. If it is not, the beam width must be increased:

\[ f_\rho = \frac{P}{A} = \frac{2822 \text{ lb}}{(2.5 \text{ in})(8 \text{ in})} = 141.1 \text{ psi} \leq F'_{cL} = 550 \text{ psi} \]

So, yes the beam width (2.5 in) is adequate.

USE a 3 x 16.
Example 7 (pg 239)

Example 1. A wood column consists of a 6 \times 6 of Douglas fir-larch, No. 1 grade. Find the safe axial compression load for unbraced lengths of: (1) 2 ft, (2) 8 ft, (3) 16 ft. using the ASD method.

\[ E_{\text{min}} = 580 \times 10^3 \text{ psi} \]

Example 8

Example Problem 10.18 (Figures 10.60 and 10.61)

An 18' tall 6x8 Southern pine column supports a roof load (dead load plus a 7-day live load) equal to 16 kips. The weak axis of buckling is braced at a point 9'6" from the bottom support. Determine the adequacy of the column.

\[ f_c = 975 \text{ psi} \quad E = 1.6 \times 10^6 \text{ psi} \quad E_{\text{min}} = 580 \times 10^3 \text{ psi} \]

Figure 10.61  (a) Strong axis. (b) Weak axis.
Example 8 (fully worked)

Example Problem 10.18 (Figures 10.60 and 10.61)

An 18' tall 6x8 Southern pine column supports a roof load (dead load plus a 7-day live load) equal to 16 kips. The weak axis of buckling is braced at a point 96" from the bottom support. Determine the adequacy of the column.

Solution:

6x8 S4S Southern pine post: \( A = 41.25 \text{ in}^2, F_c = 975 \text{ psi, } E = 1.6 \times 10^6 \text{ psi} \) \( E_{\text{min}} = 580 \times 10^3 \text{ psi} \)

Check the slenderness ratio about the weak axis:

\[
\frac{L_e}{d} = \frac{(9.5' \times 12 \text{ in./ft.})}{5.5'} = 20.7
\]

The slenderness ratio about the strong axis is:

\[
\frac{L_e}{d} = \frac{(18' \times 12 \text{ in./ft.})}{7.5'} = 28.8 \quad \text{goes governs}
\]

\[
F_{cE} = \frac{0.822 E_{\text{min}}'}{(L_e/d)^2} = \frac{0.822 (580 \times 10^3 \text{ lb/in.})}{(28.8)^2} = 575 \text{ psi}
\]

\[
F_c^* \approx F_{CD} = (975 \text{ lb./in.}^2)(1.25) = 1220 \text{ psi}
\]

where: \( C_D = 1.25 \) for 7-day-duration load \[ \frac{F_{cE}}{F_c^*} = \frac{575 \text{ psi}}{1220 \text{ psi}} = 0.471 \]

From Appendix Table 14: \( C_p = 0.412 \)

\[ F_c' = F_c^* C_p = 1220 \text{ lb./in.}^2 \times 0.412 = 503 \text{ psi} \]

\[ P_d = F_c' \times A = (503 \text{ lb./in.}^2) \times (41.25 \text{ in.}^2) = 20,700 \text{ lb.} \]

\[ P_d = 20.7 \text{ k} > P_{\text{actual}} = 16 \text{ k} \]

The column is adequate.

---

Figure 10.60  Wood column with intermediate bracing.

Figure 10.61 (a) Strong axis. (b) Weak axis.
Example 9 (pg 251)

**Example 4.** An exterior wall stud of Douglas fir-larch, stud grade, is loaded as shown in Figure 6.5a. Investigate the stud for the combined loading. (*Note:* This is the wall stud from the building example in Chapter 18.) *(Wind load duration does apply, as well as load combinations.)*
Example 10 (pg 264)

Example 2. The truss heel joint shown in Figure 7.5 is made with 2 in. nominal thickness lumber and gusset plates of $\frac{1}{2}$-in. -thick plywood. Nails are 6d common wire with the nail layout shown occurring in both sides of the joint. Find the tension load capacity for the bottom chord member (load 3 in the figure).

Example 11

A nominal 4 x 6 in. redwood beam is to be supported by two 2 x 6 in. members acting as a spaced column. The minimum spacing and edge distances for the $\frac{1}{2}$ inch bolts are shown. How many $\frac{1}{2}$ in. bolts will be required to safely carry a load of 1500 lb? Use the chart provided.

### Table 7.1: Reference Lateral Load Values for Common Wire Nails (lb/ln)

<table>
<thead>
<tr>
<th>Side Member Thickness, $t_i$ (in.)</th>
<th>Nail Length, $L$ (in.)</th>
<th>Nail Diameter, $D$ (in.)</th>
<th>Nail Pennyweight</th>
<th>Load per Nail, $Z$ (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{3}{4}$</td>
<td>2</td>
<td>0.113</td>
<td>6d</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>2$\frac{1}{2}$</td>
<td>0.131</td>
<td>8d</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.148</td>
<td>10d</td>
<td>76</td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td>2</td>
<td>0.113</td>
<td>6d</td>
<td>50</td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td>2$\frac{1}{2}$</td>
<td>0.131</td>
<td>8d</td>
<td>65</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.148</td>
<td>10d</td>
<td>77</td>
</tr>
<tr>
<td>$\frac{5}{8}$</td>
<td>2$\frac{1}{2}$</td>
<td>0.162</td>
<td>16d</td>
<td>92</td>
</tr>
</tbody>
</table>

### Table 23.4-F: Holding Power of Bolts

- **p** = safe loads parallel to grain, in pounds.
- **q** = safe loads perpendicular to grain, in pounds.

<table>
<thead>
<tr>
<th>LENGTH OF BOLT IN MAIN WOOD MEMBER (Inches)</th>
<th>DIAMETER OF BOLT (Inches)</th>
<th>( P )</th>
<th>( \frac{1}{2} )</th>
<th>( \frac{3}{4} )</th>
<th>( \frac{7}{8} )</th>
<th>( 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2$\frac{1}{2}$</td>
<td></td>
<td>4.45</td>
<td>25.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Tabulated values are on a normal load-duration basis and apply to joints made of seasoned lumber used in dry locations. See Division III for other service conditions.
2. Double shear values are for joints consisting of three wood members in which the side members are one half the thickness of the main member. Single shear values are for joints consisting of two wood members having a minimum thickness not less than that specified.
3. See Division III for wood-to-metal bolted joints.
4. The length specified is the length of the bolt in the main member of double shear joints or the length of the bolt in the thinner member of single shear joints.
This may be both the limit for live load deflection and total load deflection.

Collect data: \( F_b \) & \( F_v \)

Find \( V_{\text{max}} \) & \( M_{\text{max}} \) from constructing diagrams or using beam chart formulas

Find \( S_{\text{req'd}} \) and pick a section from a table with \( S_x \) greater or equal to \( S_{\text{req'd}} \)

Calculate \( \omega \) (self wt.) using \( A \) found and \( \gamma \). Find \( M_{\text{max-adj}} \) & \( V_{\text{max-adj}} \)

Calculate \( S_{\text{req'd-adj}} \) using \( M_{\text{max-adj}} \)

Is \( S_{\text{req'd-adj}} \) \( \geq S_{\text{req'd}} \) ?

(OR calculate \( f_b \) Is \( f_b \) \( \leq F_b \) ?)

Yes

Calculate \( A_{\text{req'd-adj}} \) using \( V_{\text{max-adj}} \)

Is \( A_{\text{req'd-adj}} \) \( \geq A_{\text{req'd}} \) ?

(OR calculate \( f_v \) Is \( f_v \) \( \leq F_v \) ?)

No  pick a new section with a larger area

Calculate \( \Delta_{\text{max}} \) using superpositioning and beam chart equations with the \( I_x \) for the section

Is \( \Delta_{\text{max}} \leq \Delta_{\text{lim}} \) ?

This may be both the limit for live load deflection and total load deflection.

No  pick a section with a larger \( I_x \)

Yes  (DONE)
### Section Properties/Standard Sizes

To the extent that other considerations will permit, the finished sizes of structural glued laminated timber as given in Table B constitute normal industry practice. Industry standards do, however, permit the use of any depth or width of glued laminated timber. Dimension lumber of 1/2 in. net thickness is normally used for laminating straight members. The modified section modulus includes size factor \((C_{n})\), and no further reduction of bending stress for size is needed.

<table>
<thead>
<tr>
<th>DEPTH, d in.</th>
<th>AREA, A in.²</th>
<th>MODIFIED SECTION MODULUS, S_n in.³</th>
<th>MOMENT OF INERTIA, I_n in.⁴</th>
<th>DEPTH, d in.</th>
<th>AREA, A in.²</th>
<th>MODIFIED SECTION MODULUS, S_n in.³</th>
<th>MOMENT OF INERTIA, I_n in.⁴</th>
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<tbody>
<tr>
<td>3 1/4&quot; WIDTH</td>
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<td>3 1/4&quot; WIDTH</td>
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<td>6.0</td>
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<td>18.8</td>
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<td>379.5</td>
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<td>60.0</td>
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<td></td>
</tr>
<tr>
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ARCH 614

Note Set 13.2

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Table developed and permission for use granted by Professor Ed Leberi, Dept. of Architecture, University of Washington.
Connections & Stresses

Notation:

\[ A = \text{area (net = with holes, bearing = in contact, etc...)} \]
\[ d = \text{diameter of a hole} \]
\[ f_p = \text{bearing stress (see P)} \]
\[ f_t = \text{tensile stress} \]
\[ f_v = \text{shear stress} \]
\[ P = \text{name for axial force vector, as is } T \]
\[ t = \text{thickness} \]
\[ \pi = \text{pi (3.1415 radians or 180°)} \]

Bolts in Shear and Bearing

Single shear - forces cause only one shear “drop” across the bolt.

\[ f_v = \text{Average shear stress through bolt cross section} \]
\[ A = \text{Bolt cross-sectional area} \]
\[ f_v = \frac{P}{A} \]

Figure 5.11 A bolted connection—single shear.
Double shear - forces cause two shear changes across the bolt.

\[ f_v = \frac{P}{2A} \]

(two shear planes)

Bearing of a bolt on a bolt hole – The bearing surface can be represented by projecting the cross section of the bolt hole on a plane (into a rectangle).

\[ f_p = \frac{P}{A} = \frac{P}{td} \]

Bearing stress on plate.
**Example 1**

A pipe storage rack is used for storing pipe in a shop. The support rack beam is fastened to the main floor beam using steel straps $\frac{1}{2}" \times 2"$ in dimension. Round bolts are used to fasten the strap to the floor beam in single shear. (a) If the weight of the pipes impose a maximum tension load of 10,000 pounds in each strap, determine the tension stress developed in the steel strap. (b) Also, what diameter bolt is necessary to fasten the strap to the floor beam if the allowable shear stress for the bolts equals $f_s = 15,000 \text{lb/in}^2$? Determine the bearing stress in the strap from the bolt diameter chosen.
Steel Design

Notation:

\(a\) = name for width dimension
\(A\) = name for area
\(A_b\) = area of a bolt
\(A_e\) = effective net area found from the product of the net area \(A_n\) by the shear lag factor \(U\)
\(A_g\) = gross area, equal to the total area ignoring any holes
\(A_{gv}\) = gross area subjected to shear for block shear rupture
\(A_n\) = net area, equal to the gross area subtracting any holes, as is \(A_{net}\)
\(A_{nt}\) = net area subjected to tension for block shear rupture
\(A_{nv}\) = net area subjected to shear for block shear rupture
\(A_w\) = area of the web of a wide flange section
\(AISC\) = American Institute of Steel Construction
\(ASD\) = allowable stress design
\(b\) = name for a (base) width
\(b_f\) = width of the flange of a steel beam cross section
\(B_1\) = factor for determining \(M_u\) for combined bending and compression
\(c\) = largest distance from the neutral axis to the top or bottom edge of a beam
\(c_l\) = coefficient for shear stress for a rectangular bar in torsion
\(C_b\) = modification factor for moment in ASD & LRFD steel beam design
\(C_c\) = column slenderness classification constant for steel column design
\(C_m\) = modification factor accounting for combined stress in steel design
\(C_v\) = web shear coefficient
\(d\) = calculus symbol for differentiation
\(d_b\) = nominal bolt diameter
\(D\) = shorthand for dead load
\(DL\) = shorthand for dead load
\(e\) = eccentricity
\(E\) = shorthand for earthquake load
\(f_c\) = axial compressive stress
\(f_b\) = bending stress
\(f_p\) = bearing stress
\(f_v\) = shear stress
\(f_{v-max}\) = maximum shear stress
\(f_y\) = yield stress
\(F\) = shorthand for fluid load
\(F_a\) = allowable axial (compressive) stress
\(F_b\) = allowable bending stress
\(F_c\) = critical unfactored compressive stress for buckling in LRFD
\(F_{cr}\) = flexural buckling stress
\(F_e\) = elastic critical buckling stress
\(F_{EXX}\) = yield strength of weld material
\(F_n\) = nominal strength in LRFD = nominal tension or shear strength of a bolt
\(F_p\) = allowable bearing stress
\(F_t\) = allowable tensile stress
\(F_u\) = ultimate stress prior to failure
\(F_y\) = allowable shear stress
\(F_y\) = yield strength
\(F_{yw}\) = yield strength of web material
\(F.S.\) = factor of safety
\(g\) = gage spacing of staggered bolt holes
\(h\) = name for a height
\(h_c\) = height of the web of a wide flange steel section
\(H\) = shorthand for lateral pressure load
\(I\) = moment of inertia with respect to neutral axis bending
\(I_{trial}\) = moment of inertia of trial section
\(I_{req'd}\) = moment of inertia required at limiting deflection
\(I_y\) = moment of inertia about the y axis
\(J\) = polar moment of inertia
$k$ = distance from outer face of W flange to the web toe of fillet
= shape factor for plastic design of steel beams
$K$ = effective length factor for columns, as is $k$
$l$ = name for length
$L$ = name for length or span length
= shorthand for live load
$L_b$ = unbraced length of a steel beam
$L_c$ = clear distance between the edge of a hole and edge of next hole or edge of the connected steel plate in the direction of the load
$L_e$ = effective length that can buckle for column design, as is $l_e$
$L_r$ = shorthand for live roof load
= maximum unbraced length of a steel beam in LRFD design for inelastic lateral-torsional buckling
$L_p$ = maximum unbraced length of a steel beam in LRFD design for full plastic flexural strength
$L'$ = length of an angle in a connector with staggered holes
$LL$ = shorthand for live load
LRFD = load and resistance factor design
$M$ = internal bending moment
$M_a$ = required bending moment (ASD)
$M_n$ = nominal flexure strength with the full section at the yield stress for LRFD beam design
$M_{max}$ = maximum internal bending moment
$M_{max-adj}$ = maximum bending moment adjusted to include self weight
$M_p$ = internal bending moment when all fibers in a cross section reach the yield stress
$M_u$ = maximum moment from factored loads for LRFD beam design
$M_y$ = internal bending moment when the extreme fibers in a cross section reach the yield stress
$n$ = number of bolts
$n.a.$ = shorthand for neutral axis
$N$ = bearing length on a wide flange steel section
= bearing type connection with threads included in shear plane
$p$ = bolt hole spacing (pitch)
$P$ = name for load or axial force vector
$P_a$ = required axial force (ASD)
$P_c$ = available axial strength
$P_{ei}$ = Euler buckling strength
$P_n$ = nominal column load capacity in steel design
$P_r$ = required axial force
$P_a$ = factored column load calculated from load factors in LRFD steel design
$Q$ = first moment area about a neutral axis
= generic axial load quantity for LRFD design
$r$ = radius of gyration
$ty$ = radius of gyration with respect to a y-axis
$R$ = generic load quantity (force, shear, moment, etc.) for LRFD design
= shorthand for rain or ice load
= radius of curvature of a deformed beam
$R_a$ = required strength (ASD)
$R_n$ = nominal value (capacity) to be multiplied by $\phi$ in LRFD and divided by the safety factor $\Omega$ in ASD
$R_d$ = factored design value for LRFD design
$s$ = longitudinal center-to-center spacing of any two consecutive holes
$S$ = shorthand for snow load
= section modulus
= allowable strength per length of a weld for a given size
$S_{req'd}$ = section modulus required at allowable stress
$S_{req'd-adj}$ = section modulus required at allowable stress when moment is adjusted to include self weight
$SC$ = slip critical bolted connection
$t$ = thickness of the connected material
$tf$ = thickness of flange of wide flange
$tw$ = thickness of web of wide flange
$T$ = torque (axial moment)
= shorthand for thermal load
= throat size of a weld
Steel Design

Structural design standards for steel are established by the Manual of Steel Construction published by the American Institute of Steel Construction, and uses Allowable Stress Design and Load and Factor Resistance Design. With the 13th edition, both methods are combined in one volume which provides common requirements for analyses and design and requires the application of the same set of specifications.

Materials

American Society for Testing Materials (ASTM) is the organization responsible for material and other standards related to manufacturing. Materials meeting their standards are guaranteed to have the published strength and material properties for a designation.
A36 – carbon steel used for plates, angles  
\( F_y = 36 \text{ ksi}, F_u = 58 \text{ ksi}, E = 29,000 \text{ ksi} \)

A572 – high strength low-alloy used for some beams  
\( F_y = 60 \text{ ksi}, F_u = 75 \text{ ksi}, E = 29,000 \text{ ksi} \)

A992 – for building framing used for most beams  
\( F_y = 50 \text{ ksi}, F_u = 65 \text{ ksi}, E = 29,000 \text{ ksi} \)  
(A572 Grade 50 has the same properties as A992)

ASD  
\[ R_a \leq \frac{R_n}{\Omega} \]

where \( R_a \) = required strength (dead or live; force, moment or stress)  
\( R_n \) = nominal strength specified for ASD  
\( \Omega \) = safety factor

Factors of Safety are applied to the limit strengths for allowable strength values:

- bending (braced, \( L_b < L_p \))  
\( \Omega = 1.67 \)

- bending (unbraced, \( L_p < L_b \) and \( L_b > L_r \))  
\( \Omega = 1.67 \) (nominal moment reduces)

- shear (beams)  
\( \Omega = 1.5 \) or 1.67

- shear (bolts)  
\( \Omega = 2.00 \) (tabular nominal strength)

- shear (welds)  
\( \Omega = 2.00 \)

- \( L_b \) is the unbraced length between bracing points, laterally

- \( L_p \) is the limiting laterally unbraced length for the limit state of yielding

- \( L_r \) is the limiting laterally unbraced length for the limit state of inelastic lateral-torsional buckling

LRFD  
\[ R_u \leq \phi R_n \]

where \( \phi \) = resistance factor

\( \gamma \) = load factor for the type of load

\( R = \) load (dead or live; force, moment or stress)

\( R_u = \) factored load (moment or stress)

\( R_n = \) nominal load (ultimate capacity; force, moment or stress)

**Nominal strength** is defined as the capacity of a structure or component to resist the effects of loads, as determined by computations using specified material strengths (such as yield strength, \( F_y \), or ultimate strength, \( F_u \)) and dimensions and formulas derived from accepted principles of structural mechanics or by field tests or laboratory tests of scaled models, allowing for modeling effects and differences between laboratory and field conditions
Factored Load Combinations

The design strength, $\phi R_n$, of each structural element or structural assembly must equal or exceed the design strength based on the ASCE-7 (2010) combinations of factored nominal loads:

$$
1.4D \\
1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R) \\
1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.5W) \\
1.2D + 1.0W + L + 0.5(L_r \text{ or } S \text{ or } R) \\
1.2D + 1.0E + L + 0.2S \\
0.9D + 1.0W \\
0.9D + 1.0E
$$

Criteria for Design of Beams

Allowable normal stress or normal stress from LRFD should not be exceeded:

Knowing $M$ and $F_b$, the minimum section modulus fitting the limit is:

$$
S_{req'd} \geq \frac{M}{F_b}
$$

Determining Maximum Bending Moment

Drawing $V$ and $M$ diagrams will show us the maximum values for design. Remember:

$$
V = \Sigma(-w)dx \\
M = \Sigma(V)dx \\
\frac{dV}{dx} = -w \\
\frac{dM}{dx} = V
$$

Determining Maximum Bending Stress

For a prismatic member (constant cross section), the maximum normal stress will occur at the maximum moment.

For a non-prismatic member, the stress varies with the cross section AND the moment.

Deflections

If the bending moment changes, $M(x)$ across a beam of constant material and cross section then the curvature will change:

The slope of the n.a. of a beam, $\theta$, will be tangent to the radius of curvature, $R$:

The equation for deflection, $y$, along a beam is:

$$
y = \frac{1}{EI} \int \theta dx = \frac{1}{EI} \iiint M(x) dx
$$
Elastic curve equations can be found in handbooks, textbooks, design manuals, etc... Computer programs can be used as well. Elastic curve equations can be superimposed ONLY if the stresses are in the elastic range.

*The deflected shape is roughly the same shape flipped as the bending moment diagram but is constrained by supports and geometry.*

**Allowable Deflection Limits**

All building codes and design codes limit deflection for beam types and damage that could happen based on service condition and severity.

\[
y_{\text{max}}(x) = \Delta_{\text{actual}} \leq \Delta_{\text{allowable}} = \frac{L}{y_{\text{value}}}
\]

<table>
<thead>
<tr>
<th>Use</th>
<th>LL only</th>
<th>DL+LL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof beams:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial (no ceiling)</td>
<td>L/180</td>
<td>L/120</td>
</tr>
<tr>
<td>Commercial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>plaster ceiling</td>
<td>L/240</td>
<td>L/180</td>
</tr>
<tr>
<td>no plaster</td>
<td>L/360</td>
<td>L/240</td>
</tr>
<tr>
<td>Floor beams:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ordinary Usage</td>
<td>L/360</td>
<td>L/240</td>
</tr>
<tr>
<td>Roof or floor (damageable elements)</td>
<td>L/480</td>
<td></td>
</tr>
</tbody>
</table>

**Lateral Buckling**

With compression stresses in the top of a beam, a sudden “popping” or buckling can happen even at low stresses. In order to prevent it, we need to brace it along the top, or laterally brace it, or provide a bigger \( I_y \).

**Local Buckling in Steel I Beams – Web Crippling or Flange Buckling**

Concentrated forces on a steel beam can cause the web to buckle (called **web crippling**). Web stiffeners under the beam loads and bearing plates at the supports reduce that tendency. Web stiffeners also prevent the web from shearing in plate girders.
The maximum support load and interior load can be determined from:

\[ P_{n\text{ (max-end)}} = (2.5k + N)F_{yw}t_w \]
\[ P_{n\text{ (interior)}} = (5k + N)F_{yw}t_w \]

where  
\( t_w = \) thickness of the web  
\( F_{yw} = \) yield strength of the web  
\( N = \) bearing length  
\( k = \) dimension to fillet found in beam section tables  

\[ \phi = 1.00 \text{ (LRFD)} \quad \Omega = 1.50 \text{ (ASD)} \]

**Beam Loads & Load Tracing**

In order to determine the loads on a beam (or girder, joist, column, frame, foundation...) we can start at the top of a structure and determine the *tributary area* that a load acts over and the beam needs to support. Loads come from material weights, people, and the environment. This area is assumed to be from half the distance to the next beam over to halfway to the next beam.

The reactions must be supported by the next lower structural element *ad infinitum*, to the ground.

**LRFD - Bending or Flexure**

For determining the flexural design strength, \( \phi_b M_n \), for resistance to pure bending (no axial load) in most flexural members where the following conditions exist, a single calculation will suffice:

\[ \Sigma \gamma_i R_i = M_u \leq \phi_b M_n = 0.9F_yZ \]

where  
\( M_u = \) maximum moment from factored loads  
\( \phi_b = \) resistance factor for bending = 0.9  
\( M_n = \) nominal moment (ultimate capacity)  
\( F_y = \) yield strength of the steel  
\( Z = \) plastic section modulus

**Plastic Section Modulus**

Plastic behavior is characterized by a yield point and an increase in strain with no increase in stress.

\[ f_y = 50\text{ksi} \quad \varepsilon_y = 0.001724 \]
**Internal Moments and Plastic Hinges**

Plastic hinges can develop when all of the material in a cross section sees the yield stress. Because all the material at that section can strain without any additional load, the member segments on either side of the hinge can rotate, possibly causing instability.

For a rectangular section:

Elastic to $f_y$: \[ M_y = \frac{I}{c} f_y = \frac{bh^2}{6} f_y = \frac{b(2c)^2}{6} f_y = \frac{2bc^2}{3} f_y \]

Fully Plastic: \[ M_{ub} \text{ or } M_p = bc^2 f_y = \frac{3}{2} M_y \]

For a non-rectangular section and internal equilibrium at $\sigma_y$, the n.a. will not necessarily be at the centroid. The n.a. occurs where $A_{\text{tension}} = A_{\text{compression}}$. The reactions occur at the centroids of the tension and compression areas.

**Instability from Plastic Hinges**

**Shape Factor:**

The ratio of the plastic moment to the elastic moment at yield:

\[ k = \frac{M_p}{M_y} \]

- $k = 3/2$ for a rectangle
- $k \approx 1.1$ for an I beam

**Plastic Section Modulus**

\[ Z = \frac{M_p}{f_y} \quad \text{and} \quad k = \frac{Z}{S} \]
Design for Shear

\[ V_u \leq \frac{V_n}{\Omega} \text{ or } V_u \leq \phi V_n \]

The nominal shear strength is dependent on the cross section shape. Case 1: With a thick or stiff web, the shear stress is resisted by the web of a wide flange shape (with the exception of a handful of W’s). Case 2: When the web is not stiff for doubly symmetric shapes, singly symmetric shapes (like channels) (excluding round high strength steel shapes), inelastic web buckling occurs. When the web is very slender, elastic web buckling occurs, reducing the capacity even more:

Case 1) For \( \frac{h}{t_w} \leq 2.24 \sqrt{\frac{E}{F_y}} \), \( V_n = 0.6 F_{yw} A_w \quad \phi \gamma = 1.00 \) (LRFD) \( \Omega = 1.50 \) (ASD)

where \( h \) equals the clear distance between flanges less the fillet or corner radius for rolled shapes

\( V_n = \) nominal shear strength

\( F_{yw} = \) yield strength of the steel in the web

\( A_w = t_w d = \) area of the web

Case 2) For \( \frac{h}{t_w} > 2.24 \sqrt{\frac{E}{F_y}} \), \( V_n = 0.6 F_{yw} A_w C_v \quad \phi \gamma = 0.9 \) (LRFD) \( \Omega = 1.67 \) (ASD)

where \( C_v \) is a reduction factor (1.0 or less by equation)

Design for Flexure

\[ M_u \leq M_n / \Omega \text{ or } M_u \leq \phi_b M_n \quad \phi_b = 0.90 \) (LRFD) \( \Omega = 1.67 \) (ASD)

The nominal flexural strength \( M_n \) is the lowest value obtained according to the limit states of

1. yielding, limited at length \( L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \), where \( r_y \) is the radius of gyration in \( y \)

2. lateral-torsional buckling (inelastic) limited at length \( L_t \)

3. flange local buckling

4. web local buckling

Beam design charts show available moment, \( M_u / \Omega \) and \( \phi_b M_n \), for unbraced length, \( L_{bn} \) of the compression flange in one-foot increments from 1 to 50 ft. for values of the bending coefficient \( C_b = 1 \). For values of \( 1 < C_b \leq 2.3 \), the required flexural strength \( M_u \) can be reduced by dividing it by \( C_b \). (\( C_b = 1 \) when the bending moment at any point within an unbraced length is larger than that at both ends of the length. \( C_b \) of 1 is conservative and permitted to be used in any case. When the free end is unbraced in a cantilever or overhang, \( C_b = 1 \). The full formula is provided below.)

NOTE: the self weight is not included in determination of \( M_u / \Omega \) or \( \phi_b M_n \)
Compact Sections

For a laterally braced compact section (one for which the plastic moment can be reached before local buckling) only the limit state of yielding is applicable. For unbraced compact beams and non-compact tees and double angles, only the limit states of yielding and lateral-torsional buckling are applicable.

Compact sections meet the following criteria:

\[
\frac{b_f}{2t_f} \leq 0.38 \sqrt{\frac{E}{F_y}} \quad \text{and} \quad \frac{h_c}{t_w} \leq 3.76 \sqrt{\frac{E}{F_y}}
\]

where:
- \(b_f\) = flange width in inches
- \(t_f\) = flange thickness in inches
- \(E\) = modulus of elasticity in ksi
- \(F_y\) = minimum yield stress in ksi
- \(h_c\) = height of the web in inches
- \(t_w\) = web thickness in inches

With lateral-torsional buckling the nominal flexural strength is

\[
M_a = C_b \left[ M_p - (M_p - 0.7F_yS_x) \left( \frac{L_b - L_p}{L_p - L_p} \right) \right] \leq M_p
\]

where \(C_b\) is a modification factor for non-uniform moment diagrams where, when both ends of the beam segment are braced:

- \(M_{max}\) = absolute value of the maximum moment in the unbraced beam segment
- \(M_A\) = absolute value of the moment at the quarter point of the unbraced beam segment
- \(M_B\) = absolute value of the moment at the center point of the unbraced beam segment
- \(M_C\) = absolute value of the moment at the three quarter point of the unbraced beam segment length.

Available Flexural Strength Plots

Plots of the available moment for the unbraced length for wide flange sections are useful to find sections to satisfy the design criteria of \( M_a \leq M_{a\Omega} \) or \( M_a \leq \phi_n M_a \). The maximum moment that can be applied on a beam (taking self weight into account), \( M_A \) or \( M_B \), can be plotted against the unbraced length, \( L_b \). The limit \( L_{a\Omega} \) is indicated by a solid dot (●), while \( L_p \) is indicated by an open dot (○). Solid lines indicate the most economical, while dashed lines indicate there is a lighter section that could be used. \( C_b\), which is a modification factor for non-zero moments at the ends, is 1 for simply supported beams (0 moments at the ends). (see figure)
Design Procedure

The intent is to find the most lightweight member (which is economical) satisfying the section modulus size.

1. Determine the unbraced length to choose the limit state (yielding, lateral torsional buckling or more extreme) and the factor of safety and limiting moments. Determine the material.
2. Draw V & M, finding V_max and M_max, for unfactored loads (ASD, V_a & M_a) or from factored loads (LRFD, V_u & M_u)
3. Calculate Z_req’d when yielding is the limit state. This step is equivalent to determining if

   \[ f_b = \frac{M_{max}}{S} \leq F_b, \quad Z_{req'd} \geq \frac{M_{max}}{F_y} / \phi \]

   to meet the design criteria that

   \[ M_u \leq M_n / \Omega \text{ or } M_u \leq \phi_f M_n \]

   If the limit state is something other than yielding, determine the nominal moment, M_n, or use plots of available moment to unbraced length, L_b.

4. For steel: use the section charts to find a trial Z and remember that the beam self weight (the second number in the section designation) will increase Z_req’d. The design charts show the lightest section within a grouping of similar Z’s.

   **** Determine the “updated” V_max and M_max including the beam self weight, and verify that the updated Z_req’d has been met.*****
5. Consider lateral stability.

6. Evaluate horizontal shear using $V_{\text{max}}$. This step is equivalent to determining if $f_v \leq F_v$ is satisfied to meet the design criteria that $V_a \leq V_n / \Omega$ or $V_a \leq \phi V_n$

   For I beams: $f_{v \text{-max}} = \frac{3V}{2A} \approx \frac{V}{A_{\text{web}}} = \frac{V}{t_w d} \quad V_n = 0.6F_{yw}A_w \quad \text{or} \quad V_n = 0.6F_{yw}A_vC_v$

   Others: $f_{v \text{-max}} = \frac{VQ}{lb}$

7. Provide adequate bearing area at supports. This step is equivalent to determining if $f_p = \frac{P}{A} \leq F_p$ is satisfied to meet the design criteria that $P_a \leq P_n / \Omega$ or $P_a \leq \phi P_n$

8. Evaluate shear due to torsion $f_v = \frac{T_d}{J} \text{ or } \frac{T}{c_ib^2} \leq F_v$ (circular section or rectangular)

9. Evaluate the deflection to determine if $\Delta_{\text{max LL}} \leq \Delta_{\text{LL-allowed}}$ and/or $\Delta_{\text{max Total}} \leq \Delta_{\text{Total allowed}}$

   **** note: when $\Delta_{\text{calculated}} > \Delta_{\text{limit}}$, $I_{\text{req'd}}$ can be found with: $I_{\text{req'd}} \geq \frac{\Delta_{\text{hav big}}}{\Delta_{\text{limit}}} \cdot I_{\text{trial}}$

   and $Z_{\text{req'd}}$ will be satisfied for similar self weight *****

FOR ANY EVALUATION:

Redesign (with a new section) at any point that a stress or serviceability criteria is NOT satisfied and re-evaluate each condition until it is satisfactory.

Load Tables for Uniformly Loaded Joists & Beams

Tables exist for the common loading situation of uniformly distributed load. The tables provide the safe distributed loads based on bending and deflection limits. For specific clear spans, they provide maximum total loads and live loads for a specific deflection limits. If the load is not uniform, an equivalent uniform load can be calculated from the maximum moment equation:

If the deflection limit needed is not that of the table, the design live load must be adjusted. For example: $W_{\text{adjusted}} = W_{\text{ll-have}} \left( \frac{L/360}{L/400} \right)$

Criteria for Design of Columns

If we know the loads, we can select a section that is adequate for strength & buckling.

If we know the length, we can find the limiting load satisfying strength & buckling.
Allowable Stress Design

The allowable stress design provisions prior to the combined design of the 13th edition of the AISC Steel Construction Manual had relationships for short and intermediate length columns (crushing and the transition to inelastic buckling), and long columns (buckling) as shown in the figure. The transition slenderness ratio is based on the yield strength and modulus of elasticity and are 126.1 ($F_y = 36$ ksi) and 107.0 ($F_y = 50$ ksi) with a limiting slenderness ratio of 200.

Design for Compression

American Institute of Steel Construction (AISC) Manual 14th ed:

$$P_a \leq P_n / \Omega \text{ or } P_a \leq \phi V_n$$

where

$$P_a = \sum \gamma_i P_i$$

$\gamma$ is a load factor
$P$ is a load type
$\phi$ is a resistance factor
$P_n$ is the nominal load capacity (strength)

$$\phi = 0.90 \text{ (LRFD)} \quad \Omega = 1.67 \text{ (ASD)}$$

For compression

$$P_n = F_{cr} A_g$$

where: $A_g$ is the cross section area and $F_{cr}$ is the flexural buckling stress

The flexural buckling stress, $F_{cr}$, is determined as follows:

when $\frac{KL}{r} \leq 4.71 \sqrt{\frac{E}{F_y}}$ or ($F_e \geq 0.44 F_y$):

$$F_{cr} = \left[ \frac{F_y}{0.658 F_e} \right] F_y$$

when $\frac{KL}{r} > 4.71 \sqrt{\frac{E}{F_y}}$ or ($F_e < 0.44 F_y$):

$$F_{cr} = 0.877 F_e$$

where $F_e$ is the elastic critical buckling stress:

$$F_e = \frac{\pi^2 E}{(KL/r)^2}$$
**Design Aids**

Tables exist for the value of the flexural buckling stress based on slenderness ratio. In addition, tables are provided in the AISC Manual for Available Strength in Axial Compression based on the effective length with respect to least radius of gyration, $r_y$. If the critical effective length is about the largest radius of gyration, $r_x$, it can be turned into an effective length about the y axis by dividing by the fraction $r_x/r_y$.

### Procedure for Analysis

1. Calculate $KL/r$ for each axis (if necessary). The largest will govern the buckling load.
2. Find $F_{cr}$ as a function of $KL/r$ from the appropriate equation (above) or table.
3. Compute $P_n = F_{cr}A_g$ or alternatively compute $f_c = P/A$ or $P_u/A$
4. Is the design satisfactory?
   
   $\text{Is } P_a \leq P_u/\phi \text{ or } P_a \leq \phi_Pu? \Rightarrow \text{yes, it is; no, it is no good}$
   
   or $\text{Is } f_c \leq F_{cr}/\phi \text{ or } \phi F_{cr}? \Rightarrow \text{yes, it is; no, it is no good}$

### Procedure for Design

1. Guess a size by picking a section.
2. Calculate $KL/r$ for each axis (if necessary). The largest will govern the buckling load.
3. Find $F_{cr}$ as a function of $KL/r$ from appropriate equation (above) or table.
4. Compute $P_n = F_{cr} A_g$ or alternatively compute $f_c = P/A$ or $P_u/A$

5. Is the design satisfactory?
   
   Is $P_a \leq P_u/\Omega$ or $P_u \leq \phi_c P_n$? yes, it is; no, pick a bigger section and go back to step 2.
   
   Is $f_c \leq F_{cr}/\Omega$ or $\phi_c F_{cr}$? yes, it is; no, pick a bigger section and go back to step 2.

6. Check design efficiency by calculating percentage of capacity used:
   
   $$\frac{P_a}{P_n/\Omega} \cdot 100\% \text{ or } \frac{P_u}{\phi_c P_n} \cdot 100\%$$

   If value is between 90-100%, it is efficient.
   
   If values is less than 90%, pick a smaller section and go back to step 2.

Columns with Bending (Beam-Columns)

In order to design an adequate section for allowable stress, we have to start somewhere:

1. Make assumptions about the limiting stress from:
   - buckling
   - axial stress
   - combined stress

2. See if we can find values for $r$ or $A$ or $Z$

3. Pick a trial section based on if we think $r$ or $A$ is going to govern the section size.

4. Analyze the stresses and compare to allowable using the allowable stress method or interaction formula for eccentric columns.

5. Did the section pass the capacity adequacy test?
   - If not, do you increase $r$ or $A$ or $Z$?
   - If so, is the difference really big so that you could decrease $r$ or $A$ or $Z$ to make it more efficient (economical)?

6. Change the section choice and go back to step 4. Repeat until the section meets the stress criteria.

Design for Combined Compression and Flexure:

The interaction of compression and bending are included in the form for two conditions based on the size of the required axial force to the available axial strength. This is notated as $P_r$ (either $P_a$ from ASD or $P_u$ from LRFD) for the axial force being supported, and $P_c$ (either $P_u/\Omega$ for ASD or $\phi_c P_n$ for LRFD). The increased bending moment due to the P-Δ effect must be determined and used as the moment to resist.
For $\frac{P_r}{P_c} \geq 0.2$:

$$\frac{P_u}{P_n} + \frac{8}{9} \left( \frac{M_x}{M_{nx}} + \frac{M_y}{M_{ny}} \right) \leq 1.0$$

(ASD)

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_u M_{nx}} + \frac{M_{uy}}{\phi_u M_{ny}} \right) \leq 1.0$$

(LRFD)

For $\frac{P_r}{P_c} < 0.2$:

$$\frac{P_u}{2P_n} + \left( \frac{M_x}{M_{nx}} + \frac{M_y}{M_{ny}} \right) \leq 1.0$$

(ASD)

$$\frac{P_u}{2\phi_c P_n} + \left( \frac{M_{ux}}{\phi_u M_{nx}} + \frac{M_{uy}}{\phi_u M_{ny}} \right) \leq 1.0$$

(LRFD)

where:

- for compression $\phi_c = 0.90$ (LRFD) $\Omega = 1.67$ (ASD)
- for bending $\phi_b = 0.90$ (LRFD) $\Omega = 1.67$ (ASD)

For a braced condition, the moment magnification factor $B_1$ is determined by $B_1 = \frac{C_m}{1 - (P_u/P_{e1})} \geq 1.0$

where $C_m$ is a modification factor accounting for end conditions.

When not subject to transverse loading between supports in plane of bending:

$$= 0.6 - 0.4 (M_1/M_2) \leq 1.0$$

where $M_1$ and $M_2$ are the end moments and $M_1 < M_2$.

$M_1/M_2$ is positive when the member is bent in reverse curvature (same direction), negative when bent in single curvature.

When there is transverse loading between the two ends of a member:

$$= 0.85$$

members with restrained (fixed) ends

$$= 1.00$$

members with unrestrained ends

$P_{e1} = $Euler buckling strength

$$\quad = \frac{\pi^2 EA}{(Kl/r)^2}$$

Criteria for Design of Connections

Connections must be able to transfer any axial force, shear, or moment from member to member or from beam to column.

Connections for steel are typically high strength bolts and electric arc welds. Recommended practice for ease of construction is to specified shop welding and field bolting.
Bolted and Welded Connections

The limit state for connections depends on the loads:

1. tension yielding
2. shear yielding
3. bearing yielding
4. bending yielding due to eccentric loads
5. rupture

Welds must resist tension AND shear stress. The design strengths depend on the weld materials.

Bolted Connection Design

Bolt designations signify material and type of connection where
- SC: slip critical
- N: bearing-type connection with bolt threads included in shear plane
- X: bearing-type connection with bolt threads excluded from shear plane

<table>
<thead>
<tr>
<th>Bolt Designation</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A307</td>
<td>similar in strength to A36 steel (also known as ordinary, common or unfinished bolts)</td>
</tr>
<tr>
<td>A325</td>
<td>high strength bolts (Group A)</td>
</tr>
<tr>
<td>A490</td>
<td>high strength bolts (higher than A325) (Group B)</td>
</tr>
</tbody>
</table>

Bearing-type connection: no frictional resistance in the contact surfaces is assumed and slip between members occurs as the load is applied. (Load transfer through bolt only).

Slip-critical connections: bolts are torqued to a high tensile stress in the shank, resulting in a clamping force on the connected parts. (Shear resisted by clamping force). Requires inspections and is useful for structures seeing dynamic or fatigue loading.

Class A indicates the faying (contact) surfaces are clean mill scale or adequate paint system, while Class B indicates blast cleaning or paint for $\mu = 0.50$.

Bolts rarely fail in bearing. The material with the hole will more likely yield first.

For the determination of the net area of a bolt hole the width is taken as $1/16”$ greater than the nominal dimension of the hole. Standard diameters for bolt holes are $1/16”$ larger than the bolt diameter. (This means the net width will be $1/8”$ larger than the bolt.)

Design for Bolts in Bearing, Shear and Tension

Available shear values are given by bolt type, diameter, and loading (Single or Double shear) in AISC manual tables. Available shear value for slip-critical connections are given for limit states of serviceability or strength by bolt type, hole type (standard, short-slotted, long-slotted or oversized), diameter, and loading. Available tension values are given by bolt type and diameter in AISC manual tables.
Allowable bearing force values are given by bolt diameter, ultimate tensile strength, $F_u$, of the connected part, and thickness of the connected part in AISC manual tables.

For shear or tension (same equation) in bolts:

$$R_u \leq R_n / \Omega \quad \text{or} \quad R_u \leq \phi R_n$$

where $R_u = \Sigma \gamma_i R_i$

- single shear (or tension) $R_n = F_n A_b$
- double shear $R_n = F_n 2A_b$

where $\phi = \text{the resistance factor}$
$F_n = \text{the nominal tension or shear strength of the bolt}$
$A_b = \text{the cross section area of the bolt}$

$\phi = 0.75 \ (LRFD) \quad \Omega = 2.00 \ (ASD)$

For bearing of plate material at bolt holes:

$$R_u \leq R_n / \Omega \quad \text{or} \quad R_u \leq \phi R_n$$

where $R_u = \Sigma \gamma_i R_i$

- deformation at bolt hole is a concern
  $$R_n = 1.2L_c t F_u \leq 2.4dt F_u$$
- deformation at bolt hole is not a concern
  $$R_n = 1.5L_c t F_u \leq 3.0dt F_u$$
- long slotted holes with the slot perpendicular to the load
  $$R_n = 1.0L_c t F_u \leq 2.0dt F_u$$

where $R_n = \text{the nominal bearing strength}$
$F_u = \text{specified minimum tensile strength}$
$L_c = \text{clear distance between the edges of the hole and the next hole or edge in the direction of the load}$
$d = \text{nominal bolt diameter}$
$t = \text{thickness of connected material}$

$\phi = 0.75 \ (LRFD) \quad \Omega = 2.00 \ (ASD)$

The minimum edge distance from the center of the outermost bolt to the edge of a member is generally $1 \frac{3}{4}$ times the bolt diameter for the sheared edge and $1 \frac{1}{4}$ times the bolt diameter for the rolled or gas cut edges.

The maximum edge distance should not exceed 12 times the thickness of thinner member or 6 in.

Standard bolt hole spacing is 3 in. with the minimum spacing of $2 \frac{3}{8}$ times the diameter of the bolt, $d_b$. Common edge distance from the center of last hole to the edge is $1 \frac{1}{4}$ in..

Tension Member Design

In steel tension members, there may be bolt holes which reduce the size of the cross section.
$g$ refers to the row spacing or *gage*
$p$ refers to the bolt spacing or *pitch*
$s$ refers to the longitudinal spacing of two consecutive holes

**Effective Net Area:**

The smallest effective area must be determined by subtracting the bolt hole areas. With staggered holes, the shortest length must be evaluated.

A series of bolts can also transfer a portion of the tensile force, and some of the effective net areas see reduced stress.

The effective net area, $A_e$, is determined from the net area, $A_n$, multiplied by a shear lag factor, $U$, which depends on the element type and connection configuration. If a portion of a connected member is not fully connected (like the leg of an angle), the unconnected part is not subject to the full stress and the shear lag factor can range from 0.6 to 1.0: $A_e = A_n U$

**For tension elements:**

\[ R_u \leq \frac{R_n}{\Omega} \text{ or } R_u \leq \phi R_n \]

where $R_n = \Sigma \gamma_i R_i$

1. yielding

\[ R_n = F_y A_g \]

\[ \phi = 0.90 \text{ (LRFD) } \quad \Omega = 1.67 \text{ (ASD)} \]

2. rupture

\[ R_n = F_u A_e \]

\[ \phi = 0.75 \text{ (LRFD) } \quad \Omega = 2.00 \text{ (ASD)} \]

where $A_g$ = the gross area of the member (excluding holes)

$A_e$ = the effective net area (with holes, etc.)

$F_y$ = the yield strength of the steel

$F_u$ = the tensile strength of the steel (ultimate)
When holes are staggered in a chain of holes (zigzagging) at diagonals, the length of each path from hole edge to edge is taken as the net area less each bolt hold area and the addition of $\frac{s^2}{4g}$ for each gage space in the chain: 

$$A_n = bt - \Sigma ht - \Sigma \left(\frac{s^2}{4g}\right)t$$

where

- $b$ is the plate width
- $t$ is the plate thickness
- $h$ is the standard hole diameter of each hole
- $s$ is the staggered hole spacing
- $g$ is the gage spacing between rows

**Welded Connections**

Weld designations include the strength in the name, i.e. E70XX has $F_y = 70$ ksi. Welds are weakest in shear and are assumed to always fail in the shear mode.

The throat size, $T$, of a fillet weld is determined trigonometry by: 

$$T = 0.707 \times \text{weld size}$$

* When the submerged arc weld process is used, welds over 3/8” will have a throat thickness of 0.11 in. larger than the formula.

Weld sizes are limited by the size of the parts being put together and are given in AISC manual table J2.4 along with the allowable strength per length of fillet weld, referred to as $S$.

The maximum size of a fillet weld:

- a) can’t be greater than the material thickness if it is $\frac{1}{4}”$ or less
- b) is permitted to be 1/16” less than the thickness of the material if it is over $\frac{1}{4}”$

The minimum length of a fillet weld is 4 times the nominal size. If it is not, then the weld size used for design is $\frac{1}{4}$ the length.

Intermittent fillet welds cannot be less than four times the weld size, not to be less than $1 \frac{1}{2}”$. 

**TABLE J2.4**

<table>
<thead>
<tr>
<th>Material Thickness of Thicker Part Joined (in.)</th>
<th>Minimum Size of Fillet Weld (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>To $\frac{1}{4}$ inclusive</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>Over $\frac{1}{4}$ to $\frac{1}{2}$</td>
<td>$\frac{1}{8}$</td>
</tr>
<tr>
<td>Over $\frac{1}{2}$ to $\frac{3}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>Over $\frac{3}{4}$</td>
<td>$\frac{1}{8}$</td>
</tr>
</tbody>
</table>

*Leg dimension of fillet welds. Single-pass welds must be used.*
For fillet welds:

\[ R_a \leq \frac{R_n}{\Omega} \text{ or } R_a \leq \phi R_n \]

where \( R_a = \sum \gamma_i R_i \)

for the weld metal: \( R_n = 0.6 F_{EXX} Tl = Sl \)

\[ \phi = 0.75 \text{ (LRFD)} \quad \Omega = 2.00 \text{ (ASD)} \]

where:

- \( T \) is throat thickness
- \( l \) is length of the weld

For a connected part, the other limit states for the base metal, such as tension yield, tension rupture, shear yield, or shear rupture must be considered.

**Framed Beam Connections**

*Coping* is the term for cutting away part of the flange to connect a beam to another beam using welded or bolted angles.

AISC provides tables that give bolt and angle available strength knowing number of bolts, bolt type, bolt diameter, angle leg thickness, hole type and coping, and the wide flange beam being connected.

Group A bolts include A325, while Group B includes A490.

There are also tables for bolted/welded double-angle connections and all-welded double-angle connections.
### Limiting Strength or Stability States

In addition to resisting shear and tension in bolts and shear in welds, the connected materials may be subjected to shear, bearing, tension, flexure and even prying action. Coping can significantly reduce design strengths and may require web reinforcement. All the following must be considered:

- shear yielding
- shear rupture
- block shear rupture - failure of a block at a beam as a result of shear and tension
- tension yielding
- tension rupture
- local web buckling
- lateral torsional buckling

---

#### Sample AISC Table for Bolt and Angle Available Strength in All-Bolted Double-Angle Connections

<table>
<thead>
<tr>
<th>Bolt and Angle Available Strength per Inch Thickness, kip/in.</th>
<th>3/4-in.</th>
<th>1-in.</th>
<th>1-1/4-in.</th>
<th>1-1/2-in.</th>
<th>2-in.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bolt and Angle Available Strength per Inch Thickness, kip/in.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Shear</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Tension</strong></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Flexure</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Prying</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Local Web Buckling</strong></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Lateral Torsional Buckling</strong></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td><strong>Threaded Bolts</strong></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Unthreaded Bolts</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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**Note:** The values provided are for design purposes and should be confirmed with the appropriate codes and standards.
Block Shear Strength (or Rupture):  
\[ R_u \leq R_n / \Omega \text{ or } R_u \leq \phi R_n \]
where  
\[ R_u = \sum \gamma_i R_i \]

\[ R_n = 0.6F_d A_{nv} + U_{bs} F_u A_{nt} \leq 0.6F_d A_{gv} + U_{bs} F_u A_{nt} \]

\[ \phi = 0.75 \text{ (LRFD)} \quad \Omega = 2.00 \text{ (ASD)} \]

where:
- \( A_{nv} \) is the net area subjected to shear
- \( A_{nt} \) is the net area subjected to tension
- \( A_{gv} \) is the gross area subjected to shear
- \( U_{bs} = 1.0 \) when the tensile stress is uniform (most cases)
  \[ = 0.5 \text{ when the tensile stress is non-uniform} \]

Gusset Plates

Gusset plates are used for truss member connections where the geometry prevents the members from coming together at the joint “point”. Members being joined are typically double angles.

Decking

Shaped, thin sheet-steel panels that span several joists or evenly spaced support behave as continuous beams. Design tables consider a “1 unit” wide strip across the supports and determine maximum bending moment and deflections in order to provide allowable loads depending on the depth of the material.

The other structural use of decking is to construct what is called a diaphragm, which is a horizontal unit tying the decking to the joists that resists forces parallel to the surface of the diaphragm.

When decking supports a concrete topping or floor, the steel-concrete construction is called composite.
Example 1 (pg 290)

Example 2. A simple beam consisting of a W 21 × 57 is subjected to bending. Find the limiting moments (a) based on elastic stress conditions and a limiting stress of $F_y = 36$ ksi, and (b) based on full development of the plastic moment.

Example 2 (pg 300)

Example 7. Design a simply supported floor beam to carry a superimposed load of 2 kips per ft [29.2 kN/m] over a span of 24 ft [7.3 m]. (The term superimposed load is used to denote any load other than the weight of a structural member itself.) The superimposed load is 25 percent dead load and 75 percent live load. The yield stress is 36 ksi [250 MPa]. The floor beam is continuously supported along its length against lateral buckling.
Example 3

**Given:**
Select an ASTM A992 W-shape beam with a simple span of 35 feet. Limit the member to a maximum nominal depth of 18 in. Limit the live load deflection to $L/360$. The nominal loads are a uniform dead load of 0.45 kip/ft and a uniform live load of 0.75 kip/ft. Assume the beam is continuously braced. Use ASD of the Unified Design method.

**Solution:**

**Material Properties:**
ASTM A992  
$F_y = 50$ ksi  
$F_u = 65$ ksi

1. The unbraced length is 0 because it says it is fully braced.
2. Find the maximum shear and moment from unfactored loads:
   \[
   w_a = 0.450 \text{ kip/ft} + 0.750 \text{ kip/ft} = 1.20 \text{ kip/ft}
   \]
   \[
   V_a = 1.20 \text{ kip/ft}(35 \text{ ft})/2 = 21 \text{ k}
   \]
   \[
   M_a = 1.20 \text{ kip/ft}(35 \text{ ft})^2/8 = 184 \text{ k-ft}
   \]
   If $M_a \leq M_n/\Omega$, the maximum moment for design is $M_{max} = 184 \text{ k-ft}$

3. Find $Z_{req'd}$:
   \[
   Z_{req'd} \geq M_{max}/F_y = 184 \text{ k-ft}(1.67)(12 \text{ in/ft})/50 \text{ ksi} = 73.75 \text{ in}^3
   \]

4. Choose a trial section, and also limit the depth to 18 in as instructed:
   W18 x 40 has a plastic section modulus of 78.4 in$^3$ and is the most light weight (as indicated by the bold text) in Table 9.1

   Include the self weight in the maximum values:
   \[
   w_{a-adjusted} = 1.20 \text{ kip/ft} + 0.04 \text{ kip/ft}
   \]
   \[
   V_{a-adjusted} = 1.24 \text{ kip/ft}(35 \text{ ft})/2 = 21.7 \text{ k}
   \]
   \[
   M_{a-adjusted} = 1.24 \text{ kip/ft}(35 \text{ ft})^3/8 = 189.9 \text{ k-ft}
   \]

   $Z_{req'd} \geq 189.9 \text{ k-ft}(1.67)(12 \text{ in/ft})/50 \text{ ksi} = 76.11 \text{ in}^3$
   And the $Z$ we have (78.4) is larger than the Z we need (76.11), so OK.

6. Evaluate shear (is $V_a \leq V_n/\Omega$):
   \[
   A_w = d t_w
   \]
   Look up section properties for W18 x 40: $d = 17.90 \text{ in}$ and $t_w = 0.315 \text{ in}$
   \[
   V_{n/\Omega} = 0.6F_y A_w/\Omega = 0.6(50 \text{ ksi})(17.90 \text{ in})(0.315 \text{ in})/1.5 = 112.8 \text{ k}
   \]
   Which is much larger than 21.7 k, so OK.

9. Evaluate the deflection with respect to the limit stated of $L/360$ for the live load. (If we knew the total load limit we would check that as well). The moment of inertia for the W18 x 40 is needed. $I_x = 612 \text{ in}^4$

   $\Delta = 35 \text{ ft}(12 \text{ in/ft})/360 = 1.17 \text{ in}$
   \[
   \Delta = 5wL^4/384EI = 5(0.75 \text{ kip/ft})(35 \text{ ft})^4(12 \text{ in/ft})^3/384(29 \times 10^3 \text{ ksi})(612 \text{ in}^4) = 1.42 \text{ in}
   \]
   This is TOO BIG (not less than the limit.

Find the moment of inertia needed:

   $I_{req'd} \geq \Delta_{live \ limit} (1/\Delta_{live})/1.17 \text{ in} = 742.8 \text{ in}^4$

From the Listing of W Shapes in Descending order of $Z$ for Beam Design, a W21 x 44 is larger (by $I_x = 843 \text{ in}^4$), and the most light weight, but it is too deep! In the next group up, the W16 x 57 works ($I_x = 758 \text{ in}^4$, but we aren’t certain there is a more economical section. Then W18x50, W12x72, and W18x55, so we stop because the W18x50 has the smallest weight with the depth restriction.

(St: $I_x = 800, 597, and 890 \text{ in}^4$)

Choose a W18 x 50
Example 4
A steel beam with a 20 ft span is designed to be simply supported at the ends on columns and to carry a floor system made with open-web steel joists at 4 ft on center. The joists span 28 feet and frame into the beam from one side only and have a self weight of 8.5 lb/ft. Use A992 (grade 50) steel and select the most economical wide-flange section for the beam. Floor loads are 50 psf LL and 14.5 psf DL.
Example 5
Select a A992 W shape flexural member \((F_y = 50 \text{ ksi}, F_u = 65 \text{ ksi})\) for a beam with distributed loads of 825 lb/ft (dead) and 1300 lb/ft (live) and a live point load at midspan of 3 k using the Available Moment tables. The beam is simply supported, 20 feet long, and braced at the ends and midpoint only \((L_b = 10 \text{ ft})\). The beam is a roof beam for an institution without plaster ceilings. (LRFD)

SOLUTION:
To use the Available Moment tables, the maximum moment required is plotted against the unbraced length. The first solid line with capacity or unbraced length above what is needed is the most economical.

DESIGN LOADS (load factors applied on figure):

\[
M_u = \frac{wl^2}{2} + Pb = \frac{3.07}{2} \left( \frac{20}{10} \right)^2 + 4.8k(10 \text{ ft}) = 662 \cdot \frac{k}{ft} \quad V_u = wl + P = 3.07 \left( \frac{20}{20} \right) + 4.8k = 66.2k
\]

Plotting 662 k-ft vs. 10 ft lands just on the capacity of the W21x83, but it is dashed (and not the most economical) AND we need to consider the contribution of self weight to the total moment. Choose a trial section of W24 x 76. Include the new dead load:

\[
M'_{u-\text{adjusted}} = 662 \cdot \frac{k}{ft} + \frac{1.2(76)}{2(1000)}(20 \text{ ft})^2 = 680.2 \cdot \frac{k}{ft} \quad V'_{u-\text{adjusted}} = 66.2k + 1.2(0.076)(20 \text{ ft}) = 68.0k
\]

Replot 680.2 k-ft vs. 10 ft, which lands above the capacity of the W21x83. We can't look up because the chart ends, but we can look for that capacity with a longer unbraced length. This leads us to a W24 x 84 as the most economical. (With the additional self weight of 84 - 76 lb/ft = 8 lb/ft, the increase in the factored moment is only 1.92 k-ft; therefore, it is still OK.)

Evaluate the shear capacity:
\[
\phi \cdot \frac{V_u}{V_u} = 0.6 \cdot \frac{66.2}{338.4} = 0.14 \text{ (safe)}
\]

Evaluate the deflection with respect to the limits of L/240 for live (unfactored) load and L/180 for total (unfactored) load:
\[
\Delta_{\text{total}} = \frac{Pb^3(3l - b)}{48EI} + \frac{6lw^4}{24EI} = \frac{3k(10 \cdot (20 - 10)(12)^{3/2})}{6(30 \cdot 10^4 \cdot \text{ksi})2370 \cdot \text{in}^3} + \frac{(2.209^{3/2})(20 \cdot 20)(12)^{1/2}}{24(30 \cdot 10^4 \cdot \text{ksi})2370 \cdot \text{in}^3} = 0.06 + 0.36 = 0.42 \text{ in}
\]

\[
\Delta_{\text{total}} \leq \Delta_{\text{limit}} \quad \Delta_{\text{total}} \leq \Delta_{\text{total-limit}}
\]

0.06 in. \leq 1 in. and 0.42 in. \leq 1.33 in.

(This section is so big to accommodate the large bending moment at the cantilever support that it deflects very little.)

\[
\therefore \text{FINAL SELECTION IS W24x84}
\]
Example 6
A floor is to be supported by trusses spaced at 5 ft. on center and spanning 60 ft. having a dead load of 53 lb/ft² and a live load of 100 lb/ft². With 3 ft-long panel points, the depth is assumed to be 3 ft with a span-to-depth ratio of 20. With 6 ft-long panel points, the depth is assumed to be 6 ft with a span-to-depth ratio of 10. Determine the maximum force in a horizontal chord and the maximum force in a web member. Use factored loads. Assume a self weight of 40 lb/ft².

### Table 7.2 Computation of Truss Joint Loads

<table>
<thead>
<tr>
<th>Truss</th>
<th>Node to Node Spacing (ft)</th>
<th>Truss Spacing (ft)</th>
<th>Floor Area per Node A (ft²)</th>
<th>W_dead (K/ft²)</th>
<th>W_live (K/ft²)</th>
<th>P_dead (W_dead * A) (K)</th>
<th>P_live (W_live * A) (K)</th>
<th>Factored Dead Load 1.2 * P_dead (K)</th>
<th>Factored Live Load 1.6 * P_live (K)</th>
<th>Factored Total Load 1.2 * P_dead + 1.6 * P_live (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 ft deep</td>
<td>53</td>
<td>100</td>
<td>0.053</td>
<td>15</td>
<td>0.795</td>
<td>1.50</td>
<td>0.954</td>
<td>2.40</td>
<td>3.35 + 0.14 = 3.49</td>
<td></td>
</tr>
<tr>
<td>6 ft deep</td>
<td>53</td>
<td>100</td>
<td>0.053</td>
<td>30</td>
<td>1.59</td>
<td>3.00</td>
<td>1.908</td>
<td>4.80</td>
<td>6.71 + 0.29 = 7.00</td>
<td></td>
</tr>
<tr>
<td>self weight</td>
<td>0.04 k/ft (distributed)</td>
<td>3</td>
<td>1</td>
<td>1.2 * P_dead = 1.2 * w_dead * tributary width = 0.14 K</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>1</td>
<td>1.2 * P_dead = 1.2 * w_dead * tributary width = 0.29 K</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NOTE – end panels only have half the tributary width of interior panels</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

### FBD 1 for 3 ft deep truss

FBD 2 of cut just to the left of midspan

FBD 3 of cut just to right of left support

### FBD 4 for 6 ft deep truss

FBD 5 of cut just to the left of midspan

FBD 6 of cut just to right of left support

### FBD 2: Maximum chord force (top or bottom) will be at midspan

\[ \Sigma M_G = 9.5P_1(30\degree) - P_1(27\degree) - P_1(24\degree) - P_1(21\degree) - P_1(18\degree) - P_1(15\degree) - P_1(12\degree) - P_1(9\degree) - P_1(6\degree) - P_1(3\degree) - T_1(3\degree) = 0 \]

\[ T_1 = P_1(150\degree)/3 = (3.49 k)(50) = 174.5 k \]

\[ \Sigma F_y = 10P_1 - 9.5P_1 - D_1 \sin 45\degree = 0 \]

\[ D_1 = 0.5(3.49 k)/0.707 = 2.5 k \] (minimum near midspan)

\[ \Sigma F_x = C_1 + T_1 + D_1 \cos 45\degree = 0 \]

\[ C_1 = 176.2 k \]

### FBD 3: Maximum web force will be in the end diagonal (just like maximum shear in a beam)

\[ \Sigma F_y = 10P_1 - 0.5P_1 - F_{AB} \sin 45\degree = 0 \]

\[ F_{AB} = 9.5P_1/\sin 45\degree = 9.5(3.49 k)/0.707 = 46.9 k \]

### FBD 5: Maximum chord (top or bottom) force will be at midspan

\[ \Sigma M_G = 4.5P_2(30\degree) - P_2(27\degree) - P_2(24\degree) - P_2(12\degree) - P_2(9\degree) - P_2(6\degree) - T_2(6\degree) = 0 \]

\[ T_2 = P_2(75\degree)/6 = (7 k)(12.5) = 87.5 k \]

\[ \Sigma F_y = 5P_2 - 4.5P_1 - D_2 \sin 45\degree = 0 \]

\[ D_2 = 0.5(7 k)/0.707 = 4.9 k \] (minimum near midspan)

\[ \Sigma F_x = C_2 + T_2 + D_2 \cos 45\degree = 0 \]

\[ C_2 = 92.4 k \]
Example 7 (pg 339)

Example 14. Open web steel joists are to be used for a floor with a unit live load of 75 psf [3.59 kN/m²] and a unit dead load of 40 psf [1.91 kN/m²] (not including the joist weight) on a span of 30 ft [9.15 m]. Joists are 2 ft [0.61 m] on center, and deflection is limited to 1/240 of the span under total load and 1/360 of the span under live load only. Determine the lightest possible joist and the lightest joist of least depth possible.

Example 8 (pg 353)

Example 3. Figure 10.5a shows an elevation of the steel framing at the location of an exterior wall. The column is laterally restrained but rotationally free at the top and bottom in both directions. (The end condition is as shown for Case (d) in Figure 10.3.) With respect to the x-axis of the section, the column is laterally braced for its full height. However, the existence of the horizontal framing in the wall plane provides lateral bracing with respect to the y-axis of the section; thus, the buckling of the column in this direction takes the form shown in Figure 10.5b. If the column is a W 12 × 53 of A36 steel, L₂ is 30 ft [9.15 m], and L₃ is 18 ft [5.49 m], what is the maximum factored compression load?
Example 9 (pg 364)

**Example 6.** Using Table 10.4, select a standard weight steel pipe to carry a dead load of 15 kips [67 kN] and a live load of 26 kips [116 kN] if the unbraced height is 12 ft [3.66 m].

**Example 10**
Investigate the acceptability of a W16 x 67 used as a beam-column under the unfactored loading shown in the figure. It is A992 steel (Fy = 50 ksi). Assume 25% of the load is dead load with 75% live load.

**SOLUTION:**

**DESIGN LOADS (shown on figure):**

Axial load = 1.2(0.25)(350k)+1.6(0.75)(350k)=525k

Moment at joint = 1.2(0.25)(60 k-ft) + 1.6(0.75)(60 k-ft) = 90 k-ft

Determine column capacity and fraction to choose the appropriate interaction equation:

\[ \frac{kl}{r_y} = \frac{15 F_y (12 \gamma_f)}{6.96 \text{in}} = 25.9 \quad \text{and} \quad \frac{kl}{r_y} = \frac{15 F_y (12 \gamma_f)}{2.46 \text{in}} = 73 \] (governs)

\[ P_e = \phi P_n = \phi F_y A_x = (30.5 \text{ksi})(9.7 \text{in}^2) = 600.85 k \]

\[ \frac{P_n}{P_e} = \frac{525k}{600.85k} = 0.87 > 0.2 \quad \text{so use} \quad \frac{P_n}{P_e} + \frac{8}{9} \left( \frac{M_{uy}}{\phi M_{ux}} + \frac{M_{uy}}{\phi M_{uy}} \right) \leq 1.0 \]

There is no bending about the y axis, so that term will not have any values.

Determine the bending moment capacity in the x direction:

The unbraced length to use the full plastic moment (Lb) is listed as 8.69 ft, and we are over that so of we don't want to determine it from formula, we can find the beam in the Available Moment vs. Unbraced Length tables. The value of φMn at Lb=15 ft is 422 k-ft.

Determine the magnification factor when \( M_1 = 0, M_2 = 90 \text{ k-ft} \):

\[ C_n = 0.6 - 0.4 \frac{M_1}{M_i} = 0.6 - 0^{3-\beta} = 0.6 \leq 1.0 \]

\[ P_{el} = \frac{\pi^2 EA}{(KL/P_e)^2} = \frac{\pi^2 (30 \times 10^3 \text{ksi})(9.7 \text{in}^2)}{(25.9)^2} = 8695.4 k \]

\[ B_1 = \frac{C_n}{1 - \alpha(P_n/P_e)} = \frac{0.6}{1 - (1.0)(525k/8695.4 k)} = 0.64 \geq 1.0 \quad \text{USE 1.0} \]

\[ M_s = (1)90 \text{ k-ft} \]

Finally, determine the interaction value:

\[ \frac{P_n}{\phi P_n} + \frac{8}{9} \left( \frac{M_{uy}}{\phi M_{ux}} + \frac{M_{uy}}{\phi M_{uy}} \right) = 0.87 + \frac{8}{9} \left( \frac{90^{3-\beta}}{422^{3-\beta}} \right) = 1.06 \leq 1.0 \]

This is NOT OK. (and outside error tolerance).

The section should be larger.
Example 11 (pg 371)

**Example 7.** It is desired to use a 10-in. W shape for a column in a situation such as that shown in Figure 10.7. The factored axial load from above on the column is 175 kips [778 kN], and the factored beam load at the column face is 35 kips [156 kN]. The column has an unbraced height of 16 ft [4.88 m] and a K factor of 1.0. **Select a trial section for the column.** Evaluate the trial W10x45 chosen in the text of A36 steel with \( d = 10.1 \) in and \( \phi M_n = 133.4 \) k-ft (16 ft unbraced length).

Example 12

10.5 Using the AISC framed beam connection bolt shear in Table 7-1, determine the shear adequacy of the connection shown in Figure 10.28. What thickness and angle length are required? Also determine the bearing capacity of the wide flange sections.

Factored end beam reaction = 90 k.

![Figure 10.28](typical-beam-column-connection.png)
Example 13

10.2. The butt splice shown in Figure 10.22 uses two 8 x 
3/8" plates to “sandwich" in the 8 x 1/2" plates being joined. 
Four 3/8” A325-5C bolts are used on both sides of the 
splice. Assuming A36 steel and standard round holes, 
determine the allowable capacity of the connection.

SOLUTION:

Shear, bearing and net tension will be checked to determine the critical conditions 
that govern the capacity of the connection. (The edge distance to the holes is 
premised to be adequate.)

Shear: Using the AISC available shear in Table 7-3 (Group A):
\[
\phi R_n = 26.4 \text{k/bolt} \times 4 \text{ bolts} = 105.6 \text{k}
\]

Bearing: Using the AISC available bearing in Table 7-4:
There are 4 bolts bearing on the center (1/2") plate, while there are 4 bolts bearing on 
a total width of two sandwich plates (3/4" total). The thinner bearing width will govern. 
Assume 3 in. spacing (center to center) of bolts
For A36 steel, \(F_u = 58 \text{ ksi}\).
\[
\phi R_n = 91.4 \text{k/bolt/in.} \times 0.5 \text{ in.} \times 4 \text{ bolts} = 182.8 \text{k}
\]

Tension: The center plate is critical, again, because its thickness is less than the combined 
thicknesses of the two outer plates. We must consider tension yielding and tension rupture:
\[
\phi R_n = \phi F_y A_g \quad \text{and} \quad \phi R_n = \phi F_u A_e \quad \text{where} \quad A_e = A_{net} U
\]

\(A_g = 8 \text{ in.} \times \frac{1}{2} \text{ in.} = 4 \text{ in}^2\)

The holes are considered 1/8 in. larger than the nominal bolt diameter = 7/8 + 1/8 = 1 in.
\(A_t = (8 \text{ in.} - 2 \text{ holes} \times 1 \text{ in.}) \times \frac{1}{2} \text{ in.} = 3 \text{ in}^2\)

The whole cross section sees tension, so the shear lag factor \(U = 1\)
\[
\phi F_e A_g = 0.9 \times 36 \text{ ksi} \times 4 \text{ in}^2 = 129.6 \text{k}
\]
\[
\phi F_u A_e = 0.75 \times 58 \text{ ksi} \times (1) \times 3 \text{ in}^2 = 130.5 \text{k}
\]

The maximum connection capacity (smallest value) so far is governed by bolt shear:
\[
\phi R_n = 105.6 \text{k}
\]

Block Shear Rupture: It is possible for the center plate to rip away from the sandwich plates 
leaving the block (shown hatched) behind:
\[
\phi R_n = \phi (0.6 F_u A_{av} + U_{bs} F_u A_{nt}) \leq \phi (0.6 F_y A_{gv} + U_{bs} F_u A_{nt})
\]

where \(A_{av}\) is the area resisting shear, \(A_{nt}\) is the area resisting tension, \(A_{gv}\) is the gross area resisting shear, 
and \(U_{bs} = 1\) when the tensile stress is uniform.

\(A_{av} = (4 + 2 \text{ in.}) \times \frac{1}{2} \text{ in.} \times 2 \text{ sides} = 6 \text{ in}^2\)

\(A_{nt} = A_{av} - 1 \frac{1}{8} \text{ holes area} \times 2 \text{ sides} = 6 \text{ in}^2 - 1.5 \times 1 \text{ in.} \times \frac{1}{2} \text{ in.} \times 2 = 4.5 \text{ in}^2\)

\(A_{nt} = 3.5 \text{ in.} \times t - 1 \text{ holes} = 3.5 \text{ in.} \times \frac{1}{2} \text{ in.} - 1 \times 1 \text{ in.} \times \frac{1}{2} \text{ in.} = 1.25 \text{ in}^2\)

\[
\phi (0.6 F_u A_{av} + U_{bs} F_u A_{nt}) = 0.75 \times (0.6 \times 58 \text{ ksi} \times 4.5 \text{ in}^2 + 1 \times 58 \text{ ksi} \times 1.25 \text{ in}^2) = 171.8 \text{k}
\]
\[
\phi (0.6 F_y A_{gv} + U_{bs} F_u A_{nt}) = 0.75 \times (0.6 \times 36 \text{ ksi} \times 6 \text{ in}^2 + 1 \times 58 \text{ ksi} \times 1.25 \text{ in}^2) = 151.6 \text{k}
\]

The maximum connection capacity is governed by bolt shear (from the boxed values). 
\[
\phi R_n = 105.6 \text{k}
\]
Example 14

10.9 Determine the maximum load carrying capacity of this lap joint, assuming A36 steel with E60XX electrodes.

Example 15

10.7 Determine the capacity of the connection in Figure 10.44 assuming A36 steel with E70XX electrodes.

Solution:

Capacity of weld:

For a \( \frac{3}{16} \)" fillet weld, \( \phi S = 6.96 \text{k/in} \)

Weld length = 22"

Weld capacity = \( 22" \times 6.96 \text{k/in} = 153.1 \text{k} \)

Capacity of plate:

\[ \phi P_n = \phi F_y A_g \quad \phi = 0.9 \]

Plate capacity = \( 0.9 \times 36 \text{k/in}^2 \times \frac{3/8" \times 6"}{3/8" \times 6"} = 72.9 \text{k} \)

\[ \therefore \text{Plate capacity governs, } \phi P_n = 72.9 \text{k} \]

The weld size used is obviously too strong. What size, then, can the weld be reduced to so that the weld strength is more compatible to the plate capacity? To make the weld capacity = plate capacity:

\[ 22" \times \text{(weld capacity per in.)} = 72.9 \text{k} \]

Weld capacity per inch = \( \frac{72.9 \text{k}}{22\text{in.}} = 3.31 \text{k/in.} \)

From Available Strength table, use \( \frac{3}{16} \)" weld

\( \phi S = 4.18 \text{k/in.} \)

Minimum size fillet = \( \frac{3}{16} \)" based on a \( \frac{3}{8} \)" thick plate.
Example 16
Verify the tensile strength of an L4 x 4 x ½, ASTM 36, with one line of (4) ½ in.-diameter bolts and standard holes. The member carries a dead load of 20 kips and a live load of 60 kips in tension. Assume that connection limit states do not govern, and $U = 0.869$.

Example 17
The steel used in the connection and beams is A992 with $F_y = 50$ ksi, and $F_u = 65$ ksi. Using A490-N bolt material, determine the maximum capacity of the connection based on shear in the bolts, bearing in all materials and pick the number of bolts and angle length (not staggered). Use A36 steel for the angles.

W21x93: $d = 21.62$ in, $t_w = 0.58$ in, $t_f = 0.93$ in
W10x54: $t_f = 0.615$ in

SOLUTION:
The maximum length the angles can be depends on how it fits between the top and bottom flange with some clearance allowed for the fillet to the flange, and getting an air wrench in to tighten the bolts. This example uses 1” of clearance:

Available length = beam depth – both flange thicknesses – 1” clearance at top & 1” at bottom

$= 21.62$ in – 2(0.93 in) – 2(1 in) = 17.76 in.

With the spaced at 3 in. and 1 ¼ in. end lengths (each end), the maximum number of bolts can be determined:

Available length $\geq 1.25$ in. + 1.25 in. + 3 in. x (number of bolts – 1)  
number of bolts $\leq (17.76$ in – 2.5 in. - (-3 in.))/3 in. = 6.1, so 6 bolts.

It is helpful to have the All-bolted Double-Angle Connection Tables 10-1. They are available for ¾", 7/8", and 1" bolt diameters and list angle thicknesses of ¼", 5/16", 3/8", and ½". Increasing the angle thickness is likely to increase the angle strength, although the limit states include shear yielding of the angles, shear rupture of the angles, and block shear rupture of the angles.
For these diameters, the available shear (double) from Table 7-1 for 6 bolts is (6)45.1 k/bolt = 270.6 kips, (6)61.3 k/bolt = 367.8 kips, and (6)80.1 k/bolt = 480.6 kips.

Tables 10-1 (not all provided here) list a bolt and angle available strength of 271 kips for the ¾" bolts, 296 kips for the 7/8" bolts, and 281 kips for the 1" bolts. It appears that increasing the bolt diameter to 1" will not gain additional load. Use 7/8" bolts.

\[ \phi R_n = 367.8 \text{ kips for double shear of 7/8" bolts} \quad \phi R_n = 296 \text{ kips for limit state in angles} \]

We also need to evaluate bearing of bolts on the beam web, and column flange where there are bolt holes. Table 7-4 provides available bearing strength for the material type, bolt diameter, hole type, and spacing per inch of material thicknesses.

a) Bearing for beam web: There are 6 bolt holes through the beam web. This is typically the critical bearing limit value because there are two angle legs that resist bolt bearing and twice as many bolt holes to the column. The material is A992 (F_u = 65 ksi), 0.58" thick, with 7/8" bolt diameters at 3 in. spacing.

\[ \phi R_n = 6 \text{ bolts} \cdot (102 \text{ k/bolt/inch}) \cdot (0.58 \text{ in}) = 355.0 \text{ kips} \]

b) Bearing for column flange: There are 12 bolt holes through the column. The material is A992 (F_u = 65 ksi), 0.615" thick, with 1" bolt diameters.

\[ \phi R_n = 12 \text{ bolts} \cdot (102 \text{ k/bolt/inch}) \cdot (0.615 \text{ in}) = 752.8 \text{ kips} \]

Although, the bearing in the beam web is the smallest at 355 kips, with the shear on the bolts even smaller at 324.6 kips, the maximum capacity for the simple-shear connector is 296 kips limited by the critical capacity of the angles.
Beam Design Flow Chart

Collect data: L, ω, γ, Δmax; find beam charts for load cases and Δ_actual equations

ASD Allowable Stress or LRFD Design?

Listing of W Shapes in Descending order of Z

Collect data: load factors, Fy, Fu, and equations for shear capacity with φv

Find V & Mu from constructing diagrams or using beam chart formulas with the factored loads

Find Vmax & Mmax from constructing diagrams or using beam chart formulas

Find Zreq'd and pick a section from a table with Zx greater or equal to Zreq'd

Determine ωself wt (last number in name) or calculate ωself wt using A found. Find Mmax-adj & Vmax-adj.

Calculate Zreq'd-adj using Mmax-adj. Is Z(xpicked) ≥ Zreq'd-adj?

No

Is Vmax-adj ≤ (0.6FywebAweb)/Ω?

Yes

No pick a new section with a larger web area

Yes

Calculate Δmax (no load factors!) using superpositioning and beam chart equations with the Ix for the section

Ireq'd ≥ Δmax/Δ_limit

No pick a new section with a larger Ix

Yes (DONE)
### Listing of W Shapes in Descending order of Zₐ for Beam Design

<table>
<thead>
<tr>
<th>Zₐ - US (in.³)</th>
<th>Iₐ - US (in.²)</th>
<th>Section</th>
<th>Iₐ - SI (10⁶mm.²)</th>
<th>Zₐ - SI (10⁸mm.³)</th>
<th>Zₐ - US (in.³)</th>
<th>Iₐ - US (in.²)</th>
<th>Section</th>
<th>Iₐ - SI (10⁶mm.²)</th>
<th>Zₐ - SI (10⁸mm.³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>514 7450</td>
<td>W33X141</td>
<td>3100</td>
<td>8420</td>
<td>289</td>
<td>3100</td>
<td>W24X104</td>
<td>1290</td>
<td>4740</td>
<td></td>
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<tr>
<td>511 5680</td>
<td>W24X176</td>
<td>2360</td>
<td>8370</td>
<td>287</td>
<td>1900</td>
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<td>791</td>
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<td>4150</td>
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### Bolt Strength Tables

#### Table 7-1
Available Shear Strength of Bolts, kips

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<td>1.77</td>
</tr>
<tr>
<td>ASTM Design.</td>
<td>Thread Cond.</td>
<td>( F_p / d^2 ) (kips)</td>
<td>( \phi F_p / d^2 ) (kips)</td>
<td>( \frac{d}{2} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Loading</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group A</td>
<td>N</td>
<td>27.0</td>
<td>40.5</td>
<td>S</td>
</tr>
<tr>
<td></td>
<td>X</td>
<td>34.0</td>
<td>51.0</td>
<td>S</td>
</tr>
<tr>
<td>Group B</td>
<td>N</td>
<td>36.0</td>
<td>51.0</td>
<td>S</td>
</tr>
<tr>
<td></td>
<td>X</td>
<td>42.0</td>
<td>63.0</td>
<td>S</td>
</tr>
<tr>
<td>A307</td>
<td>–</td>
<td>13.5</td>
<td>20.3</td>
<td>S</td>
</tr>
</tbody>
</table>

\( \Omega = 2.00 \), \( \phi = 0.75 \)

#### Table 7-2
Available Tensile Strength of Bolts, kips

<table>
<thead>
<tr>
<th>Nominal Bolt Diameter, ( d ), in.</th>
<th>( \frac{d}{8} )</th>
<th>( \frac{d}{16} )</th>
<th>( \frac{d}{24} )</th>
<th>1/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Bolt Area, in.²</td>
<td>0.307</td>
<td>0.442</td>
<td>0.601</td>
<td>0.785</td>
</tr>
<tr>
<td>ASTM Design.</td>
<td>( F_p / \Omega ) (kips)</td>
<td>( \phi F_p / \Omega ) (kips)</td>
<td>( \frac{d}{2} )</td>
<td>( \frac{d}{4} )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Loading</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Group A</td>
<td>45.0</td>
<td>67.5</td>
<td>13.8</td>
<td>26.0</td>
</tr>
<tr>
<td></td>
<td>48.5</td>
<td>68.9</td>
<td>17.3</td>
<td>26.0</td>
</tr>
<tr>
<td>A307</td>
<td>22.5</td>
<td>33.0</td>
<td>6.90</td>
<td>10.4</td>
</tr>
<tr>
<td>Group B</td>
<td>45.0</td>
<td>67.5</td>
<td>13.8</td>
<td>26.0</td>
</tr>
<tr>
<td>Group A</td>
<td>45.0</td>
<td>67.5</td>
<td>13.8</td>
<td>26.0</td>
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<tr>
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<td>22.5</td>
<td>33.0</td>
<td>6.90</td>
<td>10.4</td>
</tr>
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\( \Omega = 2.00 \), \( \phi = 0.75 \)
### Table 7-3
Slip-Critical Connections
Available Shear Strength, kips
(Class A Faying Surface, $\mu = 0.30$)

#### Group A Bolts
A449, A490
A354 Grade BC

<table>
<thead>
<tr>
<th>Hole Type</th>
<th>Loading</th>
<th>$t_{\phi}$</th>
<th>$t_{\phi}^2$</th>
<th>$t_{\phi}$</th>
<th>$t_{\phi}^2$</th>
<th>$t_{\phi}$</th>
<th>$t_{\phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>STD/SSLT</td>
<td>S</td>
<td>4.29</td>
<td>6.44</td>
<td>6.33</td>
<td>9.49</td>
<td>8.81</td>
<td>13.2</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>8.59</td>
<td>12.9</td>
<td>12.7</td>
<td>19.0</td>
<td>17.6</td>
<td>26.4</td>
</tr>
<tr>
<td>OVS/SSLP</td>
<td>S</td>
<td>3.66</td>
<td>5.47</td>
<td>5.39</td>
<td>8.07</td>
<td>7.51</td>
<td>11.2</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>7.32</td>
<td>10.9</td>
<td>10.8</td>
<td>16.1</td>
<td>15.0</td>
<td>22.5</td>
</tr>
<tr>
<td>LSL</td>
<td>S</td>
<td>3.01</td>
<td>4.51</td>
<td>4.44</td>
<td>6.64</td>
<td>6.18</td>
<td>9.25</td>
</tr>
<tr>
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<td>D</td>
<td>8.02</td>
<td>12.0</td>
<td>11.1</td>
<td>18.5</td>
<td>16.5</td>
<td>24.2</td>
</tr>
</tbody>
</table>

#### Group B Bolts
A490, A490M
F2280
A354 Grade BD

<table>
<thead>
<tr>
<th>Hole Type</th>
<th>Loading</th>
<th>$t_{\phi}$</th>
<th>$t_{\phi}^2$</th>
<th>$t_{\phi}$</th>
<th>$t_{\phi}^2$</th>
<th>$t_{\phi}$</th>
<th>$t_{\phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>STD/SSLT</td>
<td>S</td>
<td>5.42</td>
<td>8.14</td>
<td>7.91</td>
<td>11.9</td>
<td>11.1</td>
<td>16.6</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>10.8</td>
<td>16.3</td>
<td>15.8</td>
<td>23.7</td>
<td>22.1</td>
<td>33.2</td>
</tr>
<tr>
<td>OVS/SSLP</td>
<td>S</td>
<td>4.62</td>
<td>6.92</td>
<td>6.74</td>
<td>10.1</td>
<td>9.44</td>
<td>14.1</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>9.25</td>
<td>13.8</td>
<td>13.5</td>
<td>20.2</td>
<td>18.8</td>
<td>29.2</td>
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<tr>
<td>LSL</td>
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<td>3.80</td>
<td>5.70</td>
<td>5.54</td>
<td>8.31</td>
<td>7.76</td>
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<tr>
<td></td>
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<td>7.60</td>
<td>11.4</td>
<td>11.1</td>
<td>18.6</td>
<td>16.6</td>
<td>23.3</td>
</tr>
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### Table 7-3 (continued)
Slip-Critical Connections
Available Shear Strength, kips
(Class A Faying Surface, $\mu = 0.30$)

#### Group B Bolts
A449, A490
A354 Grade BC

<table>
<thead>
<tr>
<th>Hole Type</th>
<th>Loading</th>
<th>$t_{\phi}$</th>
<th>$t_{\phi}^2$</th>
<th>$t_{\phi}$</th>
<th>$t_{\phi}^2$</th>
<th>$t_{\phi}$</th>
<th>$t_{\phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>STD/SSLT</td>
<td>S</td>
<td>18.1</td>
<td>27.1</td>
<td>23.1</td>
<td>34.6</td>
<td>27.3</td>
<td>41.0</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>36.2</td>
<td>54.2</td>
<td>46.1</td>
<td>69.0</td>
<td>54.2</td>
<td>82.0</td>
</tr>
<tr>
<td>OVS/SSLP</td>
<td>S</td>
<td>15.4</td>
<td>23.1</td>
<td>19.6</td>
<td>29.4</td>
<td>23.3</td>
<td>34.9</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>30.8</td>
<td>46.1</td>
<td>39.3</td>
<td>58.8</td>
<td>46.6</td>
<td>69.7</td>
</tr>
<tr>
<td>LSL</td>
<td>S</td>
<td>12.7</td>
<td>19.0</td>
<td>16.2</td>
<td>24.2</td>
<td>19.2</td>
<td>28.7</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>25.3</td>
<td>39.0</td>
<td>32.3</td>
<td>48.4</td>
<td>38.3</td>
<td>57.4</td>
</tr>
</tbody>
</table>

---

**Note:** Each table represents the minimum group bolt pretension values for different hole types and loading conditions. The tables include nominal bolt diameter, various loadings, and the minimum pretension values for both Group A and Group B bolts. The calculations consider standard, oversized, and short-slotted hole transverse to the line of force conditions, with options for single or double shear. The tables are provided for both A354 Grade BC and A354 Grade BD, with corresponding bolt specifications. For Class B faying surfaces, the available strength is multiplied by 1.67. The tables ensure adherence to ASTM standards such as A325, A449, A354, A490, and A490M, ensuring structural integrity in critical applications.
Table 7-4
Available Bearing Strength at Bolt Holes Based on Bolt Spacing
kips/in. thickness

<table>
<thead>
<tr>
<th>Hole Type</th>
<th>Bolt Spacing, s, in.</th>
<th>F_b, ksi</th>
<th>Nominal Bolt Diameter, d, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ASD</td>
<td>LRFD</td>
<td>ASD</td>
</tr>
<tr>
<td>STD</td>
<td>2(\sqrt{s}d_0)</td>
<td>58</td>
<td>34.1</td>
</tr>
<tr>
<td>3 in.</td>
<td>58</td>
<td>38.2</td>
<td>57.3</td>
</tr>
<tr>
<td>SSLT</td>
<td>2(\sqrt{s}d_0)</td>
<td>58</td>
<td>43.5</td>
</tr>
<tr>
<td>3 in.</td>
<td>58</td>
<td>48.8</td>
<td>73.1</td>
</tr>
<tr>
<td>OVS</td>
<td>2(\sqrt{s}d_0)</td>
<td>58</td>
<td>29.7</td>
</tr>
<tr>
<td>3 in.</td>
<td>58</td>
<td>33.3</td>
<td>50.0</td>
</tr>
<tr>
<td>LSLT</td>
<td>2(\sqrt{s}d_0)</td>
<td>58</td>
<td>43.5</td>
</tr>
<tr>
<td>3 in.</td>
<td>58</td>
<td>48.8</td>
<td>73.1</td>
</tr>
<tr>
<td>STD, SSLT, OVS, LSLT</td>
<td>s (\geq s_{max})</td>
<td>58</td>
<td>43.5</td>
</tr>
<tr>
<td>LSLT</td>
<td>s (\geq s_{max})</td>
<td>58</td>
<td>36.3</td>
</tr>
<tr>
<td>Spacing for full bearing strength s_{max}^a, in.</td>
<td></td>
<td></td>
<td>ASD</td>
</tr>
<tr>
<td>STD, SSLT, LSLT</td>
<td>1(\sqrt{s/n})</td>
<td>2(\sqrt{s/n})</td>
<td>2(\sqrt{s/n})</td>
</tr>
<tr>
<td>OVS</td>
<td>2(\sqrt{s/n})</td>
<td>2(\sqrt{s/n})</td>
<td>2(\sqrt{s/n})</td>
</tr>
<tr>
<td>LSLT</td>
<td>2(\sqrt{s/n})</td>
<td>1(\sqrt{s/n})</td>
<td>3(\sqrt{s/n})</td>
</tr>
<tr>
<td>Minimum Spacing</td>
<td>s (= 2\sqrt{s}d_0), in.</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

STD = standard hole
SSLT = short-slotted hole oriented transverse to the line of force
SSLP = short-slotted hole oriented parallel to the line of force
OVS = oversized hole
LSLT = long-slotted hole oriented transverse to the line of force

Note: Spacing indicated is from the center of the hole or slot to the center of the adjacent hole or slot in the line of force. Hole deformation is considered. When hole deformation is not considered, see AISC Specification Section J.3.10.

Minimum Spacing = 2\(\sqrt{s}d_0\), in.

\(s_{max}\) = maximum spacing

\(\Omega = 2.00, \phi = 0.75\) indicates spacing less than minimum spacing required per AISC Specification Section J.3.10.

Note: Spacing indicated is from the center of the hole or slot to the center of the adjacent hole or slot in the line of force. Hole deformation is considered. When hole deformation is not considered, see AISC Specification Section J.3.10.

Decimal value has been rounded to the nearest sixteenth of an inch.
### Available Bearing Strength at Bolt Holes Based on Edge Distance

**kips/in. thickness**

<table>
<thead>
<tr>
<th>Hole Type</th>
<th>Edge Distance, ( L_e ), in.</th>
<th>Nominal Bolt Diameter, ( d ), in.</th>
<th>( F_b ), kpsi</th>
<th>1/16</th>
<th>1/16</th>
<th>1/32</th>
<th>1/32</th>
<th>1/8</th>
<th>1/8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ASD LRF D ASD LRF D ASD LRF D ASD LRF D ASD LRF D</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>STD</strong></td>
<td>( 1\frac{1}{4} ) 58</td>
<td>31.5 47.3 29.4 44.0</td>
<td>27.2 40.8</td>
<td>25.0</td>
<td>37.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>65 35.3 53.0 32.9 49.4</td>
<td>30.5 45.7</td>
<td>28.0</td>
<td>42.0</td>
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</tr>
<tr>
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<td>53.4 79.9</td>
<td>51.1</td>
<td>78.7</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
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<td>59.2 89.6</td>
<td>57.3</td>
<td>85.9</td>
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<td></td>
</tr>
<tr>
<td><strong>SSLT</strong></td>
<td>( 1\frac{1}{4} ) 58</td>
<td>28.3 42.4 26.1 39.2</td>
<td>23.9 35.9</td>
<td>20.7</td>
<td>31.0</td>
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<td></td>
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<td>26.8 40.2</td>
<td>23.2</td>
<td>34.7</td>
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</tr>
<tr>
<td></td>
<td>2 58 43.5 65.3 52.2 78.3</td>
<td>50.0 75.0</td>
<td>48.6</td>
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<td>52.4</td>
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<tr>
<td><strong>OVS</strong></td>
<td>( 1\frac{1}{4} ) 58</td>
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<td>25.0 37.5</td>
<td>21.8</td>
<td>32.6</td>
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</tr>
<tr>
<td></td>
<td>65 32.9 48.4 30.5 45.7</td>
<td>28.0 42.0</td>
<td>24.4</td>
<td>36.8</td>
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<td>43.5 65.3 52.2 78.3</td>
<td>51.1 76.7</td>
<td>47.9</td>
<td>71.8</td>
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</tr>
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<td>5.4 9.1</td>
<td>5.4 9.1</td>
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</tr>
<tr>
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<td>65 23.8 37.4 12.2 18.3</td>
<td>6.9 11.3</td>
<td>6.9 11.3</td>
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<td></td>
</tr>
<tr>
<td><strong>LSL</strong></td>
<td>( 1\frac{1}{4} ) 58</td>
<td>42.4 63.6 37.0 55.5</td>
<td>31.5 47.3</td>
<td>28.1</td>
<td>39.2</td>
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<tr>
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<td>65 47.5 71.3 41.4 56.2</td>
<td>35.3 53.0</td>
<td>29.3</td>
<td>43.9</td>
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<td>22.7 34.0</td>
<td>20.8</td>
<td>31.5</td>
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<td></td>
</tr>
<tr>
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<td>65 29.5 44.2 27.4 41.1</td>
<td>25.4 38.1</td>
<td>23.4</td>
<td>38.0</td>
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<tr>
<td><strong>LSL</strong></td>
<td>( 1\frac{1}{4} ) 58</td>
<td>26.3 43.6 24.5 36.7</td>
<td>22.7 34.0</td>
<td>20.8</td>
<td>31.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>65 47.5 71.3 41.4 56.2</td>
<td>35.3 53.0</td>
<td>29.3</td>
<td>43.9</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

### Table 7-5 (continued)

**Available Bearing Strength at Bolt Holes Based on Edge Distance**

<table>
<thead>
<tr>
<th>Hole Type</th>
<th>Edge Distance, ( L_e ), in.</th>
<th>Nominal Bolt Diameter, ( d ), in.</th>
<th>( F_b ), kpsi</th>
<th>1/16</th>
<th>1/16</th>
<th>1/32</th>
<th>1/32</th>
<th>1/8</th>
<th>1/8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ASD LRF D ASD LRF D ASD LRF D ASD LRF D ASD LRF D</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>STD</strong></td>
<td>( 1\frac{1}{4} ) 58</td>
<td>22.8 34.3 20.7 31.0</td>
<td>18.5 27.7</td>
<td>16.3</td>
<td>24.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>65 25.6 38.4 22.2 34.7</td>
<td>20.7 31.1</td>
<td>18.3</td>
<td>27.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>SSLT</strong></td>
<td>( 1\frac{1}{4} ) 58</td>
<td>48.9 73.4 46.8 70.1</td>
<td>44.6 66.9</td>
<td>42.4</td>
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<tr>
<td><strong>OVS</strong></td>
<td>( 1\frac{1}{4} ) 58</td>
<td>17.4 26.1 15.2 22.8</td>
<td>13.1 19.6</td>
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<td><strong>LSL</strong></td>
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<td>18.5 27.7 16.3 24.5</td>
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<tr>
<td><strong>LST</strong></td>
<td>( 1\frac{1}{4} ) 58</td>
<td>20.7 31.0 15.2 22.8</td>
<td>18.9 27.7</td>
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<td>65 23.2 34.7 17.1 25.6</td>
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### Edge distance for full bearing strength

- **STD** = standard hole
- **SSLT** = short-slotted hole oriented transverse to the line of force
- **OVS** = oversized hole
- **LSL** = long-slotted hole oriented parallel to the line of force
- **OVS** = long-slotted hole oriented parallel to the line of force
- **LSL** = long-slotted hole oriented transverse to the line of force

**ASD LRF D** indicates spacing less than minimum spacing required per AISC Specification Section J.3.3.

**Note:** Spacing indicated is from the center of the hole or slot to the center of the adjacent hole or slot in the line of force. Hole deformation is considered. When hole deformation is not considered, see AISC Specification Section J.3.10.

**STD** = standard hole

**SSLT** = short-slotted hole oriented transverse to the line of force

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**\( \Omega = 2.00 \)** **\( \phi = 0.75 \)**

**Decimal value has been rounded to the nearest sixteenth of an inch.**
Method of Sections for Truss Analysis

Joint Configurations (special cases to recognize for faster solutions)

Case 1) Two Bodies Connected

\[ \text{Case 2) Three Bodies Connected with Two Bodies in Line} \]

\[ \text{Case 3) Three Bodies Connected and a Force} - \text{2 Bodies aligned & 1 Body and a Force are Aligned} \]

\[ \text{Four Bodies Connected - 2 Bodies Aligned and the Other 2 Bodies Aligned} \]

\[ \text{F}_{AB} \text{ has to be equal and opposite to } F_{BC} \]

\[ \text{F}_{AB} \text{ and } F_{BC} \text{ have to be equal, and } F_{BD} \text{ has to have zero force.} \]

\[ \text{F}_{AB} \text{ has to equal } F_{BC}, \text{ and } [F_{BD} \text{ has to equal } P] \text{ or } [F_{BD} \text{ has to equal } F_{BE}] \]

Method of Sections (relies on internal forces being in equilibrium with external forces on a section)

1. Determine support reaction forces.

2. Cut a section in such a way that force action lines intersect.

3. Solve for equilibrium. Sum moments about an intersection of force lines of action
Advantages: Quick when you only need one or two forces (only 3 equations needed)
Disadvantages: Not always easy to find a place to cut a section or see where force lines intersect

- **Compound Truss:** A truss assembled of simple trusses and additional links. It has $b=2n-3$, is statically determinate, rigid and completely constrained with a pin and roller. It can be identified by triangles with pins in the middle of some sides.

- **Statically Indeterminate Trusses:**
  Occur if there are more members than equations for all the joints
  OR if there are more reaction supports unknowns than 3

- **Diagonal Tension Counters:** Crossed bracing of cables or slender members is commonly used in bridge trusses, buildings and towers. These trusses look indeterminate, but can be solved statically because the bracing cannot hold a compressive force. The members are excluded in the analysis.

**Method:**
1. Determine support reaction forces.
2. Cut a section in such a way that the tension counters are exposed.
3. Solve for force equilibrium in $y$ with one counter. If the value is positive (in tension), this is the solution.
4. Solve for force equilibrium in $y$ with the other counter.
Example 1 (pg 26 & 33)
Using the method of sections, determine member forces in JH and EN.
Example 2
Using the method of sections, determine member forces in BC, CD and BD.

SOLUTION:
Find the support reactions from rigid body equilibrium, or in this case, from load tracing with symmetrical loads.

Draw a section line through the members of interest, cutting through no more than 3 members to separate the truss into two pieces. In this case, BC and CD can be cut through, while BD will need another section.

Draw one of the sections, exposing the member forces. Drawing them “out” or “away” from the cut assumes tension. BC is drawn in compression. So is DC, but because it has a 45 degree angle, the components will have the same magnitude.

Find a point to sum moments where two unknown forces intersect. This may be on a point of the section or off the section. $X$ is such a location where the line of action of BC intersects that of DE. For every 15 ft to the left, the line slopes down 5 ft, so $X$ is located $(10 \text{ ft}/5 \text{ ft})15 \text{ ft} = 30 \text{ ft}$ to the left of B.

\[
\sum M_X = -450lb(15 \text{ ft}) + 300lb(30 \text{ ft}) + DC_y(30 \text{ ft}) = 0
\]

DC$_y$ = -75 lb, so DC = DC$_y$/sin45 = 106 lb tension

(compression was assumed, but the answer was negative indicating our assumption wasn’t verified).

(Notice that DC$_x$ and DC$_y$ “slid” down to D and then the lever arm for DC$_x$ was 0. The components can also slide to the other end point of the member to locate the lever arms)

Summing at D where DC and DE intersect means there will be no lever arms. Sliding the components of BC to B means there will be no lever arm for BC$_y$:

\[
\sum M_D = 450lb(15 \text{ ft}) - BC_x(10 \text{ ft}) = 0
\]

BC$_x$ = 675lb, so BC = BC$_x$/$\sqrt{10}/3$ = 711.5 lb compression

Draw a section line that passes through BD and cuts through no more than three members.

If we hadn’t already found BC, we could sum moments at point $X$ again to eliminate BC and AD from our equation, leaving BD.

But it is obvious that we have only one unknown y force, which is BD:

\[
\sum F_y = 450lb - BD - 711.5lb\left(\frac{1}{\sqrt{10}}\right) = 0
\]

BD = 225 lb tension
**Example 3**

A 64-foot parallel chord truss (Figure ) supports horizontal and vertical loads as shown. Using the method of sections, determine the member forces $BC$, $HG$, and $GD$. 

![Diagram of a 64-foot parallel chord truss with forces indicated.]
Example 4
Using the method of sections, determine member forces in FE, EB, BC, AB and FB.

SOLUTION:
A section can’t pass through 5 members, so there will have to be two sections. The first passes through FE, EB and BC.

FE is shown assumed to be in compression, while the other forces are drawn assumed to be in tension.

There can be only two intersections when two of the three forces are parallel – at E and B:

\[ \Sigma M_E = -100 \text{lb}(6 \text{ ft}) + BC(8 \text{ ft}) = 0 \]
BC = 75 lb (T)

\[ \Sigma M_B = -100 \text{lb}(12 \text{ ft}) + FE(8 \text{ ft}) = 0 \]
FE = 150 lb (C)

Because EB is the only unknown force with a y component, it is useful to sum forces in the y direction (although it also has the only remaining unknown x component):

\[ \Sigma F_y = 100 \text{lb} - EB \left( \frac{8 \text{ ft}}{\sqrt{100 \text{ ft}}} \right) = 0 \]
(or \[ \Sigma F_y = 150 \text{lb} - 75 \text{lb} - EB \left( \frac{6 \text{ ft}}{\sqrt{100 \text{ ft}}} \right) = 0 \])
EB = 125 lb (T)

A second section can be drawn through AB, FB and FE.

There are three points of intersection of the unknown forces - at A, F and B. B is not on the section, but we know where it is.

\[ \Sigma M_A = 200 \text{lb}(6 \text{ ft}) - FB(6 \text{ ft}) = 0 \quad \text{FB} = 200 \text{lb (C)} \]

\[ \Sigma M_F = 200 \text{lb}(6 \text{ ft}) - AB_y(6 \text{ ft}) = 0 \quad \text{(sliding AB components to A)} \]
AB = AB_y \left( \frac{\sqrt{100}}{8} \right) = 250 \text{lb (T)}

or \[ \Sigma M_F = 200 \text{lb}(6 \text{ ft}) - AB_y(8 \text{ ft}) = 0 \quad \text{(sliding AB components to B)} \]
AB = AB_y \left( \frac{\sqrt{100}}{6} \right) = 300 \text{lb (T)}

\[ \Sigma M_B = 200 \text{lb}(6 \text{ ft}) - FE(8 \text{ ft}) = 0 \]
FE = 150 lb (C)
Reinforced Concrete Design

Notation:

- \( a \) = depth of the effective compression block in a concrete beam
- \( A \) = name for area
- \( A_g \) = gross area, equal to the total area ignoring any reinforcement
- \( A_s \) = area of steel reinforcement in concrete beam design
- \( A'_s \) = area of steel compression reinforcement in concrete beam design
- \( A_{sl} \) = area of steel reinforcement in concrete column design
- \( A_v \) = area of concrete shear stirrup reinforcement
- \( \text{ACI} \) = American Concrete Institute
- \( b \) = width, often cross-sectional
- \( b_{E} \) = effective width of the flange of a concrete T beam cross section
- \( b_f \) = width of the flange
- \( b_w \) = width of the stem (web) of a concrete T beam cross section
- \( c \) = distance from the top to the neutral axis of a concrete beam (see \( x \))
- \( cc \) = shorthand for clear cover
- \( C \) = name for centroid
- \( C_c \) = name for a compression force
- \( C_s \) = compressive force in the compression steel in a doubly reinforced concrete beam
- \( C_{s} \) = compressive force in the concrete of a doubly reinforced concrete beam
- \( d \) = effective depth from the top of a reinforced concrete beam to the centroid of the tensile steel
- \( d' \) = effective depth from the top of a reinforced concrete beam to the centroid of the compression steel
- \( d_b \) = bar diameter of a reinforcing bar
- \( D \) = shorthand for dead load
- \( DL \) = shorthand for dead load
- \( e \) = eccentricity
- \( E \) = modulus of elasticity or Young’s modulus
- \( E_c \) = modulus of elasticity of concrete
- \( E_s \) = modulus of elasticity of steel
- \( f \) = symbol for stress
- \( f'_c \) = concrete design compressive stress
- \( f'_s \) = compressive stress in the compression reinforcement for concrete beam design
- \( f_y \) = yield stress or strength
- \( f_{yt} \) = yield stress or strength of transverse reinforcement
- \( F \) = shorthand for fluid load
- \( G \) = relative stiffness of columns to beams in a rigid connection, as is \( \Psi \)
- \( h \) = cross-section depth
- \( H \) = shorthand for lateral pressure load
- \( h_f \) = depth of a flange in a T section
- \( I_{\text{transformed}} \) = moment of inertia of a multi-material section transformed to one material
- \( k \) = effective length factor for columns
- \( l_{b} \) = length of beam in rigid joint
- \( l_{c} \) = length of column in rigid joint
- \( l_d \) = development length for reinforcing steel
- \( l_{dh} \) = development length for hooks
- \( l_n \) = clear span from face of support to face of support in concrete design
- \( L \) = name for length or span length, as is \( l \)
- \( L_r \) = shorthand for live load
- \( LL \) = shorthand for live load
- \( M \) = internal bending moment
- \( M_{n} \) = nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete beam design
- \( M_a \) = maximum moment from factored loads for LRFD beam design
- \( n \) = modulus of elasticity transformation coefficient for steel to concrete
- \( n.a. \) = shorthand for neutral axis (N.A.)
Reinforced Concrete Design

Structural design standards for reinforced concrete are established by the *Building Code and Commentary (ACI 318-14)* published by the American Concrete Institute International, and uses strength design (also known as *limit state* design).

\[
f'_c = \text{concrete compressive design strength at 28 days (units of psi when used in equations)}
\]
Materials
Concrete is a mixture of cement, coarse aggregate, fine aggregate, and water. The cement hydrates with the water to form a binder. The result is a hardened mass with “filler” and pores. There are various types of cement for low heat, rapid set, and other properties. Other minerals or cementitious materials (like fly ash) may be added.

ASTM designations are:
- Type I: Ordinary portland cement (OPC)
- Type II: Moderate heat of hydration and sulfate resistance
- Type III: High early strength (rapid hardening)
- Type IV: Low heat of hydration
- Type V: Sulfate resistant

The proper proportions, by volume, of the mix constituents determine strength, which is related to the water to cement ratio (w/c). It also determines other properties, such as workability of fresh concrete. Admixtures, such as retardants, accelerators, or superplasticizers, which aid flow without adding more water, may be added. Vibration may also be used to get the mix to flow into forms and fill completely.

Slump is the measurement of the height loss from a compacted cone of fresh concrete. It can be an indicator of the workability.

Proper mix design is necessary for durability. The pH of fresh cement is enough to prevent reinforcing steel from oxidizing (rusting). If, however, cracks allow corrosive elements in water to penetrate to the steel, a corrosion cell will be created, the steel will rust, expand and cause further cracking. Adequate cover of the steel by the concrete is important.

Deformed reinforcing bars come in grades 40, 60 & 75 (for 40 ksi, 60 ksi and 75 ksi yield strengths). Sizes are given as # of 1/8” up to #8 bars. For #9 and larger, the number is a nominal size (while the actual size is larger).

Reinforced concrete is a composite material, and the average density is considered to be 150 lb/ft^3. It has the properties that it will creep (deformation with long term load) and shrink (a result of hydration) that must be considered.

Construction
Because fresh concrete is a viscous suspension, it is cast or placed and not poured. Formwork must be able to withstand the hydraulic pressure. Vibration may be used to get the mix to flow around reinforcing bars or into tight locations, but excess vibration will cause segregation, honeycombing, and excessive bleed water which will reduce the water available for hydration and the strength, subsequently.

After casting, the surface must be worked. Screeding removes the excess from the top of the forms and gets a rough level. Floating is the process of working the aggregate under the surface and to “float” some paste to the surface. Troweling takes place when the mix has hydrated to the point of supporting weight and the surface is smoothed further and consolidated. Curing is allowing the hydration process to proceed with adequate moisture. Black tarps and curing
compounds are commonly used. *Finishing* is the process of adding a texture, commonly by using a broom, after the concrete has begun to set.

**Behavior**

Plane sections of composite materials can still be assumed to be plane (strain is linear), but the stress distribution is not the same in both materials because the *modulus of elasticity* is different. \( \frac{f = E \cdot \varepsilon}{R(\text{or } \rho)} \) is the radius of curvature

\[
f_1 = E_1 \varepsilon = \frac{E_1 y}{R} \quad f_2 = E_2 \varepsilon = \frac{E_2 y}{R}
\]

where \( R(\text{or } \rho) \) is the radius of curvature

In order to determine the stress, we can define \( n \) as the ratio of the elastic moduli:

\[
n = \frac{E_2}{E_1}
\]

\( n \) is used to transform the width of the second material such that it sees the equivalent element stress.

**Transformed Section \( y \) and \( I \)**

In order to determine stresses in all types of material in the beam, we transform the materials into a single material, and calculate the location of the neutral axis and modulus of inertia for that material.

ex: When material 1 above is concrete and material 2 is steel

\[
to \text{ transform steel into concrete} \\
\quad n = \frac{E_2}{E_1} = \frac{E_{\text{steel}}}{E_{\text{concrete}}}
\]

\[
to \text{ find the neutral axis of the equivalent concrete member we transform the width of the steel by multiplying by } n
\]

\[
to \text{ find the moment of inertia of the equivalent concrete member, } I_{\text{transformed}}, \text{ use the new geometry resulting from transforming the width of the steel}
\]

\[
\text{concrete stress: } f_{\text{concrete}} = -\frac{My}{I_{\text{transformed}}} \\
\text{steel stress: } f_{\text{steel}} = -\frac{Myn}{I_{\text{transformed}}}
\]
Reinforced Concrete Beam Members

Strength Design for Beams

The strength design method is similar to LRFD. There is a *nominal* strength that is reduced by a factor $\phi$ which must exceed the factored design stress. For beams, the concrete only works in compression over a rectangular “stress” block above the n.a. from elastic calculation, and the steel is exposed and reaches the yield stress, $f_y$.

For stress analysis in reinforced concrete beams:
- the steel is transformed to concrete
- any concrete in tension is assumed to be cracked and to have no strength
- the steel can be in tension, and is placed in the bottom of a beam that has positive bending moment

![Diagram showing stress distribution and analysis for reinforced concrete beams.](image-url)
The neutral axis is where there is no stress and no strain. The concrete above the n.a. is in compression. The concrete below the n.a. (shown as x, but also sometimes named c) is considered ineffective. The steel below the n.a. is in tension.

Because the n.a. is defined by the moment areas, we can solve for x knowing that d is the distance from the top of the concrete section to the centroid of the steel:

\[ bx \cdot \frac{x}{2} - nA_s (d - x) = 0 \]

\( x \) can be solved for when the equation is rearranged into the generic format with a, b & c in the binomial equation:

\[ ax^2 + bx + c = 0 \quad \text{by} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

**T-sections**

If the n.a. is *above* the bottom of a flange in a T section, \( x \) is found as for a rectangular section.

If the n.a. is *below* the bottom of a flange in a T section, \( x \) is found by including the flange and the stem of the web \((b_w)\) in the moment area calculation:

\[ b_fh_f \left( x - \frac{h_f}{2} \right) + \left( x - h_f \right) b_w \left( x - \frac{h_f}{2} \right) - nA_s (d - x) = 0 \]

**Load Combinations (Alternative values are allowed)**

1.4D

1.2D + 1.6L + 0.5(L, or S or R)

1.2D + 1.6(L, or S or R) + (1.0L or 0.5W)

1.2D + 1.0W + 1.0L + 0.5(L, or S or R)

1.2D + 1.0E + 1.0L + 0.2S

0.9D + 1.0W

0.9D + 1.0E

**Internal Equilibrium**

\[ a = 0.85f'_c \]

actual stress  Whitney stress block
C = compression in concrete = stress x area
= 0.85 $f'_c$ba

T = tension in steel = stress x area = $A_s f_y$

$C = T$ and $M_n = T(d-a/2)$

where

- $f'_c$ = concrete compression strength
- a = height of stress block
- $\beta_1$ = factor based on $f'_c$
- c = location to the neutral axis
- b = width of stress block
- $f_y$ = steel yield strength
- $A_s$ = area of steel reinforcement
- d = effective depth of section
- $\delta$ = depth to neutral axis

With $C=T$, $A_s f_y = 0.85 f'_c b a$ so $a$ can be determined with $a = \frac{A_s f_y}{0.85 f'_c b} = \beta_1 c$

**Criteria for Beam Design**

For flexure design:

$M_u \leq \phi M_n \quad \phi = 0.9$ for flexure (when the section is tension controlled)

so for design, $M_u$ can be set to $\phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2)$

**Reinforcement Ratio**

The amount of steel reinforcement is limited. Too much reinforcement, or over-reinforcing will not allow the steel to yield before the concrete crushes and there is a sudden failure. A beam with the proper amount of steel to allow it to yield at failure is said to be under reinforced.

The reinforcement ratio is just a fraction: $\rho = \frac{A_s}{bd}$ (or $\rho$). The amount of reinforcement is limited to that which results in a concrete strain of 0.003 and a minimum tensile strain of 0.004.

When the strain in the reinforcement is 0.005 or greater, the section is tension controlled. (For smaller strains the resistance factor reduces to 0.65 because the stress is less than the yield stress in the steel.) Previous codes limited the amount to $0.75 \rho_{balanced}$ where $\rho_{balanced}$ was determined from the amount of steel that would make the concrete start to crush at the exact same time that the steel would yield based on strain ($\varepsilon_y$) of 0.002.

The strain in tension can be determined from $\varepsilon_y = \frac{d-c}{c}(0.003)$. At yield, $\varepsilon_y = \frac{f_y}{E_s}$.

The resistance factor expressions for transition and compression controlled sections are:

- $\phi = 0.75 + (\varepsilon_y - \varepsilon_y) \times \frac{0.15}{(0.005 - \varepsilon_y)}$ for spiral members (not less than 0.75)
- $\phi = 0.65 + (\varepsilon_y - \varepsilon_y) \times \frac{0.25}{(0.005 - \varepsilon_y)}$ for other members (not less than 0.65)
Flexure Design of Reinforcement

One method is to “wisely” estimate a height of the stress block, \(a\), and solve for \(A_s\), and calculate a new value for \(a\) using \(M_u\).

1. guess \(a\) (less than n.a.)
2. \(A_s = \frac{0.85 f'_c b a}{f_y}\)
3. solve for \(a\) from setting \(M_u = \phi A_s f_y (d-a/2)\):
   \[
   a = 2 \left( d - \frac{M_u}{\phi A_s f_y} \right)
   \]
4. repeat from 2. until \(a\) found from step 3 matches \(a\) used in step 2.

Design Chart Method:

1. calculate \(R_n = \frac{M_n}{b d^2}\)
2. find curve for \(f'_c\) and \(f_y\) to get \(\rho\)
3. calculate \(A_s\) and \(a\), where:
   \[
   A_s = \rho bd \quad \text{and} \quad a = \frac{A_s f_y}{0.85 f'_c b}
   \]

Any method can simplify the size of \(d\) using \(h = 1.1d\)

Maximum Reinforcement

Based on the limiting strain of 0.005 in the steel, \(x(\text{or} c) = 0.375d\) so

\[
a = \beta_1 (0.375d) \quad \text{to find} \quad A_{s,\text{max}}
\]

(\(\beta_1\) is shown in the table above)

Minimum Reinforcement

Minimum reinforcement is provided even if the concrete can resist the tension. This is a means to control cracking.

Minimum required: \(A_s = \frac{3\sqrt{f'_c}}{f_y} (b W d)\)

but not less than \(A_s = \frac{200}{f_y} (b W d)\)

where \(f'_c\) is in psi.  This can be translated to \(\rho_{\text{min}} = \frac{3\sqrt{f'_c}}{f_y}\) but not less than \(\frac{200}{f_y}\)
**Lightweight Concrete**

Lightweight concrete has strength properties that are different from normalweight concretes, and a modification factor, $\lambda$, must be multiplied to the strength value of $f'_{c}$ for concrete for some specifications (ex. shear). Depending on the aggregate and the lightweight concrete, the value of $\lambda$ ranges from 0.75 to 0.85, 0.85, or 0.85 to 1.0. $\lambda$ is 1.0 for normalweight concrete.

**Cover for Reinforcement**

Cover of concrete over/under the reinforcement must be provided to protect the steel from corrosion. For indoor exposure, 1.5 inch is typical for beams and columns, 0.75 inch is typical for slabs, and for concrete cast against soil, 3 inch minimum is required.

**Bar Spacing**

Minimum bar spacings are specified to allow proper consolidation of concrete around the reinforcement. The minimum spacing is the maximum of 1 in, a bar diameter, or 1.33 times the maximum aggregate size.

**T-beams and T-sections (pan joists)**

Beams cast with slabs have an effective width, $b_{E}$, that sees compression stress in a wide flange beam or joist in a slab system with positive bending.

For interior T-sections, $b_{E}$ is the smallest of $L/4$, $b_{w} + 16t$, or center to center of beams.

For exterior T-sections, $b_{E}$ is the smallest of $b_{w} + L/12$, $b_{w} + 6t$, or $b_{w} + \frac{1}{2}(\text{clear distance to next beam})$.

When the web is in tension the minimum reinforcement required is the same as for rectangular sections with the web width ($b_{w}$) in place of $b$. $M_{n} = C_w(d-a/2) + C_f(d-h_f/2)$ ($h_f$ is height of flange or $t$).

When the flange is in tension (negative bending), the minimum reinforcement required is the greater value of $A_y = \frac{6f'_{c}}{f_y}(b_{w}d)$ or $A_y = \frac{3f'_{c}}{f_y}(b_{f}d)$

where $f'_{c}$ is in psi, $b_{w}$ is the beam width, and $b_{f}$ is the effective flange width.
Compression Reinforcement

If a section is *doubly reinforced*, it means there is steel in the beam seeing compression. The force in the compression steel that *may not be yielding* is

\[ C_s = A_s' (f'_s - 0.85 f'_c) \]

The total compression that balances the tension is now:

\[ T = C_c + C_s. \]

And the moment taken about the centroid of the compression stress is

\[ M_n = T(d-a/2) + C_s(a-d') \]

where \( A_s' \) is the area of compression reinforcement, and \( a-d' \) is the effective depth to the centroid of the compression reinforcement.

Because the compression steel may not be yielding, the neutral axis \( x \) must be found from the force equilibrium relationships, and the stress can be found based on strain to see if it has yielded.

**Slabs 304**

One way slabs can be designed as “one unit”-wide beams. Because they are thin, control of deflections is important, and minimum depths are specified, as is minimum reinforcement for shrinkage and crack control when not in flexure. Reinforcement is commonly small diameter bars and welded wire fabric.

Maximum spacing between bars is also specified for shrinkage and crack control as five times the slab thickness not exceeding 18”.

For required flexure reinforcement the spacing limit is three times the slab thickness not exceeding 18”.

Shrinkage and temperature reinforcement (and minimum for flexure reinforcement):

Minimum for slabs with grade 40 or 50 bars:

\[ \rho = \frac{A_s}{bt} = 0.002 \quad \text{or} \quad A_{s-min} = 0.002bt \]

Minimum for slabs with grade 60 bars:

\[ \rho = \frac{A_s}{bt} = 0.0018 \quad \text{or} \quad A_{s-min} = 0.0018bt \]
Shear Behavior

Horizontal shear stresses occur along with bending stresses to cause tensile stresses where the concrete cracks. Vertical reinforcement is required to bridge the cracks which are called shear stirrups (or stirrups).

The maximum shear for design, $V_u$, is the value at a distance of $d$ from the face of the support.

Nominal Shear Strength

The shear force that can be resisted is the shear stress $\times$ cross section area: $V_c = \tau_c \times b_w d$

The shear stress for beams (one way) $\tau_c = 2\lambda \sqrt{f'_c}$ so $\phi V_c = \phi 2\lambda \sqrt{f'_c} b_w d$

where $b_w =$ the beam width or the minimum width of the stem.
$\phi = 0.75$ for shear
$\lambda =$ modification factor for lightweight concrete

One-way joists are allowed an increase to $1.1\times V_c$ if the joists are closely spaced.

Stirrups are necessary for strength (as well as crack control): $V_s = \frac{A_v f_y}{s} \leq 8\sqrt{f'_c} b_w d$ (max)

where $A_v =$ area of all vertical legs of stirrup
$f_y =$ yield strength of transvers reinforcement (stirrups)
$s =$ spacing of stirrups
$d =$ effective depth

For shear design:

$$V_U \leq \phi V_C + \phi V_S \quad \phi = 0.75 \text{ for shear}$$

Spacing Requirements

Stirrups are required when $V_u$ is greater than $\frac{\phi V_c}{2}$. A minimum is required because shear failure of a beam without stirrups is sudden and brittle and because the loads can vary with respect to the design values.

Economical spacing of stirrups is considered to be greater than $d/4$. Common spacings of $d/4$, $d/3$ and $d/2$ are used to determine the values of $\phi V_s$ at which the spacings can be increased.

$$\phi V_s = \frac{\phi A_v f_y d}{s}$$
This figure shows the size of $V_n$ provided by $V_c + V_s$ (long dashes) exceeds $V_u/\phi$ in a step-wise function, while the spacing provided (short dashes) is at or less than the required $s$ (limited by the maximum allowed).  (Note that the maximum shear permitted from the stirrups is $8\sqrt{f'_c b_w d}$.

The minimum recommended spacing for the first stirrup is 2 inches from the face of the support.

**Torsional Shear Reinforcement**

On occasion beam members will see twist along the axis caused by an eccentric shape supporting a load, like on an L-shaped spandrel (edge) beam. The torsion results in shearing stresses, and closed stirrups may be needed to resist the stress that the concrete cannot resist.
Development Length for Reinforcement

Because the design is based on the reinforcement attaining the yield stress, the reinforcement needs to be properly bonded to the concrete for a finite length (both sides) so it won’t slip. This is referred to as the development length, \( l_d \). Providing sufficient length to anchor bars that need to reach the yield stress near the end of connections are also specified by hook lengths. Detailing reinforcement is a tedious job. The equations for development length must be modified if the bar is epoxy coated or is cast with more than 12 in. of fresh concrete below it. Splices are also necessary to extend the length of reinforcement that come in standard lengths. The equations for splices are not provided here.

Development Length in Tension

With the proper bar to bar spacing and cover, the common development length equations are:

- #6 bars and smaller: \( l_d = \frac{d_b f_y}{25\lambda \sqrt{f'_c}} \) or 12 in. minimum
- #7 bars and larger: \( l_d = \frac{d_b f_y}{20\lambda \sqrt{f'_c}} \) or 12 in. minimum

Development Length in Compression

\[ l_d = \frac{d_b f_y}{50\lambda \sqrt{f'_c}} \leq 0.0003 f_y d_b \] or 8 in. minimum

Hook Bends and Extensions

The minimum hook length is \( l_{dh} = \frac{d_b f_y}{50\lambda \sqrt{f'_c}} \) but not less than the larger of \( 8d_b \) and 6 in.

![Figure 9-17: Minimum requirements for 90° bar hooks.](image1)

![Figure 9-18: Minimum requirements for 180° bar hooks.](image2)
**Modulus of Elasticity & Deflection**

$E_c$ for deflection calculations can be used with the transformed section modulus in the elastic range. After that, the cracked section modulus is calculated and $E_c$ is adjusted.

Code values:

$$E_c = 57,000 \sqrt{f'_c} \text{ (normal weight)} \quad E_c = w_c^{1.5} \sqrt{f'_c}, \ w_c = 90 \text{ lb/ft}^3 - 160 \text{ lb/ft}^3$$

Deflections of beams and one-way slabs need not be computed if the overall member thickness meets the minimum specified by the code, and are shown in Table 7.3.1.1 (see Slabs). The span lengths for continuous beams or slabs is taken as the clear span, $l_n$.

**Criteria for Flat Slab & Plate System Design**

Systems with slabs and supporting beams, joists or columns typically have multiple bays. The horizontal elements can act as one-way or two-way systems. Most often the flexure resisting elements are continuous, having positive and negative bending moments. These moment and shear values can be found using beam tables, or from code specified approximate design factors. Flat slab two-way systems have drop panels (for shear), while flat plates do not.

**Criteria for Column Design**

(American Concrete Institute) ACI 318-02 Code and Commentary:

$$P_u \leq \phi P_n$$

where

- $P_u$ is a factored load
- $\phi$ is a resistance factor
- $P_n$ is the nominal load capacity (strength)

Load combinations, ex:

- 1.4D (D is dead load)
- 1.2D + 1.6L (L is live load)

For compression, $\phi_c = 0.75$ and $P_n = 0.85P_o$ for spirally reinforced, $\phi_c = 0.65$ and $P_n = 0.8P_o$ for tied columns where $P_o = 0.85 f'_c (A_g - A_{st}) + f_y A_{st}$ and $P_o$ is the name of the maximum axial force with no concurrent bending moment.

Columns which have reinforcement ratios, $\rho_g = \frac{A_{st}}{A_g}$, in the range of 1% to 2% will usually be the most economical, with 1% as a minimum and 8% as a maximum by code.

Bars are symmetrically placed, typically.

Spiral ties are harder to construct.
Columns with Bending (Beam-Columns)

Concrete columns rarely see only axial force and must be designed for the combined effects of axial load and bending moment. The interaction diagram shows the reduction in axial load a column can carry with a bending moment.

Design aids commonly present the interaction diagrams in the form of load vs. equivalent eccentricity for standard column sizes and bars used.

Rigid Frames

Monolithically cast frames with beams and column elements will have members with shear, bending and axial loads. Because the joints can rotate, the effective length must be determined from methods like that presented in the handout on Rigid Frames. The charts for evaluating $k$ for non-sway and sway frames can be found in the ACI code.

Frame Columns

Because joints can rotate in frames, the effective length of the column in a frame is harder to determine. The stiffness $(EI/L)$ of each member in a joint determines how rigid or flexible it is. To find $k$, the relative stiffness, $G$ or $\Psi$, must be found for both ends, plotted on the alignment charts, and connected by a line for braced and unbraced frames.

$$G = \Psi = \frac{\sum EI/l_c}{\sum EI/l_b}$$

where

- $E$ = modulus of elasticity for a member
- $I$ = moment of inertia of for a member
- $l_c$ = length of the column from center to center
- $l_b$ = length of the beam from center to center

- For pinned connections we typically use a value of 10 for $\Psi$.
- For fixed connections we typically use a value of 1 for $\Psi$. 
Slenderness effects can be neglected if $\frac{kl}{r} \leq 22$ for columns not braced against sidesway, $\frac{kl}{r} \leq 34 + 12(M_1/M_2)$ and less than 40 for columns braced against sidesway where $M_1/M_2$ is negative if the column is bent in single curvature, and positive for double curvature.
Example 1

(a) Determine the ultimate moment capacity of a beam with dimensions \( b = 10 \) in. and \( d_{\text{effective}} = 15 \) in. and that has three No. 9 bars (3.0 in.) of tension-reinforcing steel. Assume that \( h = 18 \) in., \( F_y = 40 \) ksi, and \( f_c' = 5 \) ksi. (b) Assume also that the section is used as a cantilever beam 10 ft long, where the service loads are dead load = 400 lb/ft and live load = 300 lb/ft. Is the beam adequate in bending? Calculate the ultimate moment capacity of the beam first.

Solution:

(a) \( a = A_F F_y / 0.85f'_c b = (3)(40,000)/(0.85)(5000)(10) = 2.82 \) in.
\[ \phi M_u = \phi A_F F_y [d - a/2] = 0.9(3)(40,000)(15 - (2.82)/2) = 1,466,640 \text{ in.-lb} \]

Check for overreinforcement, \( c = 0.375 \cdot 15 = 5.625 \). Depth of stress block \( a = 0.80 \cdot 5.625 \) in. = 4.5 in. \( A_{x,\text{max}} = (0.85)(5\text{ksi})(4.5\text{in.})(10\text{in.})/(40\text{ksi}) = 4.78 \) in.². The beam is not over reinforced.

Check for minimum steel: \( A_{x,\text{min}} = 3\sqrt{f'_c / F_y} bd = 0.16 \) in.², so beam is sufficiently reinforced.

(b) \( U = 1.2D + 1.6L = 1.2(400) + 1.6(300) = 960 \) lb/ft
\[ M_u = w_uL^2/2 = (960)(10^2)/2 = 48,000 \text{ ft-lb} = 576,000 \text{ in.-lb} \]

Since \( M_u = 576,000 < \phi M_u = 1,466,640 \), the beam is adequate in bending.

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Example 2 (pg 423)

**Example 1.** The service load bending moments on a beam are 58 kip-ft [78.6 kN-m] for dead load and 38 kip-ft [51.5 kN-m] for live load. The beam is 10 in. [254 mm] wide, \( f'_c \) is 3000 psi [27.6 MPa], and \( f_y \) is 60 ksi [414 MPa]. Determine the depth of the beam and the tensile reinforcing required.
Example 2 (continued)
Example 3
A simply supported beam 20 ft long carries a service dead load of 300 lb/ft and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:
Find the design moment, \( M_u \), from the factored load combination of 1.2D + 1.6L. It is good practice to guess a beam size to include self weight in the dead load, because “service” means dead load of everything except the beam itself.

Guess a size of 10 in x 12 in. Self weight for normal weight concrete is the density of 150 lb/ft\(^3\) multiplied by the cross section area: \( \text{self weight} = \frac{2}{3} \text{ft}(12\text{in})(10\text{in}) = 125 \text{lb/ft} \)

\[ w_u = 1.2(300 \text{ lb/ft} + 125 \text{ lb/ft}) + 1.6(500 \text{ lb/ft}) = 1310 \text{ lb/ft} \]

The maximum moment for a simply supported beam is
\[ M_u = \frac{w_u l^2}{8} \]

\[ M_u = \frac{1310(20\text{ ft})^2}{8} = 65,500 \text{ lb-ft} \]

\[ M_n \text{ required} = \frac{M_u}{\phi} = \frac{65,500}{0.9} = 72,778 \text{ lb-ft} \]

To use the design chart aid, find \( R_n = \frac{M_n}{bd^2} \), estimating that \( d \) is about 1.75 inches less than \( h \):

\[ d = 12\text{ in} - 1.75 \text{ in} - (0.375\text{ in}) = 10.25 \text{ in} \]

\( R_n = \frac{72,778^{\text{12-10}}} {(10\text{in})(10.25\text{in})^2} \cdot (12\text{ in/ft}) = 831 \text{ psi} \)

\( \rho \) corresponds to approximately 0.023 (which is less than that for 0.005 strain of 0.0319), so the estimated area required, \( A_s \), can be found:

\[ A_s = \rho bd = (0.023)(10\text{in})(10.25\text{in}) = 2.36 \text{ in}^2 \]

The number of bars for this area can be found from handy charts.

(Whether the number of bars actually fit for the width with cover and space between bars must also be considered. If you are at \( \rho_{\text{max}} \), do not choose an area bigger than the maximum!)

Try \( A_s = 2.37 \text{ in}^2 \) from 3#8 bars

\[ d = 12 \text{ in} - 1.5 \text{ in (cover)} - \frac{1}{8} (0.8\text{ in diameter bar}) = 10 \text{ in} \]

Check \( \rho = \frac{2.37 \text{ in}^2}{(10\text{ in})(10\text{ in})} = 0.0237 \) which is less than \( \rho_{\text{max}} = 0.0319 \) OK (We cannot have an over reinforced beam!!)

Find the moment capacity of the beam as designed, \( \phi M_n \)

\[ a = A_s f_y/0.85f'_c = 2.37 \text{ in}^2 (40 \text{ ksi})(0.85(5 \text{ ksi})10 \text{ in}) = 2.23 \text{ in} \]

\[ \phi M_n = \phi A_s f_y (d-a/2) = 0.9(2.37 \text{ in}^2)(40 \text{ ksi})(10 \text{ in} - \frac{2.23 \text{ in}}{2} \cdot \frac{1}{12\text{ ft}}) = 63.2 \text{ k-ft} \not> 65.5 \text{ k-ft needed} \] (not OK)

So, we can increase \( d \) to 13 in, and \( \phi M_n = 70.3 \text{ k-ft} \) (OK). Or increase \( A_s \) to 2 # 10's (2.54 \text{ in}^2), for \( a = 2.39 \text{ in} \) and \( \phi M_n \) of 67.1 k-ft (OK). Don’t exceed \( \rho_{\text{max}} \) or \( \rho_{\text{max}} = 0.005 \) if you want to use \( \phi = 0.9 \).
Example 4
A simply supported beam 20 ft long carries a service dead load of 425 lb/ft (including self weight) and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:

Find the design moment, $M_n$, from the factored load combination of 1.2D + 1.6L. 

If self weight is not included in the service loads, you need to guess a beam size to include self weight in the dead load, because “service” means dead load of everything except the beam itself.

$$w_u = 1.2(425 \text{ lb/ft}) + 1.6(500 \text{ lb/ft}) = 1310 \text{ lb/ft}$$

The maximum moment for a simply supported beam is

$$M_u = \frac{w_u l^2}{8}$$

$$M_u = \frac{1310 \text{ lb/ft} \cdot (20 \text{ ft})^2}{8} = 65,500 \text{ lb-ft}$$

$$M_n \text{ required} = \frac{M_u}{\phi} = \frac{65,500 \text{ lb-ft}}{0.9} = 72,778 \text{ lb-ft}$$

To use the design chart aid, we can find $R_n = \frac{M}{bd^2}$, and estimate that $h$ is roughly 1.5-2 times the size of $b$, and $h = 1.1d$ (rule of thumb): $d = h/1.1 = (2b)/1.1$, so $d \approx 1.8b$ or $b \approx 0.55d$.

We can find $R_n$ at the maximum reinforcement ratio for our materials, keeping in mind $\rho_{\text{max}}$ at a strain = 0.005 is 0.0319 off of the chart at about 1070 psi, with $\rho_{\text{max}} = 0.037$. Let’s substitute $b$ for a function of $d$:

$$R_n = 1070 \text{ psi} = \frac{72,778 \text{ lb-ft}}{(0.55d)(d)^2(12/12)} \cdot (1/12)$$

Rearranging and solving for $d = 11.4$ inches

That would make $b$ a little over 6 inches, which is impractical. 10 in is commonly the smallest width.

So if $h$ is commonly 1.5 to 2 times the width, $b$, $h$ ranges from 14 to 20 inches. (10x1.5=15 and 10x2 = 20)

Choosing a depth of 14 inches, $d = 14 - 1.5$ (clear cover) - $\frac{1}{2}$ (8/8 in bar diameter guess) - 3/8 in (stirrup diameter) = 11.625 in.

Now calculating an updated $R_n = \frac{72,778 \text{ lb-ft}}{(10\text{in})(11625\text{ in}^2)} \cdot (12/12) = 646.2 \text{ psi}$

$\rho$ now is 0.020 (under the limit at 0.005 strain of 0.0319), so the estimated area required, $A_s$, can be found:

$$A_s = \rho bd = (0.020)(10\text{in})(11.625\text{in}) = 1.98 \text{ in}^2$$

The number of bars for this area can be found from handy charts.

(Whether the number of bars actually fit for the width with cover and space between bars must also be considered. If you are at $\rho_{\text{max}}-0.005$ do not choose an area bigger than the maximum!)

Try $A_s = 2.37 \text{ in}^2$ from 3#8 bars. (or 2.0 in$^2$ from 2 #9 bars. 4#7 bars don’t fit...)

$d$(actually) = 14 in. – 1.5 in (cover) – $\frac{1}{2}$ (8/8 in bar diameter) – 3/8 in. (stirrup diameter) = 11.625 in.

Check $\rho = 2.37 \text{ in}^2/(10 \text{in})(11.625 \text{in}) = 0.0203$ which is less than $\rho_{\text{max}}-0.005 = 0.0319$ OK (We cannot have an over reinforced beam!!)

Find the moment capacity of the beam as designed, $\phi M_n$

$$a = A_s f_y /0.85f'y = 2.37 \text{ in}^2 (40 \text{ ksi}) / (0.85(5 \text{ ksi})10 \text{ in}) = 2.23 \text{ in}$$

$$\phi M_n = \phi A_s f_y (d-a/2) = 0.9(2.37\text{in}^2)(40\text{ksi})(11.625\text{in} - \frac{2.23\text{in}}{2}) \cdot \left( \frac{1}{12/12} \right) = 74.7 \text{ k-ft} > 65.5 \text{ k-ft needed}$$

OK! Note: If the section doesn’t work, you need to increase $d$ or $A_s$ as long as you don’t exceed $\rho_{\text{max}}-0.005$
Example 5
A simply supported beam 25 ft long carries a service dead load of 2 k/ft, an estimated self weight of 500 lb/ft and a live load of 3 k/ft. Design an appropriate beam (for flexure only). Use grade 60 steel and concrete strength of 3000 psi.

SOLUTION:

Find the design moment, \( M_u \), from the factored load combination of 1.2D + 1.6L. If self weight is estimated, and the selected size has a larger self weight, the design moment must be adjusted for the extra load.

\[
w_u = 1.2(2 \text{ k/ft} + 0.5 \text{ k/ft}) + 1.6(3 \text{ k/ft}) = 7.8 \text{ k/ft}
\]

So, \( M_u = \frac{w_u l^2}{8} = \frac{7.8 \sqrt{25} \text{ ft}^2}{8} = 609.4 \text{ k-ft} \)

\( M_r \) required = \( \frac{M_u}{\phi} \)

To use the design chart aid, we can find \( R_n = \frac{M_n}{bd^2} \), and estimate that \( h \) is roughly 1.5-2 times the size of \( b \), and \( h = 1.1d \) (rule of thumb): \( d = \frac{h}{1.1} = \frac{(2b)}{1.1} \), so \( d \approx 1.8b \) or \( b \approx 0.55d \).

We can find \( R_n \) at the maximum reinforcement ratio for our materials off of the chart at about 700 psi with \( \rho_{\text{max-0.005}} = 0.0135 \). Let's substitute \( b \) for a function of \( d \):

\[
R_n = 700 \text{ psi} = \frac{677.1k-\beta(1000h/k)}{(0.55d/kd)^2} \cdot (12/\rho_p) \]

Rearranging and solving for \( d = 27.6 \) inches

That would make \( b \) 15.2 in. (from \( 0.55d \)). Let's try 15. So,

\[
h = d + 1.5 \text{ (clear cover)} + \frac{h}{2} (1\text{" diameter bar guess}) + \frac{3}{8} \text{ in (stirrup diameter)} = 27.6 + 2.375 = 29.975 \text{ in.}
\]

Choosing a depth of 30 inches, \( d \approx 30 - 1.5 \text{ (clear cover)} - \frac{h}{2} (1\text{" diameter bar guess}) - \frac{3}{8} \text{ in (stirrup diameter)} = 27.625 \text{ in.}

Now calculating an updated \( R_n = \frac{677.1 \text{ k-in}}{(15\text{in})(27.625\text{in})^2} \cdot (12/\rho_p) \approx 710 \text{ psi} \) This is larger than \( R_n \) for the 0.005 strain limit!

We can't just use \( \rho_{\text{max-0.005}} \). The way to reduce \( R_n \) is to increase \( b \) or \( d \) or both. Let's try increasing \( h \) to 31 in., then \( R_n = 661 \text{ psi} \) with \( d = 28.625 \text{ in.} \) That puts us under \( \rho_{\text{max-0.005}} \). We'd have to remember to keep UNDER the area of steel calculated, which is hard to do.

From the chart, \( \rho = 0.013 \), less than the \( \rho_{\text{max-0.005}} \) of 0.0135, so the estimated area required, \( A_s \), can be found:

\[ A_s = \rho bd = (0.013)(15\text{in})(29.625\text{in}) = 5.8 \text{ in}^2 \]

The number of bars for this area can be found from handy charts. Our charts say there can be 3 – 6 bars that fit when \( \frac{3}{4} \)" aggregate is used. We'll assume 1 inch spacing between bars. The actual limit is the maximum of 1 in, the bar diameter or 1.33 times the maximum aggregate size.

Try \( A_s = 6.0 \text{ in}^2 \) from 6#9 bars. Check the width: 15 – 3 (1.5 in cover each side) – 0.75 (two #3 stirrup legs) – 6"*1.128 – 51.128 in. = 1.16 in NOT OK.

Try \( A_s = 5.08 \text{ in}^2 \) from 4#10 bars. Check the width: 15 – 3 (1.5 in cover each side) – 0.75 (two #3 stirrup legs) – 4"*1.27 – 3"*1.27 in. = 2.36 OK.

\( d \)(actually) = 31 in. – 1.5 in (cover) – \( \frac{h}{2} \) (1.27 in bar diameter) – 3/8 in. (stirrup diameter) = 28.49 in.

Find the moment capacity of the beam as designed, \( \phi M_n \)

\[
a = A_d/0.85 f' = 5.08 \text{ in}^2 (60 \text{ ksi})/0.85(3 \text{ ksi})15 \text{ in} = 8.0 \text{ in}
\]

\[
\phi M_n = \phi A_d f_d (d/2) = 0.9(5.08\text{in}^2)(60\text{ksi})(2.849\text{in} - \frac{8.0\text{in}}{2}) \cdot (\frac{1}{12\sqrt{h}}) = 559.8 \text{ k-ft} < 609 \text{ k-ft needed!! (NO GOOD)}
\]

More steel isn’t likely to increase the capacity much unless we are close. It looks like we need more steel and lever arm. Try \( h = 32 \text{ in.} \) AND \( b = 16 \text{ in.} \), then \( M_n \) (with the added self weight of 33.3 lb/ft) = 680.2 k-ft, \( \rho \approx 0.012 \), \( A_s = 0.012(16\text{in})(29.42\text{in})=5.66 \text{ in}^2 \). 6#9’s won’t fit, but 4#11’s will: \( \rho = 0.0132 \sqrt{a} \), \( a = 9.18 \text{ in} \), and \( \phi M_n = 697.2 \text{ k-ft} \) which is finally larger than 680.2 k-ft OK
Example 6 (pg 437)

A T-section is to be used for a beam to resist positive moment. The following data are given: beam span is 18 ft (5.49 m), beams are 9 ft (2.74 m) center to center, slab thickness is 4 in. (0.102 m), beam stem dimensions are \( b_w = 15 \text{ in.} \) (0.381 m) and \( b = 22 \text{ in.} \) (0.559 m), \( f'_c = 4 \text{ksi} \) (27.6 MPa), \( f'_y = 60 \text{ksi} \) (414 MPa). Find the required area of steel and select the reinforcing bars for a dead load moment of 125 kip-ft (170 kN-m) plus a live load moment of 100 kip-ft (136 kN-m).
Example 7
Design a T-beam for a floor with a 4 in slab supported by 22-ft-span-length beams cast monolithically with the slab. The beams are 8 ft on center and have a web width of 12 in. and a total depth of 22 in.; \( f'_c = 3000 \text{ psi} \) and \( f_y = 60 \text{ ksi} \). Service loads are 125 psf and 200 psf dead load which does not include the weight of the floor system.

**SOLUTION:**

1. Establish the design moment:
   
   slab weight = \( \frac{96(4)}{144}(0.150) = 0.400 \text{ kip/ft} \)
   
   stem weight = \( \frac{12(18)}{144}(0.150) = 0.225 \text{ kip/ft} \)

   total = 0.625 kip/ft

   service DL = \( 8(0.200) = 1.60 \text{ kips/ft} \)

   service LL = \( 8(0.125) = 1.00 \text{ kips/ft} \)

2. Assume an effective depth \( d = h - 3 \text{ in.} \):
   
   \( d = 22 - 3 = 19 \text{ in.} \)

3. Determine the effective flange width:
   
   \( \frac{1}{4} \text{ span length} = 0.25(22)(12) = 66 \text{ in.} \)

   \( b_w + 16h_f = 12 + 16(4) = 76 \text{ in.} \)

   beam spacing = 96 in.

   Use an effective flange width \( b = 66 \text{ in.} \).

4. Determine whether the beam behaves as a true T-beam or as a rectangular beam by computing the practical moment strength \( \phi M_{nf} \) with the full effective flange assumed to be in compression. This assumes that the bottom of the compressive stress block coincides with the bottom of the flange, as shown in Figure 3-10. Thus:
   
   \[ \phi M_{nf} = \phi (0.85f'_c)bh_f \left( d - \frac{h_f}{2} \right) \]

   \[ = 0.9(0.85)(3)(66) \frac{(19 - 4/2)}{12} = 858 \text{ ft-kips} \]

5. Since 858 ft-kips \( > 258 \text{ ft-kips} \), the total effective flange need not be completely utilized in compression (i.e., \( a < h_f \)) and the T-beam behaves as a wide rectangular beam with a width \( b \) of 66 in.

6. Design as a rectangular beam with \( b \) and \( d \) as known values (see Section 2-15):

   required \( R_n = \frac{M_n}{\phi bd^2} = \frac{258(12)}{0.9(66)(19)^2} = 0.1444 \text{ ksi} \)

7. From Table A-8, select the required steel ratio to provide a \( R_n \) of 0.1444 ksi.

   \( \text{required} \rho = \frac{0.0024}{66} = 0.0024 \)

8. Calculate the required steel area:

   \( \text{required} A_s = \rho bd \)

   \( = 0.0024(66)(19) = 3.01 \text{ in.}^2 \)

9. Select the steel bars. Use 3#9 (\( A_s = 3.00 \text{ in.}^2 \))

   \( \text{minimum} b_w = 7.125 \text{ in.} \) (O.K.)

   Check the effective depth \( d: \)

   \( d = 22 - 1.5 - 0.38 - \frac{1.129}{2} = 19.56 \text{ in.} \)

   19.49 in. \( > 19 \text{ in.} \) (O.K.)

10. Check \( A_s,_{\text{min}} \) From Table A-5:

    \( A_{s,\text{min}} = 0.0033bd \)

    \( = 0.0033(12)(19) = 0.75 \text{ in.}^2 \)

    0.75 in. \( ^2 \) \( < 3.00 \text{ in.}^2 \)

11. Check \( A_s,_{\text{max}} \):

    \( A_{s,\text{max}} = 0.0135(66)(19) \)

    \( = 16.93 \text{ in.}^2 > 3.00 \text{ in.}^2 \) (O.K.)

12. Verify the moment capacity:

    (Is \( M_n \leq \phi M_{nf} \))

    \( a = (3.00)(60)/[0.85(3)(66)] = 1.07 \text{ in.} \)

   \( \phi M_n = 0.9(3.00)(60)(19.56 - \frac{1.07}{2})^{0.12} \)

   \( = 256.91 \text{ ft-kips} \) (Not O.K.)

    Choose more steel, \( A_s = 3.16 \text{ in.}^2 \) from 4-#8’s

    \( d = 19.62 \text{ in.}, a = 1.13 \text{ in} \)

   \( \phi M_n = 271.0 \text{ ft-kips}, \text{ which is OK} \)

13. Sketch the design...
Example 8

Design a T-beam for the floor system shown for which \( b_w \) and \( d \) are given. \( M_D = 200 \text{ ft-k} \), \( M_L = 425 \text{ ft-k} \), \( f'c = 3000 \text{ psi} \) and \( f_y = 60 \text{ ksi} \), and simple span = 18 ft.

**SOLUTION**

**Effective Flange Width**

(a) \( \frac{1}{4} \times 18' = 4.5'' = 54'' \)

(b) \( 15'' + (2)(8)(3) = 63'' \)

(c) \( 60'' = 72'' \)

**Moments Assuming \( \phi = 0.90 \)**

\[
M_u = \frac{920(12,000)}{0.90} = 1022 \text{ ft-k}
\]

First assume \( a \leq h_f \) (which is very often the case). Then the design would proceed like that of a rectangular beam with a width equal to the effective width of the T beam flange.

\[
M_u = \frac{920(12,000)}{0.90} = 1022 \text{ ft-k}
\]

Assuming \( \phi = 0.90 \)

\[
A_{df} = \frac{(0.85)(3)(54 - 15)(3)}{60} = 4.97 \text{ in}^2
\]

\[
M_{df} = \frac{(0.9)(4.97)(60)(24 - \frac{3}{2})}{0.85f_c} = 4039 \text{ in.-k} = 503 \text{ ft-k}
\]

Designing a rectangular beam with \( b_w = 15 \text{ in} \) and \( d = 24 \text{ in} \) to resist 417 k-ft

\[
M_{awr} = \frac{(12)(417)(1000)}{(0.9)(15)(24)^2} = 643.5
\]

\[
\rho_{aw} = 0.0126 \text{ from Appendix Table A.12}
\]

\[
A_{awr} = (0.0126)(15)(24) = 4.54 \text{ in}^2
\]

\[
A_s = 4.97 + 4.54 = 9.51 \text{ in}^2
\]

Check minimum reinforcing:

\[
A_{s \text{ min}} = \frac{3\sqrt{3000(15)(24)}}{60,000} = 0.986 \text{ in}^2
\]

but not less than

\[
A_{s \text{ min}} = \frac{200b_fd}{60,000} = 1.2 \text{ in}^2
\]

Only 2 rows fit, so try 8-\#10 bars, \( A_s = 10.16 \text{ in}^2 \) for equilibrium:

\[
T = C_w + C_f
\]

\[
T = Af_f + (10.16)(60) = 609.6 \text{ k}
\]

\[
C_f = 0.85f'(b-b_w)h_f \text{ and } C_w = T - C_f
\]

\[
C_w = T - C_f = 609.6 - (0.85)(3)(54-15)(3) = 311.25 \text{ k}
\]

\[
a = \frac{311.25/(0.85*3*15)}{8.14} = 8.14 \text{ in}
\]

Check strain (\( \epsilon_t \)) and \( \phi \):

\[
e_t = \frac{d - c}{c} (0.003) = \frac{24 - 9.58}{9.58} (0.003) = 0.0045 > 0.005!
\]

We could try 10-\#9 bars at 10 in\(^2\), \( T = 600 \text{ k}, C_w = 301.65 \text{ k}, \ a = 7.89, \ \phi = 0.906; \ \phi = 0.9! \)

Finally check the capacity:

\[
M_c = C_s (d - \frac{a}{2}) + C_f (d - \frac{h_f}{2})
\]

\[
= [301.65(24-7.89/2) + 298.35(24-3/2)]1\text{ft/12in}\]

\[
= 1063.5 \text{ k-ft}
\]

So: \( \phi M_c = 0.9(1063.5) = 957.2 \text{ k-ft} \geq 920 \text{ k-ft} \) (OK)

---

*Figure 5.7 Separation of T beam into rectangular parts.*

---

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Example 9 (pg 448)

Example 7. A one-way solid concrete slab is to be used for a simple span of 14 ft [4.27 m]. In addition to its own weight, the slab carries a superimposed dead load of 30 psf [1.44 kPa] plus a live load of 100 psf [4.79 kPa]. Using \( f'_c = 3 \text{ ksi} [20.7 \text{ MPa}] \) and \( f_y = 40 \text{ ksi} [276 \text{ MPa}] \), design the slab for minimum overall thickness.

<table>
<thead>
<tr>
<th>Bar Spacing (in.)</th>
<th>No. 3</th>
<th>No. 4</th>
<th>No. 5</th>
<th>No. 6</th>
<th>No. 7</th>
<th>No. 8</th>
<th>No. 9</th>
<th>No. 10</th>
<th>No. 11</th>
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<td>1.76</td>
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<td>2.71</td>
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<td>0.39</td>
<td>0.50</td>
<td>0.63</td>
<td>0.78</td>
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</table>
Example 10

e 2-9

Design a simple-span one-way slab to carry a uniformly distributed live load of 400 psi. The span is 10 ft (center to center of supports). Use $f'_c = 4000$ psi and $f_y = 60,000$ psi. Select the thickness to be not less than the ACI minimum thickness requirement.

Solution:

Determine the required minimum $h$ and use this to estimate the slab dead weight.

1. From ACI Table 9.5(a), for a simply supported, solid, one-way slab,

   $\text{minimum } h = \frac{\ell}{20} = \frac{10(12)}{20} = 6.0 \text{ in.}$

   Try $h = 6 \text{ in.}$ and design a 12-in.-wide segment.

2. Determine the slab weight dead load:

   $w = \frac{6(12)}{144} = 0.150 \text{ kip/ft}$

   The total design load is

   $w_t = 1.2w_{DL} + 1.6w_{LL} = 1.2(0.075) + 1.6(0.400) = 0.730 \text{ kip/ft}$

3. Determine the design moment:

   $M_n = \frac{w_t e^2}{8} = \frac{0.73(10)^2}{8} = 9.125 \text{ ft-kips}$

4. Establish the approximate $d$. Assuming No. 6 bars and minimum concrete cover on the bars of $\frac{3}{4} \text{ in.}$,

   assumed $d = 6.0 - 0.75 - 0.375 = 4.88 \text{ in.}$

5. Determine the required $R_n$:

   required $R_n = \frac{M_n}{\phi bd^2}$

   $= \frac{9.125(12)}{0.9(12)(4.88)^2} = 0.4257 \text{ ksi}$

6. From Table A-10, for a required $R_n = 0.4257$, the required $\rho = 0.0077$. (Note that the required $\rho$ selected is the next higher value from Table A-10.) Thus

   $\rho_{\text{max}} = 0.0181 > 0.0077$ (O.K.)

   Use $\rho = 0.0077$.

7. required $A_s = \rho bd = 0.0077(12)(4.88) = 0.45 \text{ in.}^2/\text{ft}$

8. Select the main steel (from Table A-4). Select No. 5 bars at $7\frac{1}{2} \text{ in. o.c.}$ ($A_s = 0.50 \text{ in.}^2$). The assumption on bar size was satisfactory. The code requirements for maximum spacing have been discussed in Section 2-13. Minimum spacing of bars in slabs, practically, should not be less than 4 in. although the ACI Code allows bars to be placed closer together, as discussed in Example 2-7. Check the maximum spacing (ACI Code, Section 7.6.5):

   maximum spacing $= 3h$ or 18 in.

   $3h = 3(6) = 18 \text{ in.}$

   $7\frac{1}{2} \text{ in.} < 18 \text{ in.}$ (O.K.)

Therefore use No. 5 bars at $7\frac{1}{2} \text{ in. o.c.}$

9. Select shrinkage and temperature reinforcement (ACI Code, Section 7.12):

   required $A_s = 0.0018bh$

   $= 0.0018(12)(6) = 0.13 \text{ in.}^2/\text{ft}$

   Select No. 3 bars at 10 in. o.c. ($A_s = 0.13 \text{ in.}^2$) or No. 4 bars at 18 in. o.c. ($A_s = 0.13 \text{ in.}^2$):

   maximum spacing $= 5h$ or 18 in.

   Use No. 3 bars at 10 in. o.c.

10. The main steel area must exceed the area required for shrinkage and temperature steel (ACI Code, Section 10.5.4):

    $0.50 \text{ in.}^2 > 0.13 \text{ in.}^2$ (O.K.)

11. Verify the moment capacity:

    $\phi M_n \leq \phi M_n$

    $a = \frac{(0.50)(60)}{0.85(4)(12)} = 0.74 \text{ in.}$

    $\phi M_n = 0.9(0.50)(60)(5.0625)(0.74)^{1/2} \approx 10.6 \text{ ft-kips}$ (O.K)

12. A design sketch is drawn:
Example 11 (pg 461)

Example 8. Design the required shear reinforcement for the simple beam shown in Figure 13.18. Use $f'_c = 3$ ksi [20.7 MPa] and $f_s = 40$ ksi [276 MPa] and single U-shaped stirrups.
Example 12
For the simply supported concrete beam shown in Figure 5-61, determine the stirrup spacing (if required) using No. 3 U stirrups of Grade 60 (\(f_y = 60\) ksi). Assume \(f_{c'} = 3000\) psi.

\[ f_{c'} = 3000\ \text{psi.} \]
\[ F_y = 60\ \text{ksi.} \]

For #3 bars, \(A_v = 0.11\) in.\(^2\), with 2 legs, then \(A_v = 0.22\) in.\(^2\).

Solution:

\[ V_u = 50\ \text{kips} \quad \text{(neglecting weight of the beam)} \]

\[
\phi V_c = \phi V_{c'} + \phi V_s
\]
\[
= 0.75(12)\sqrt{3000} \left(\frac{32.5}{1000}\right) = 32.0\ \text{kips} < V_u \quad \therefore \text{Need Stirrups}
\]

Note: If \(V_u = \frac{1}{2} \phi V_c\), minimum stirrups would still be required.

\[ V_u \leq \phi V_c + \phi V_s \]
\[ \therefore \phi V_s = V_u - \phi V_c = 50 - 32.0 = 18.0\ \text{kips} \quad (\phi 4\sqrt{f_{c'} b_w d = 64.1\ \text{kips}})
\]

\[ S_{\text{req'd}} \leq \frac{\phi A_v F_y d}{\phi V_{c'}} = \frac{(0.75)(0.22)(60)(32.5)}{18.0} = 17.875\ \text{in.}
\]

\[ s_{\text{max}} = \frac{d}{2} = \frac{32.5}{2} = 16.2\ \text{in.} \quad \text{controls}
\]
\[ = 24\ \text{in.}
\]

\[ S_{\text{req'd}} \leq \frac{A_v F_y}{50 b_w} = \frac{0.22(60,000)}{50(12)} = 22.0\ \text{in.} \quad \text{but 16" (d/2) would be the maximum as well.}
\]

\[ \therefore \text{Use #3 U @ 16" max spacing} \]

\[ P_u = 100\ \text{kips.} \]
Example 13
Design the shear reinforcement for the simply supported reinforced concrete beam shown with a dead load of 1.5 k/ft and a live load of 2.0 k/ft. Use 5000 psi concrete and Grade 60 steel. Assume that the point of reaction is at the end of the beam.

SOLUTION:

Shear diagram:
Find self weight = 1 ft x (27/12 ft) x 150 lb/ft³ = 338 lb/ft = 0.338 k/ft
\( w_c = 1.2 \) (1.5 k/ft + 0.338 k/ft) + 1.6 (2 k/ft) = 5.41 k/ft (= 0.451 k/in)
\( V_u(\text{max}) \) is at the ends = \( w_c L/2 = 5.41 \text{ k/ft} \times 24 \text{ ft} / 2 = 64.9 \text{ k} \)
\( V_u(\text{support}) \) = \( V_u(\text{max}) - w_c \text{ (distance)} = 64.9 \text{ k} - 5.41 \text{ k/ft} \times (6/12 \text{ ft}) = 62.2 \text{ k} \)
\( V_u \) for design is \( d \) away from the support = \( V_u(\text{support}) - w_c(d) = 62.2 \text{ k} - 5.41 \text{ k/ft} \times (23.5/12 \text{ ft}) = 51.6 \text{ k} \)

Concrete capacity:
We need to see if the concrete needs stirrups for strength or by requirement because \( V_u \leq \phi V_c + \phi V_s \) (design requirement)
\( \phi V_c = \phi \lambda \sqrt{\frac{f_c}{F_c}} b_d = 0.75(2)(1.0)\sqrt{5000} \text{ psi (12 in) (23.5 in)} = 299106 \text{ lb} = 29.9 \text{ kips} \ (< 51.6 \text{ k}) \)

Stirrup design and spacing
We need stirrups: \( A_v = V_u/s/f_d \)
\( \phi V_s \geq V_u - \phi V_c = 51.6 \text{ k} - 29.9 \text{ k} = 21.7 \text{ k} \)

Spacing requirements are in Table 3-8 and depend on \( \phi V_c/2 = 15.0 \text{ k} \) and \( 2\phi V_c = 59.8 \text{ k} \)

| \( 2 \text{ legs for a #3 is} 0.22 \text{ in}^2, \text{ so } s_{\text{req}} \leq \phi A_vf_d/\phi V_s = 0.75(0.22 \text{ in}^2)(60 \text{ ksi})(23.5 \text{ in})/21.7 \text{ k} = 10.72 \text{ in} \) Use \( s = 10'' \) |
| \( \text{our maximum falls into the d/2 or 24''}, \text{ so d/2 governs with 11.75 in} \text{ Our 10'' is ok.} |
| \( \text{This spacing is valid until} V_u = \phi V_c \text{ and that happens at (64.9 k - 29.9 k)/0.451 k/in} = 78 \text{ in} \)

We can put the first stirrup at a minimum of 2 in from the support face, so we need 10" spaces for (78 - 2 - 6 in)/10 in = 7 even (8 stirrups altogether ending at 78 in)

After 78" we can change the spacing to the required (but not more than the maximum of d/2 = 11.75 in ≤ 24in);
\( s = A_{v,\text{req}} / (50b_w) = 0.22 \text{ in}^2 \times (60,000 \text{ psi})/50 \times (12 \text{ in}) = 22 \text{ in} \leq A_vf_d/0.75{\sqrt{f_c}} b_w = 0.22 \text{ in}^2 \times (60,000 \text{ psi})/0.75{\sqrt{5000} \text{ psi (12 in)}} = 20.74 \text{ in} \)

We need to continue to 111 in, so (111 - 78 in)/11 in = 3 even
Example 14 (pg 483)

Example 1. A solid one-way slab is to be used for a framing system similar to that shown in Figure 14.1. Column spacing is 30 ft. with evenly spaced beams occurring at 10 ft. center to center. Superimposed loads on the structure (floor live load plus other construction dead load) are a dead load of 38 psf [1.82 kPa] and a live load of 100 psf [4.79 kPa]. Use $f'_{c} = 3$ ksi [20.7 MPa] and $f_y = 40$ ksi [275 MPa]. Determine the thickness for the slab and select its reinforcement.

$$a_{\min} = 0.12 \text{ in}^2/\text{ft}$$

No. 3 at 11 temperature reinforcement

No. 3 at 9
No. 3 at 11
No. 3 at 9
No. 3 at 8
No. 3 at 8
No. 3 at 8
No. 3 at 8
No. 3 at 8
Example 15

Example 6-1

The floor system shown in Figure 6-4 consists of a continuous one-way slab supported by continuous beams. The service loads on the floor are 25 psf dead load (does not include weight of slab) and 250 psf live load. Use $f'_{c} = 3000$ psi (normal-weight concrete) and $f_{y} = 60,000$ psi. The bars are uncoated.

Design the continuous one-way floor slab.

Solution:

The primary difference in this design from previous flexural designs is that, because of continuity, the ACI coefficients and equations will be used to determine design shears and moments.

A. Continuous one-way floor slab

1. Determine the slab thickness. The slab will be designed to satisfy the ACI minimum thickness requirements from Table 9.5(a) of the code and this thickness will be used to estimate slab weight.

   With both ends continuous,
   \[ h_{min} = \frac{1}{28} \ell_{n} = \frac{1}{28} (11)(12) = 4.71 \text{ in.} \]

   With one end continuous,
   \[ h_{min} = \frac{1}{24} \ell_{n} = \frac{1}{24} (11)(12) = 5.5 \text{ in.} \]

   Try a $5\frac{1}{2}$-in.-thick slab. Design a 12-in.-wide segment ($b = 12$ in.).

2. Determine the load:

   slab dead load = \( \frac{5.5}{12} \times 150 = 68.8 \) psf

   total dead load = 25.0 + 68.8 = 93.8 psf

   \[ w_{u} = 1.2 w_{DL} + 1.6 w_{LL} \]
   \[ = 1.2(93.8) + 1.6(250) \]
   \[ = 112.6 + 400.0 \]
   \[ = 512.6 \text{ psf (design load)} \]

   Because we are designing a slab that is 12 in. wide, the foregoing loading is the same as 512.6 lb/ft or 0.513 kip/ft.

3. Determine the moments and shears. Moments are determined using the ACI moment equations. Refer to Figures 6-1 and 6-4. Thus

   \[ +M_{n} = \frac{1}{14} w_{u} \ell_{n}^{2} = \frac{1}{14} (0.513)(11)^{2} = 4.43 \text{ ft-kips (end span)} \]

   \[ +M_{u} = \frac{1}{16} w_{u} \ell_{n}^{2} = \frac{1}{16} (0.513)(11)^{2} = 3.88 \text{ ft-kips (interior span)} \]

   \[ -M_{n} = \frac{1}{10} w_{u} \ell_{n}^{2} = \frac{1}{10} (0.513)(11)^{2} = 6.20 \text{ ft-kips (end span - first interior support)} \]

   \[ -M_{u} = \frac{1}{11} w_{u} \ell_{n}^{2} = \frac{1}{11} (0.513)(11)^{2} = 5.64 \text{ ft-kips (interior span - both supports)} \]

   \[ -M_{n} = \frac{1}{24} w_{u} \ell_{n}^{2} = \frac{1}{24} (0.513)(11)^{2} = 2.58 \text{ ft-kips (end span - exterior support)} \]
Example 15 (continued)

Similarly, the shears are determined using the ACI shear equations. In the end span at the face of the first interior support,

\[ V_u = 1.15 \frac{w_e L_n}{2} = 1.15(0.513) \left( \frac{11}{2} \right) = 3.24 \text{kips} \quad \text{(end span – first interior support)} \]

whereas at all other supports,

\[ V_u = \frac{w_n L_n}{2} = (0.513) \left( \frac{11}{2} \right) = 2.82 \text{kips} \]

4. Design the slab. Assume #4 bars for main steel with 3/4 in. cover: \( d = 5.5 - 0.75 - \frac{1}{2}(0.5) = 4.5 \text{ in.} \)

5. Design the steel. (All moments must be considered.) For example, the negative moment in the end span at the first interior support:

\[ R_n = \frac{M_u}{\phi bd^2} = \frac{6.20(12)(1000)}{0.9(12)(4.5)^2} = 340 \text{ft-kips} \quad \text{so } \rho \geq 0.006 \]

\[ A_s = \rho bd = 0.006(12)(4.5) = 0.325 \text{ in}^2 \text{ per ft. width of slab} \quad \therefore \text{Use #4 at 7 in. (16.5 in. max. spacing)} \]

The minimum reinforcement required for flexure is the same as the shrinkage and temperature steel.

(Verify the moment capacity is achieved: \( a = 0.67 \text{ in. and } \phi M_n = 6.38 \text{ ft-kips} > 6.20 \text{ ft-kips} \))

For grade 60 the minimum for shrinkage and temperature steel is:

\[ A_{s,min} = 0.0018bt = 0.0018(12)(5.5) = 0.12 \text{ in}^2 \text{ per ft. width of slab} \quad \therefore \text{Use #3 at 11 in. (18 in. max spacing)} \]

6. Check the shear strength. (\( \lambda = 1 \) for normal weight material)

\[ \phi V_c = \phi 2\lambda \sqrt{f'_c bd} = 0.75(2) \sqrt{3000(12)(4.5)} = 4436.6lb = 4.44 \text{kips} \]

\[ V_u \leq \phi V_c \quad \therefore \text{Therefore the thickness is O.K.} \]

7. Development length for the flexure reinforcement is required. (Hooks are required at the spandrel beam.) For example, #6 bars:

\[ l_d = \frac{d_F}{25\lambda \sqrt{f'_c}} \quad \text{or 12 in. minimum} \]

With grade 40 steel and 3000 psi concrete:

\[ l_d = \frac{\gamma in(40,000 \text{psi})}{25(1)\sqrt{3000 \text{psi}}} = 21.9 \text{in} \]

(which is larger than 12 in.)

8. Sketch:
Example 16
A building is supported on a grid of columns that is spaced at 30 ft on center in both the north-south and east-west directions. Hollow core planks with a 2 in. topping span 30 ft in the east-west direction and are supported on precast L and inverted T beams. Size the hollow core planks assuming a live load of 100 lb/ft². Choose the shallowest plank with the least reinforcement that will span the 30 ft while supporting the live load.

SOLUTION:
The shallowest that works is an 8 in. deep hollow core plank.
The one with the least reinforcing has a strand pattern of 68-S, which contains 6 strands of diameter 8/16 in. = ½ in. The S indicates that the strands are straight. The plank supports a superimposed service load of 124 lb/ft² at a span of 30 ft with an estimated camber at erection of 0.8 in. and an estimated long-time camber of 0.2 in.

The weight of the plank is 81 lb/ft².

3.6 Hollow-Core Load Tables (cont.)

<table>
<thead>
<tr>
<th>Strand Pattern Designation</th>
<th>Section Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-0” x 8”</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>No Topping</th>
<th>2 in. topping</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 215 in.²</td>
<td></td>
</tr>
<tr>
<td>i = 1666 in.²</td>
<td>3071 in.²</td>
</tr>
<tr>
<td>yₕ = 4.00 in.</td>
<td>5.29 in.</td>
</tr>
<tr>
<td>yₜ = 4.00 in.</td>
<td>4.71 in.</td>
</tr>
<tr>
<td>Sₕ = 417 in.³</td>
<td>581 in.³</td>
</tr>
<tr>
<td>Sₜ = 417 in.³</td>
<td>652 in.³</td>
</tr>
<tr>
<td>wr = 224 lb/ft²</td>
<td>324 lb/ft²</td>
</tr>
<tr>
<td>DL = 56 lb/ft²</td>
<td>81 lb/ft²</td>
</tr>
<tr>
<td>V/S = 1.92 in.</td>
<td></td>
</tr>
</tbody>
</table>

**Key**
38S = Safe superimposed service load, lb/ft²
0.1 = Estimated camber at erection, in.
0.2 = Estimated long-time camber, in.

Table of safe superimposed service load, lb/ft², and cambers, in.

| Strand designation code | Span, ft | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
|------------------------|---------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 66-S                   |         | 400 | 365 | 333 | 306 | 292 | 256 | 224 | 197 | 173 | 153 | 135 | 119 | 105 | 93  | 82  | 68  | 56  | 45  | 36  | 25  |     |     |     |     |     |     |
| 76-S                   |         | 474 | 456 | 436 | 401 | 360 | 297 | 236 | 184 | 144 | 130 | 116 | 103 | 88  | 74  | 62  | 51  | 41  | 31  |     |     |     |     |     |     |     |     |     |
| 58-S                   |         | 445 | 426 | 402 | 373 | 346 | 288 | 225 | 170 | 135 | 102 | 83  | 65  | 56  | 45  | 36  | 25  |     |     |     |     |     |     |     |     |     |     |     |
| 68-S                   |         | 483 | 462 | 435 | 395 | 366 | 342 | 319 | 299 | 262 | 215 | 195 | 177 | 158 | 140 | 124 | 110 | 97  | 84  | 73  | 62  | 53  | 44  | 36  | 28  |     |     |
| 78-S                   |         | 472 | 435 | 402 | 375 | 348 | 325 | 305 | 289 | 273 | 257 | 245 | 232 | 220 | 207 | 186 | 167 | 149 | 133 | 119 | 106 | 94  | 83  | 73  | 64  | 55  | 46  | 38  |

Strength is based on strain compatibility; bottom tension is limited to 7.5√T; see pages 3–8 through 3–11 for explanation.
See item 3, note 4, Section 3.3.2 for explanation of vertical line.
Example 17 (pg 510)

Example 1. A square tied column with $f'_c = 5$ ksi and steel with $f_y = 60$ ksi sustains an axial compression load of 150 kips dead load and 250 kips live load with no computed bending moment. Find the minimum practical column size if reinforcing is a maximum of 4% and the maximum size if reinforcing is a minimum of 1%. Also, design for $e = 6$ in.
Example 18
Determine the capacity of a 16” x 16” column with 8 #10 bars, tied. Grade 40 steel and 4000 psi concrete.

SOLUTION:

Find $\phi P_n$ with $\phi = 0.65$ and $P_n = 0.80 P_o$ for tied columns and

$$ P_n = 0.85 f'_c (A_g - A_{st}) + f_s A_{st} $$

Steel area (found from reinforcing bar table for the bar size):

$$ A_{st} = 8 \text{ bars} \times (1.27 \text{ in}^2) = 10.16 \text{ in}^2 $$

Concrete area (gross):

$$ A_g = 16 \text{ in} \times 16 \text{ in} = 256 \text{ in}^2 $$

Grade 40 reinforcement has $f_y = 40,000$ psi and $f'_c = 4000$ psi

$$ \phi P_n = (0.65)(0.80)[0.85(4000 \text{ psi})(256 \text{ in}^2 - 10.16 \text{ in}^2) + (40,000 \text{ psi})(10.16 \text{ in}^2)] = 646,026 \text{ lb} = 646 \text{ kips} $$

Example 19
16” x 16” precast reinforced columns support inverted T girders on corbels as shown. The unfactored loads on the corbel are 81 k dead, and 72 k live. The unfactored loads on the column are 170 k dead and 150 k live. Determine the reinforcement required using the interaction diagram provided. Assume that half the moment is resisted by the column above the corbel and the other half is resisted by the column below. Use grade 60 steel and 5000 psi concrete.
Example 20

**EXAMPLE 5-4**

Design a short square tied column to carry an axial dead load of 300 kip and a live load of 200 kip. Assume that the applied moments on the column are negligible. Use $f'_c = 4,000 \text{ psi}$ and $f' = 60,000 \text{ psi}$.

**Solution**

Step 1 The factored load, $P_u$, is:

$$P_u = 1.2P_D + 1.6P_L$$
$$P_u = 1.2(300) + 1.6(200)$$
$$P_u = 680 \text{ kip}$$

Assume $\rho_g = 0.03$.

Step 2 The required area of the column, $A_g$, is:

$$A_g = \frac{P_u}{0.8\phi(0.85f'_c(1 - \rho_g) + f'\rho_g)}$$
$$A_g = \frac{680}{0.80(0.65)(0.85(4)(1 - 0.03) + 60(0.03))}$$
$$A_g = 257 \text{ in}^2$$

Step 3 For a square column, the size, $h$, is:

$$h = \sqrt{A_g} = \sqrt{257}$$
$$\therefore h = 16.0 \text{ in.}$$

Try a 16 in. $\times$ 16 in. column:

$$A_g = (16)(16) = 256 \text{ in}^2$$

Step 4 The required amount of steel, $A_{st}$, is:

$$A_{st} = \frac{P_u - 0.8\phi(0.85f'_c A_g)}{0.8\phi(f' - 0.85f'_c)}$$
$$A_{st} = \frac{680 - 0.8 \times 0.65(0.85 \times 4 \times 256)}{0.8 \times 0.65(60 - 0.85 \times 4)} = 7.73 \text{ in}^2$$

Step 5 Select the size and number of bars. For a square column with bars uniformly distributed along the edges, we keep the number of bars as multiples of four. Using Table A2-9, 8 #9 bars ($A_s = 8 \text{ in}^2$) are selected.

From Table A5-1 — Maximum of 12 #9 bars  — ok

Step 6 Because the longitudinal bars are #9, select #3 bars for the ties. The maximum spacing of the ties ($s_{max}$) is:

$$s_{max} = \text{min}(16d_h, 48d, d_{\min})$$
$$s_{max} = \text{min}(16(1.128), 48(1), 16)$$
$$s_{max} = \text{min}(18.0, 18.0, 16.0)$$
$$\therefore s_{max} = 16 \text{ in.}$$

The selected ties are #3 @ 16 in.
Example 21

Design a 10 ft long circular spiral column for a braced system to support the service dead and live loads of 300 k and 460 k, respectively, and the service dead and live moments of 100 ft-k each. The moment at one end is zero. Use $f'_c = 4,000$ psi and $f_y = 60,000$ psi.

Solution

1. $P_y = 1.2(300) + 1.6(460) = 1096 k$
   $M_y = 1.2(100) + 1.6(100) = 280 ft-k$

2. Assume $p_y = 0.01$, from Equation 16.10:

   
   \[
   A_y = \frac{P_y}{0.60[0.85/(1-p_y) + f_y]} \\
   = \frac{1096}{0.60[0.85(4)(1-0.01) + 600(0.01)]} \\
   = 460.58 \text{ in.}^2
   \]

   \[
   \frac{\pi h^2}{4} = 460.58
   \]

   or $h = 24.22 \text{ in.}$

   Use $h = 24 \text{ in.}, A_y = 452 \text{ in.}^2$

3. Assume #9 size of bar and 3/8 in. spiral center-to-center distance

   \[
   = 24 - 2(\text{cover}) - 2(\text{spiral diameter}) - 1 \text{ (bar diameter)}
   = 24 - 2(1.5) - 2(3/8) - 1.128 = 19.12 \text{ in.}
   \]

   \[
   \gamma = \frac{19.12}{24} = 0.8
   \]

   Use the interaction diagram Appendix D.21

4. $K_y = \frac{P_y}{\phi f_c' A_y} = \frac{1096}{(0.75)(4)(452)} = 0.808$

5. Assume $\phi = 0.75$, $M_y = 0.75 \phi A_y h$

5. At the intersection point of $K_y$ and $K_y$, $p_y = 0.02$

6. The point is above the strain line $= 1$, hence $\phi = 0.75$ \text{ OK}

7. $A_y = (0.02)(452) = 9.04 \text{ in.}^2$
   
   From Appendix D.2, select 12 bars of #8, $A_y = 9.48 \text{ in.}^2$
   
   From Appendix D.14 for a core diameter of 24 - 3 = 21 in. 17 bars of #8 can be arranged in a row

8. Selection of spirals
   
   From Appendix D.13, size = 3/8 in.
   
   pitch = 2\frac{1}{8} in.
   
   Clear distance = 2.25 - 3/8 = 1.875 > 1 in. \text{ OK}

9. $K = 1, l = 10 \times 12 = 120 \text{ in.}, r = 0.25(24) = 6 \text{ in.}$

   \[
   \frac{K l}{r} = \frac{1120}{6} = 20
   \]

   \[
   \left( \frac{M_1}{M_2} \right) = 0
   \]

   $34 - 12 \left( \frac{M_1}{M_2} \right) = 34$

   \[
   \text{ACI 6.2: In nonsway frames it shall be permitted to ignore slenderness effects for compression members that satisfy: } \frac{K l}{r} \leq 34 - 12 \left( \frac{M_1}{M_2} \right)
   \]

   since $(K l/r) < 34$, short column.
Column Interaction Diagrams

Figure D.15: Column interaction diagram for tied column with bars on end faces only. (Courtesy of the American Concrete Institute, Farmington Hills, MI)

Figure D.16: Column interaction diagram for tied column with bars on end faces only. (Courtesy of the American Concrete Institute, Farmington Hills, MI)

Figure D.17: Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI)

Figure D.18: Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI)
FIGURE D.19 Column interaction diagram for tied column with bars on all faces. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

FIGURE D.21 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

FIGURE D.20 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

FIGURE D.22 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)
Beam / One-Way Slab Design Flow Chart

1. Collect data: L, α, γ, Δlimt, hmin; find beam charts for load cases and Δactual equations (self weight = area x density)

2. Collect data: load factors, f_y, f_c

3. Find V_u & M_u from constructing diagrams or using beam chart formulas with the factored loads (V_u-max is at d away from face of support)

4. Assume b & d (based on h_min for slabs)
   - Determine M_n required, choose method
   - Chart (R_n vs ρ)
   - Select ρ_min ≤ ρ ≤ ρ_max
   - Find R_n off chart with f_y, f_c and select ρ_min ≤ ρ ≤ ρ_max
   - Choose b & d combination based on R_n and h_min (slabs), estimate h with 1" bars (#8)

5. Calculate A_s = ρbd
   - Select bar size and spacing to fit width or 12 in strip of slab and not exceed limits for crack control

6. Increase h, find d*

7. Find new d / adjust h; Is ρ_min ≤ ρ ≤ ρ_max?
   - YES
   - Calculate a, ϕM_n
   - Is M_u ≤ ϕM_n?
     - YES (on to shear reinforcement for beams)
     - NO
   - NO

8. Increase h, find d

9. YES
Beam / One-Way Slab Design Flow Chart - continued

Beam, Adequate for Flexure

Determine shear capacity of plain concrete based on $f'c$, b & d, $\phi Vc$

Is $V_u$ (at d for beams) $\leq \phi Vc$? NO

Beam? NO

Determine $\phi V_s = (V_u - \phi Vc)$

Is $\phi V_s \leq \phi \sqrt{f'c b_s d}$? NO

Increase h and re-evaluate flexure ($A_s$ and $\phi M_n$ of previous page)*

NO

Beam? NO

Is $V_u < 1/2 \phi Vc$? NO

YES

Determine $s$ & $A_u$

Find where $V = \phi Vc$ and provide minimum $A_s$ and change $s$

Find where $V = 1/2 \phi Vc$ and provide stirrups just past that point

Yes (DONE)

YES

Find where $V = \phi Vc$ and provide minimum $A_s$ and change $s$

Find where $V = 1/2 \phi Vc$ and provide stirrups just past that point

Yes (DONE)
APPENDIX E — STEEL REINFORCEMENT INFORMATION

As an aid to users of the ACI Building Code, information on sizes, areas, and weights of various steel reinforcement is presented.

### ASTM STANDARD REINFORCING BARS

<table>
<thead>
<tr>
<th>Bar size, no.</th>
<th>Nominal diameter, in.</th>
<th>Nominal area, in.(^2)</th>
<th>Nominal weight, lb/ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.375</td>
<td>0.11</td>
<td>0.376</td>
</tr>
<tr>
<td>4</td>
<td>0.500</td>
<td>0.20</td>
<td>0.668</td>
</tr>
<tr>
<td>5</td>
<td>0.625</td>
<td>0.31</td>
<td>1.043</td>
</tr>
<tr>
<td>6</td>
<td>0.750</td>
<td>0.44</td>
<td>1.502</td>
</tr>
<tr>
<td>7</td>
<td>0.875</td>
<td>0.60</td>
<td>2.044</td>
</tr>
<tr>
<td>8</td>
<td>1.000</td>
<td>0.79</td>
<td>2.670</td>
</tr>
<tr>
<td>9</td>
<td>1.128</td>
<td>1.00</td>
<td>3.400</td>
</tr>
<tr>
<td>10</td>
<td>1.270</td>
<td>1.27</td>
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</tr>
<tr>
<td>11</td>
<td>1.410</td>
<td>1.56</td>
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<tr>
<td>14</td>
<td>1.693</td>
<td>2.25</td>
<td>7.650</td>
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<tr>
<td>18</td>
<td>2.257</td>
<td>4.00</td>
<td>13.600</td>
</tr>
</tbody>
</table>

### ASTM STANDARD PRESTRESSING TENDONS

<table>
<thead>
<tr>
<th>Type*</th>
<th>Nominal diameter, in.</th>
<th>Nominal area, in.(^2)</th>
<th>Nominal weight, lb/ft</th>
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</thead>
<tbody>
<tr>
<td>1/4 (0.250)</td>
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<td>0.036</td>
<td>0.122</td>
</tr>
<tr>
<td>5/16 (0.313)</td>
<td></td>
<td>0.058</td>
<td>0.197</td>
</tr>
<tr>
<td>3/8 (0.375)</td>
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<td>0.080</td>
<td>0.272</td>
</tr>
<tr>
<td>7/16 (0.438)</td>
<td></td>
<td>0.108</td>
<td>0.367</td>
</tr>
<tr>
<td>1/2 (0.500)</td>
<td></td>
<td>0.144</td>
<td>0.490</td>
</tr>
<tr>
<td>(0.600)</td>
<td></td>
<td>0.216</td>
<td>0.737</td>
</tr>
<tr>
<td>3/8 (0.375) Seven-wire strand (Grade 250)</td>
<td></td>
<td>0.085</td>
<td>0.290</td>
</tr>
<tr>
<td>7/16 (0.438)</td>
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<td>0.115</td>
<td>0.390</td>
</tr>
<tr>
<td>1/2 (0.500)</td>
<td></td>
<td>0.153</td>
<td>0.520</td>
</tr>
<tr>
<td>(0.600)</td>
<td></td>
<td>0.217</td>
<td>0.740</td>
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<tr>
<td>Prestressing wire</td>
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<td>0.029</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td>0.196</td>
<td>0.030</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>0.250</td>
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<td>0.170</td>
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<td>0.060</td>
<td>0.200</td>
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</tr>
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<td>7/8</td>
<td>0.60</td>
<td>2.04</td>
</tr>
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<td></td>
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<td>0.78</td>
<td>2.67</td>
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<td></td>
<td>1-1/8</td>
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<td>3.38</td>
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<td>4.17</td>
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<tr>
<td></td>
<td>1-3/8</td>
<td>1.48</td>
<td>5.05</td>
</tr>
<tr>
<td>Prestressing bars (deformed)</td>
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<td>0.98</td>
</tr>
<tr>
<td></td>
<td>3/4</td>
<td>0.42</td>
<td>1.49</td>
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<td></td>
<td>1</td>
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<tr>
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<td>1-3/8</td>
<td>1.58</td>
<td>5.56</td>
</tr>
</tbody>
</table>

* Availability of some tendon sizes should be investigated in advance.

ACI 318 Building Code and Commentary
# APPENDIX E

## ASTM STANDARD WIRE REINFORCEMENT

<table>
<thead>
<tr>
<th>W &amp; D size</th>
<th>Nominal diameter, in.</th>
<th>Nominal area, in.$^2$</th>
<th>Nominal weight, lb/ft</th>
<th>Area, in.$^2$/ft of width for various spacings</th>
<th>Center-to-center spacing, in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Plain</td>
<td>Deformed</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W31</td>
<td>D31</td>
<td>0.628</td>
<td>0.310</td>
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<td>1.86</td>
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<td>W30</td>
<td>D30</td>
<td>0.618</td>
<td>0.300</td>
<td>1.020</td>
<td>1.80</td>
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<td>W28</td>
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<td>W26</td>
<td>D26</td>
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<td>0.934</td>
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<td>W24</td>
<td>D24</td>
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<td>W22</td>
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<td>0.220</td>
<td>0.748</td>
<td>1.32</td>
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<tr>
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<td>D20</td>
<td>0.504</td>
<td>0.200</td>
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<td>0.544</td>
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<td>0.476</td>
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<td>D12</td>
<td>0.390</td>
<td>0.120</td>
<td>0.408</td>
<td>0.72</td>
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<td>W11</td>
<td>D11</td>
<td>0.374</td>
<td>0.110</td>
<td>0.374</td>
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<tr>
<td>W10.5</td>
<td>D10</td>
<td>0.366</td>
<td>0.105</td>
<td>0.357</td>
<td>0.63</td>
</tr>
<tr>
<td>W10</td>
<td>D10</td>
<td>0.356</td>
<td>0.100</td>
<td>0.340</td>
<td>0.60</td>
</tr>
<tr>
<td>W9.5</td>
<td>D9</td>
<td>0.348</td>
<td>0.095</td>
<td>0.323</td>
<td>0.57</td>
</tr>
<tr>
<td>W9</td>
<td>D9</td>
<td>0.338</td>
<td>0.090</td>
<td>0.306</td>
<td>0.54</td>
</tr>
<tr>
<td>W8.5</td>
<td>D8</td>
<td>0.329</td>
<td>0.085</td>
<td>0.289</td>
<td>0.51</td>
</tr>
<tr>
<td>W8</td>
<td>D8</td>
<td>0.319</td>
<td>0.080</td>
<td>0.272</td>
<td>0.48</td>
</tr>
<tr>
<td>W7.5</td>
<td>D7</td>
<td>0.309</td>
<td>0.075</td>
<td>0.255</td>
<td>0.45</td>
</tr>
<tr>
<td>W7</td>
<td>D7</td>
<td>0.298</td>
<td>0.070</td>
<td>0.238</td>
<td>0.42</td>
</tr>
<tr>
<td>W6.5</td>
<td>D6</td>
<td>0.288</td>
<td>0.065</td>
<td>0.221</td>
<td>0.39</td>
</tr>
<tr>
<td>W6</td>
<td>D6</td>
<td>0.276</td>
<td>0.060</td>
<td>0.204</td>
<td>0.36</td>
</tr>
<tr>
<td>W5.5</td>
<td>D5</td>
<td>0.264</td>
<td>0.055</td>
<td>0.187</td>
<td>0.33</td>
</tr>
<tr>
<td>W5</td>
<td>D5</td>
<td>0.252</td>
<td>0.050</td>
<td>0.170</td>
<td>0.30</td>
</tr>
<tr>
<td>W4.5</td>
<td>D4</td>
<td>0.240</td>
<td>0.045</td>
<td>0.153</td>
<td>0.27</td>
</tr>
<tr>
<td>W4</td>
<td>D4</td>
<td>0.225</td>
<td>0.040</td>
<td>0.136</td>
<td>0.24</td>
</tr>
<tr>
<td>W3.5</td>
<td>D3.5</td>
<td>0.211</td>
<td>0.035</td>
<td>0.119</td>
<td>0.21</td>
</tr>
<tr>
<td>W3</td>
<td>D3</td>
<td>0.195</td>
<td>0.030</td>
<td>0.102</td>
<td>0.18</td>
</tr>
<tr>
<td>W2.9</td>
<td>D2.9</td>
<td>0.192</td>
<td>0.029</td>
<td>0.098</td>
<td>0.174</td>
</tr>
<tr>
<td>W2.5</td>
<td>D2.5</td>
<td>0.178</td>
<td>0.025</td>
<td>0.085</td>
<td>0.15</td>
</tr>
<tr>
<td>W2</td>
<td>D2</td>
<td>0.159</td>
<td>0.020</td>
<td>0.068</td>
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</tr>
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<td>W1.4</td>
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<td>0.135</td>
<td>0.014</td>
<td>0.049</td>
<td>0.084</td>
</tr>
</tbody>
</table>

ACI 318 Building Code and Commentary
# STEEL REINFORCEMENT INFORMATION

**Table 3.7.1**

<table>
<thead>
<tr>
<th>Nominal Diameter (in.)</th>
<th>Weight (lb/ft)</th>
<th>Number of Bars</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>#3</td>
<td>0.375</td>
<td>0.376</td>
</tr>
<tr>
<td>#4</td>
<td>0.500</td>
<td>0.668</td>
</tr>
<tr>
<td>#5</td>
<td>0.625</td>
<td>1.043</td>
</tr>
<tr>
<td>#6</td>
<td>0.750</td>
<td>1.502</td>
</tr>
<tr>
<td>#7</td>
<td>0.875</td>
<td>2.044</td>
</tr>
<tr>
<td>#8</td>
<td>1.000</td>
<td>2.670</td>
</tr>
<tr>
<td>#9</td>
<td>1.128</td>
<td>3.400</td>
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<td>#10</td>
<td>1.270</td>
<td>4.303</td>
</tr>
<tr>
<td>#11</td>
<td>1.410</td>
<td>5.313</td>
</tr>
<tr>
<td>#14*</td>
<td>1.693</td>
<td>7.65</td>
</tr>
<tr>
<td>#18*</td>
<td>2.257</td>
<td>13.60</td>
</tr>
</tbody>
</table>

* #14 and #18 bars are used primarily as column reinforcement and are rarely used in beams.

**Table 3-7 Areas of Bars per Foot Width of Slab—A₆ (in.²/ft)**

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
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</thead>
<tbody>
<tr>
<td>#3</td>
<td>0.22</td>
<td>0.19</td>
<td>0.17</td>
<td>0.15</td>
<td>0.13</td>
<td>0.12</td>
<td>0.11</td>
<td>0.10</td>
<td>0.09</td>
<td>0.09</td>
<td>0.08</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>#4</td>
<td>0.40</td>
<td>0.34</td>
<td>0.30</td>
<td>0.27</td>
<td>0.24</td>
<td>0.22</td>
<td>0.20</td>
<td>0.18</td>
<td>0.17</td>
<td>0.16</td>
<td>0.15</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>#5</td>
<td>0.62</td>
<td>0.53</td>
<td>0.46</td>
<td>0.41</td>
<td>0.37</td>
<td>0.34</td>
<td>0.31</td>
<td>0.29</td>
<td>0.27</td>
<td>0.25</td>
<td>0.23</td>
<td>0.22</td>
<td>0.21</td>
</tr>
<tr>
<td>#6</td>
<td>0.88</td>
<td>0.75</td>
<td>0.66</td>
<td>0.59</td>
<td>0.53</td>
<td>0.48</td>
<td>0.44</td>
<td>0.41</td>
<td>0.38</td>
<td>0.35</td>
<td>0.33</td>
<td>0.31</td>
<td>0.29</td>
</tr>
<tr>
<td>#7</td>
<td>1.20</td>
<td>1.03</td>
<td>0.90</td>
<td>0.80</td>
<td>0.72</td>
<td>0.65</td>
<td>0.60</td>
<td>0.55</td>
<td>0.51</td>
<td>0.48</td>
<td>0.45</td>
<td>0.42</td>
<td>0.40</td>
</tr>
<tr>
<td>#8</td>
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<td>1.35</td>
<td>1.18</td>
<td>1.05</td>
<td>0.95</td>
<td>0.86</td>
<td>0.79</td>
<td>0.73</td>
<td>0.68</td>
<td>0.63</td>
<td>0.59</td>
<td>0.56</td>
<td>0.53</td>
</tr>
<tr>
<td>#9</td>
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<td>1.71</td>
<td>1.50</td>
<td>1.33</td>
<td>1.20</td>
<td>1.09</td>
<td>1.00</td>
<td>0.92</td>
<td>0.86</td>
<td>0.80</td>
<td>0.75</td>
<td>0.71</td>
<td>0.67</td>
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<tr>
<td>#10</td>
<td>2.54</td>
<td>2.18</td>
<td>1.91</td>
<td>1.69</td>
<td>1.52</td>
<td>1.39</td>
<td>1.27</td>
<td>1.17</td>
<td>1.09</td>
<td>1.02</td>
<td>0.95</td>
<td>0.90</td>
<td>0.85</td>
</tr>
<tr>
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<td>3.12</td>
<td>2.67</td>
<td>2.34</td>
<td>2.08</td>
<td>1.87</td>
<td>1.70</td>
<td>1.56</td>
<td>1.44</td>
<td>1.34</td>
<td>1.25</td>
<td>1.17</td>
<td>1.10</td>
<td>1.04</td>
</tr>
</tbody>
</table>

**Table 3-4 Maximum Bar Spacing in One-Way Slabs for Crack Control (in.)*

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>Cover (in.)</th>
<th>Exterior Exposure (z = 129 kips/lin.)</th>
<th>Interior Exposure (z = 156 kips/lin.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3/4</td>
<td>1</td>
<td>1-1/2</td>
</tr>
<tr>
<td>#4</td>
<td>--</td>
<td>14.7</td>
<td>7.5</td>
</tr>
<tr>
<td>#5</td>
<td>--</td>
<td>13.4</td>
<td>7.0</td>
</tr>
<tr>
<td>#6</td>
<td>--</td>
<td>12.2</td>
<td>6.5</td>
</tr>
<tr>
<td>#7</td>
<td>16.3</td>
<td>11.1</td>
<td>6.1</td>
</tr>
<tr>
<td>#8</td>
<td>14.7</td>
<td>10.2</td>
<td>5.8</td>
</tr>
<tr>
<td>#9</td>
<td>13.3</td>
<td>9.4</td>
<td>5.4</td>
</tr>
<tr>
<td>#10</td>
<td>12.0</td>
<td>8.6</td>
<td>5.0</td>
</tr>
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<td>10.9</td>
<td>7.9</td>
<td>4.7</td>
</tr>
</tbody>
</table>

*Valid for f₆ = 0.6fᵧ = 36 ksi, and single layer of reinforcement. Spacing should not exceed 3 times slab thickness nor 18 in. (ACI 7.6.5). No value indicates spacing greater than 18 in.
### Table 3-3 Maximum Number of Bars in a Single Layer

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>Maximum size coarse aggregate—3/4 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beam width, $b_w$ (in.)</td>
</tr>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>#5</td>
<td>3</td>
</tr>
<tr>
<td>#6</td>
<td>3</td>
</tr>
<tr>
<td>#7</td>
<td>3</td>
</tr>
<tr>
<td>#8</td>
<td>3</td>
</tr>
<tr>
<td>#9</td>
<td>2</td>
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<td>#10</td>
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<tr>
<td>#11</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>Maximum size coarse aggregate—1 in.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beam width, $b_w$ (in.)</td>
</tr>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>#5</td>
<td>3</td>
</tr>
<tr>
<td>#6</td>
<td>3</td>
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<td>#7</td>
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<td>#8</td>
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</tr>
<tr>
<td>#10</td>
<td>2</td>
</tr>
<tr>
<td>#11</td>
<td>2</td>
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</table>

### Figure 3-2 Cover and Spacing Requirements for Tables 3-2 and 3-3

### Table 3-2 Minimum Number of Bars in a Single Layer (ACI 10.6)*

<table>
<thead>
<tr>
<th>Bar Size</th>
<th>INTERIOR EXPOSURE ($z = 175$ kips/in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beam width, $b_w$ (in.)</td>
</tr>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>#5</td>
<td>1</td>
</tr>
<tr>
<td>#6</td>
<td>1</td>
</tr>
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<td>#7</td>
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<td>#8</td>
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</tr>
<tr>
<td>#9</td>
<td>2</td>
</tr>
<tr>
<td>#10</td>
<td>2</td>
</tr>
<tr>
<td>#11</td>
<td>2</td>
</tr>
</tbody>
</table>

---

*Note: Refer to ACI 10.6 for detailed requirements.*
Torsion

Notation:

- $a$ = name for width dimension
- $A$ = area bounded by the centerline of a thin walled section subjected to torsion
- $b$ = name for height dimension
- $c$ = radial distance to shear stress location
- $c_i$ = inner radial distance to shear stress location
- $c_o$ = outer radial distance to shear stress location
- $c_1$ = coefficient for shear stress for a rectangular bar in torsion
- $c_2$ = coefficient for shear twist for a rectangular bar in torsion
- $G$ = shear modulus
- $J$ = polar moment of inertia
- $L$ = length
- $s$ = length of a segment of a thin walled section
- $t$ = name for thickness
- $T$ = torque (axial moment)
- $\phi$ = angle of twist
- $\pi$ = $\pi$ (3.1415 radians or 180°)
- $\rho$ = radial distance
- $\tau$ = engineering symbol for shearing stress
- $\Sigma$ = summation symbol

Deformation in Torsionally Loaded Members

Axi-symmetric cross sections subjected to axial moment or torque will remain plane and undistorted.

At a section, internal torque (resisting applied torque) is made up of shear forces parallel to the area and in the direction of the torque. The distribution of the shearing stresses depends on the angle of twist, $\phi$. The cross section remains plane and undistorted.

Shearing Strain

Shearing strain is the angle change of a straight line segment along the axis.

$$\gamma = \frac{\rho \phi}{L}$$

where

$\rho$ is the radial distance from the centroid to the point under strain.

The maximum strain is at the surface, a distance $c$ from the centroid:

$$\gamma_{max} = \frac{c \phi}{L}$$

$G$ is the Shear Modulus or Modulus of Rigidity:

$$\tau = G \cdot \gamma$$
Shearing Strain and Stress

In the linear elastic range: the torque is the summation of torsion stresses over the area:

\[ T = \frac{\tau J}{\rho} \]

gives:
\[ \tau = \frac{T \rho}{J} \]

Maximum torsional stress, \( \tau_{\text{max}} \), occurs at the outer diameter (or perimeter).

Polar Moment of Inertia

For axi-symmetric shapes, there is only one value for polar moment of inertia, \( J \), determined by the radius, \( c \):

\[
\begin{align*}
\text{solid section:} & \quad J = \frac{\pi c^4}{2} \\
\text{hollow section:} & \quad J = \frac{\pi(c_o^4 - c_i^4)}{2}
\end{align*}
\]

Combined Torsion and Axial Loading

Just as with combined axial load and shear, combined torsion and axial loading result in maximum shear stress at a 45° oblique “plane” of twist.

Shearing Strain

In the linear elastic range: \( \phi = \frac{T L}{J G} \) and for composite shafts:
\[ \phi = \sum_i \frac{T_i L_i}{J_i G_i} \]

Torsion in Noncircular Shapes

\( J \) is no longer the same along the lateral axes. Plane sections do not remain plane, but distort. \( \tau_{\text{max}} \) is still at the furthest distance away from the centroid. For rectangular shapes:

\[ \tau_{\text{max}} = \frac{T}{c_1 a b^2} \quad \phi = \frac{T L}{c_2 a b^3 G} \]

For \( a/b > 5 \):
\[ c_1 = c_2 = \frac{1}{3} \left( 1 - 0.630 \frac{b}{a} \right) \]

<table>
<thead>
<tr>
<th>( a/b )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.208</td>
<td>0.1406</td>
</tr>
<tr>
<td>1.2</td>
<td>0.219</td>
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<tr>
<td>1.5</td>
<td>0.231</td>
<td>0.1958</td>
</tr>
<tr>
<td>2.0</td>
<td>0.246</td>
<td>0.229</td>
</tr>
<tr>
<td>2.5</td>
<td>0.258</td>
<td>0.249</td>
</tr>
<tr>
<td>3.0</td>
<td>0.267</td>
<td>0.263</td>
</tr>
<tr>
<td>4.0</td>
<td>0.282</td>
<td>0.281</td>
</tr>
<tr>
<td>5.0</td>
<td>0.291</td>
<td>0.291</td>
</tr>
<tr>
<td>10.0</td>
<td>0.312</td>
<td>0.312</td>
</tr>
<tr>
<td>( \infty )</td>
<td>0.333</td>
<td>0.333</td>
</tr>
</tbody>
</table>
Open Sections

For long narrow shapes where \( a/b \) is very large \((a/b \to \infty)\) \( c_1 = c_2 = 1/3 \) and:

\[
\tau_{\text{max}} = \frac{T}{\gamma_3 ab^2} \quad \phi = \frac{TL}{\gamma_3 ab^3 G}
\]

Shear Flow of Closed Thin Walled Sections

\( q \) is the internal shearing force per unit length, and is constant on a cross section even though the thickness of the wall may vary. \( \bar{A} \) is the area bounded by the centerline of the wall section; \( s_i \) is a length segment of the wall and \( t_i \) is the corresponding thickness of the length segment.

\[
\tau = \frac{T}{2t\bar{A}} \quad \phi = \frac{TL}{4t\bar{A}^2} \sum_i \frac{s_i}{t_i}
\]

Shear Flow in Open Sections

The shear flow must wrap around at all edges, and the total torque is distributed among the areas making up the cross section in proportion to the torsional rigidity of each rectangle \((ab^2/3)\). The total angle of twist is the sum of the \( \phi \) values from each rectangle. \( t_i \) is the thickness of each rectangle and \( b_i \) is the length of each rectangle.

\[
\tau_{\text{max}} = \frac{Tt_{\text{max}}}{\gamma_3 \Sigma b_i t_i^3} \quad \phi = \frac{TL}{\gamma_3 G \Sigma b_i t_i^3}
\]
Example 1

Example 8.9.1

Compare the torsional resisting moment $T$ and the torsional constant $J$ for the sections of Fig. 8.9.4 all having about the same cross-sectional area. The maximum shear stress $\tau$ is 14 ksi.

**SOLUTION**

(a) Circular thin-wall section.

$T = \frac{\tau J}{\rho} = \frac{(14 \text{ ksi})(393.7 \text{ in}^4)}{5.25 \text{ in}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 87.5 \text{ k-ft}$

$J = \frac{\pi}{2} \left( c_o^4 - c_i^4 \right) = \frac{\pi}{2} ((5.25 \text{ in})^4 - (4.75 \text{ in})^4) = 393.7 \text{ in}^4$

(b) Rectangular box section. $\tau = \frac{T}{2t\bar{A}}$

$T = \tau 2t\bar{A} = (14 \text{ ksi}) (2)(0.5 \text{ in})(72 \text{ in}^2) \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 84 \text{ k-ft}$

$\bar{A} \approx (12 \text{ in}) (6 \text{ in}) = 72 \text{ in}^2$

(c) Channel section. Since for this open section,

$\tau_{\text{max}} = \frac{T}{\frac{1}{3} \sum b_i t_i^3} = \frac{T}{J}$

$T = \frac{\tau J}{\rho} = \frac{(14 \text{ ksi})(4.08 \text{ in}^4)}{1 \text{ in}} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 4.8 \text{ k-ft}$

the maximum shear stress will be in the flange. Also,

$J = \sum \frac{bt^3}{3} = \frac{1}{3} \left[ 10 \text{ in}(0.5 \text{ in})^3 + (5.5 \text{ in})(1 \text{ in})^3 + (5.5 \text{ in})(1 \text{ in})^3 \right] = 4.08 \text{ in}^4$
One-Way Frame Analysis

Notation:

\[ D = \text{shorthand for dead load} \]
\[ l_n = \text{clear span from face of support to face of support in concrete design} \]
\[ L = \text{shorthand for live load} \]

\[ w_d = \text{load per unit length on a beam from dead load} \]
\[ w_l = \text{load per unit length on a beam from live load} \]
\[ w_u = \text{load per unit length on a beam from load factors} \]

2.3 FRAME ANALYSIS BY COEFFICIENTS

The ACI Code provides a simplified method of analysis for both one-way construction (ACI 8.3.3) and two-way construction (ACI 13.6). Both simplified methods yield moments and shears based on coefficients. Each method will give satisfactory results within the span and loading limitations stated in Chapter 1. The direct design method for two-way slabs is discussed in Chapter 4.

2.3.1 Continuous Beams and One-Way Slabs

When beams and one-way slabs are part of a frame or continuous construction, ACI 8.3.3 provides approximate moment and shear coefficients for gravity load analysis. The approximate coefficients may be used as long as all of the conditions illustrated in Fig. 2-2 are satisfied: (1) There must be two or more spans, approximately equal in length, with the longer of two adjacent spans not exceeding the shorter by more than 20 percent; (2) loads must be uniformly distributed, with the service live load not more than 3 times the dead load \((L/D \leq 3)\); and (3) members must have uniform cross section throughout the span. Also, no redistribution of moments is permitted (ACI 8.4). The moment coefficients defined in ACI 8.3.3 are shown in Figs. 2-3 through 2-6. In all cases, the shear in end span members at the interior support is taken equal to \(1.15w_u\ell_n/2\). The shear at all other supports is \(w_u\ell_n/2\) (see Fig. 2-7). \(w_u\ell_n\) is the combined factored load for dead and live loads, \(w_u = 1.2w_d + 1.6w_l\). For beams, \(w_u\) is the uniformly distributed load per unit length of beam (plf), and the coefficients yield total moments and shears on the beam. For one-way slabs, \(w_u\) is the uniformly distributed load per unit area of slab (psf), and the moments and shears are for slab strips one foot in width. The span length \(\ell_n\) is defined as the clear span of the beam or slab. For negative moment at a support with unequal adjacent spans, \(\ell_n\) is the average of the adjacent clear spans. Support moments and shears are at the faces of supports.

![Figure 2-2 Conditions for Analysis by Coefficients (ACI 8.3.3)](image-url)
Simplified Design

Figure 2-3 Positive Moments—All Cases

Figure 2-4 Negative Moments—Beams and Slabs

Figure 2-5 Negative Moments—Slabs with spans ≤ 10 ft

Figure 2-6 Negative Moments—Beams with Stiff Columns (ΣK_c/ΣK_b > 8)

Figure 2-7 End Shears—All Cases
# Thickness and Cover Requirements for Fire Protection

**Simplified Design, PCA 1993**

## Table 10-1 Minimum Thickness for Floor and Roof Slabs and Cast-In-Place Walls, in.

(Adjusted Bearing and Nonload-Bearing)

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>Fire resistance rating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 hr.</td>
</tr>
<tr>
<td>Siliceous aggregate</td>
<td>3.5</td>
</tr>
<tr>
<td>Carbonate aggregate</td>
<td>3.2</td>
</tr>
<tr>
<td>Sand-lightweight</td>
<td>2.7</td>
</tr>
<tr>
<td>Lightweight</td>
<td>2.5</td>
</tr>
</tbody>
</table>

## Table 10-2 Minimum Concrete Column Dimensions, in.

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>Fire resistance rating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 hr.</td>
</tr>
<tr>
<td>Siliceous aggregate</td>
<td>8</td>
</tr>
<tr>
<td>Carbonate aggregate</td>
<td>8</td>
</tr>
<tr>
<td>Sand-lightweight</td>
<td>8</td>
</tr>
</tbody>
</table>

## Table 10-3 Minimum Cover for Reinforced Concrete Floor or Roof Slabs, in.

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>Fire resistance rating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 hr.</td>
</tr>
</tbody>
</table>

*See Table 10-5

## Table 10-4 Minimum Cover to Main Reinforcing Bars in Reinforced Concrete Beams, in.

(Applicable to All Types of Structural Concrete)

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>Fire resistance rating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 hr.</td>
</tr>
<tr>
<td>Siliceous aggregate</td>
<td>3/4</td>
</tr>
</tbody>
</table>

*See Table 10-5

**For beam widths between the tabulated values, the minimum cover can be determined by interpolation.

## Table 10-6 Minimum Cover for Reinforced Concrete Columns, in.

<table>
<thead>
<tr>
<th>Concrete type</th>
<th>Fire resistance rating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 hr.</td>
</tr>
<tr>
<td>Siliceous aggregate</td>
<td>1½</td>
</tr>
<tr>
<td>Carbonate aggregate</td>
<td>1½</td>
</tr>
<tr>
<td>Sand-lightweight</td>
<td>1½</td>
</tr>
</tbody>
</table>
11.12.5 Openings in Slabs

The effect of openings (vertical holes through slabs) on the shear strength of slabs must be investigated when the openings are within the column strip areas of slabs or within middle strip areas when the openings are closer than 10 times the slab thickness (10h) from a column. A reduction in shear strength is made by considering as ineffective that portion of the critical section b_o which is enclosed by straight lines projecting from the column centroid to the edges of the opening. Ineffective portions of critical sections b_o are illustrated in Fig. 18-10. For slabs with shear reinforcement, the ineffective portion of the perimeter b_o is one-half of that without shear reinforcement. The one-half factor is interpreted to apply equally to shearhead reinforcement and bar or wire reinforcement.

![Diagram of Openings in Slabs](image)

Figure 18-10 Effect of Slab Openings on Shear Strength

13.4 OPENINGS IN SLAB SYSTEMS

Openings of any size are permitted in slab systems without beams if special analysis indicates that both strength and serviceability of the slab system, considering the effects of the opening, are satisfied. Without special analysis, openings up to a certain size are permitted as illustrated in Fig. 18-11. The size of openings located within intersecting middle strip areas is unlimited. Within the area of the slab common to intersecting column
strips, size of openings is the most restrictive, due to their effect on slab shear strength or load transfer near slab-column connections. See discussion on effect of slab openings on shear strength (11.12.5) and Fig. 18-10. Without special analysis, size of openings within intersecting column strips is limited to one-sixteenth of the slab span length in either direction \((1/8 \ell/2 = \ell/16)\). Within the slab area common to one column and one middle strip, opening size is limited to one-eighth the span length in either direction \((1/4 \ell/2 = \ell/8)\).

The total amount of reinforcement required for the panel without openings, in both directions, must be maintained; reinforcement interrupted by any opening must be replaced, one-half on each side of the opening.

Figure 18-11 Openings in Slab Systems without Beams
Foundation Design

### Notation:
- $a$ = name for width dimension
- $A$ = name for area
- $b$ = width of retaining wall stem at base
  = width resisting shear stress
- $b_o$ = perimeter length for two-way shear in concrete footing design
- $B$ = spread footing or retaining wall base dimension in concrete design
- $cc$ = shorthand for clear cover
- $d$ = effective depth from the top of a reinforced concrete member to the centroid of the tensile steel
  = name for diameter
- $e$ = eccentric distance of application of a force ($P$) from the centroid of a cross section
- $f$ = symbol for stress
- $f'_c$ = concrete design compressive stress
- $F_{horizontal-resisting}$ = total force resisting horizontal sliding
- $F_{sliding}$ = total sliding force
- $F_x$ = force in the x direction
- $F.S.$ = shorthand for factor of safety
- $h_f$ = height of a concrete spread footing
- $H$ = height of retaining wall
- $H_A$ = horizontal force due to active soil pressure
- $l_d$ = development length for reinforcing steel
- $L$ = name for length or span length
- $M$ = moment due to a force
- $M_n$ = nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete beam design
- $M_{overturning}$ = total overturning moment
- $M_{resisting}$ = total moment resisting overturning about a point
- $M_u$ = maximum moment from factored loads for LRFD beam design
- $n$ = name for number
- $N$ = name for normal force to a surface
- $o$ = point of overturning of a retaining wall, commonly at the “toe”
- $p$ = pressure
- $p_A$ = active soil pressure
- $P$ = name for axial force vector
  = force due to a pressure
- $P_D$ = dead load axial force
- $P_L$ = live load axial force
- $P_u$ = factored axial force
- $q$ = soil bearing pressure
- $q_a$ = allowable soil bearing stress in allowable stress design, as is $q_{allowable}$
- $q_g$ = gross soil bearing pressure
- $q_{net}$ = net allowed soil bearing pressure, as is $q_n$
- $q_u$ = ultimate soil bearing strength in allowable stress design
  = factored soil bearing capacity in concrete footing design from load factors, as is $q_{nu}$
- $R$ = name for reaction force vector
- $SF$ = shorthand for factor of safety
- $t$ = thickness of retaining wall stem at top
- $T$ = name of a tension force
- $V$ = name for volume
- $V_c$ = shear force capacity in concrete
- $V_u$ = factored shear for reinforced concrete design
- $w$ = name for width
- $w_u$ = load per unit length on a beam from load factors
- $W$ = name for force due to weight
- $x$ = horizontal distance
- $y$ = the distance in the y direction from a reference axis to the centroid of a shape
- $\lambda$ = modification factor for lightweight concrete
- $\phi$ = resistance factor
- $\gamma_c$ = density or unit weight of concrete
- $\gamma_s$ = density or unit weight of soil
- $\pi$ = pi (3.1415 radians or 180°)
- $\rho$ = reinforcement ratio in concrete beam design = $\Lambda_{bd}$
- $\mu$ = coefficient of static friction
Foundations

A foundation is defined as the engineered interface between the earth and the structure it supports that transmits the loads to the soil or rock. The design differs from structural design in that the choices in material and framing system are not available, and quality of materials cannot be assured. Foundation design is dependent on geology and climate of the site.

Soil Mechanics

Soil is another building material and the properties, just like the ones necessary for steel and concrete and wood, must be known before designing. In addition, soil has other properties due to massing of the material, how soil particles pack or slide against each other, and how water affects the behavior. The important properties are

- specific weight (density)
- allowable soil pressure
- factored net soil pressure – allowable soil pressure less surcharge with a factor of safety
- shear resistance
- backfill pressure
- cohesion & friction of soil
- effect of water
- settlement
- rock fracture behavior

Structural Strength and Serviceability

There are significant serviceability considerations with soil. Soils can settle considerably under foundation loads, which can lead to redistribution of moments in continuous slabs or beams, increases in stresses and cracking. Excessive loads can cause the soil to fail in bearing and in shear. The presence of water can cause soils to swell or shrink and freeze and thaw, which causes heaving. Fissures or fault lines can cause seismic instabilities.

A geotechnical engineer or engineering service can use tests on soil bearings from the site to determine the ultimate bearing capacity, $q_u$. Allowable stress design is utilized for soils because of the variability do determine the allowable bearing capacity, $q_a = q_u / (safety\ factor)$.

Values of $q_a$ range from 3000 – 4000 psi for most soils, while clay type soils have lower capacities and sandy soils to rock have much higher capacities.
Soil acts somewhat like water, in that it exerts a lateral pressure because of the weight of the material above it, but the relationship is not linear. Soil can have an active pressure from soil behind a retaining wall and a passive pressure from soil in front of the footing. Active pressure is typically greater than passive pressure.

Foundation Materials 353

Typical foundation materials include:

- plain concrete
- reinforced concrete
- steel
- wood
- composites, ie. steel tubing filled with concrete

**Foundation Design**

**Generalized Design Steps**

Design of foundations with variable conditions and variable types of foundation structures will be different, but there are steps that are typical to every design, including:

1. Calculate loads from structure, surcharge, active & passive pressures, etc.
2. Characterize soil – hire a firm to conduct soil tests and produce a report that includes soil material properties
3. Determine footing location and depth – shallow footings are less expensive, but the variability of the soil from the geotechnical report will drive choices
4. Evaluate soil bearing capacity – the factor of safety is considered here
5. Determine footing size – these calculations are based on working loads and the allowable soil pressure
6. Calculate contact pressure and check stability
7. Estimate settlements
8. Design the footing structure – design for the material based on applicable structural design codes which may use allowable stress design, LRFD or limit state design (concrete).
Shallow Foundation Types

Considered simple and cost effective because little soil is removed or disturbed.

*Spread footing* – A single column bears on a square or rectangular pad to distribute the load over a bigger area.

*Wall footing* – A continuous wall bears on a wide pad to distribute the load.

*Eccentric footing* – A spread or wall footing that also must resist a moment in addition to the axial column load.

*Combined footing* – Multiple columns (typically two) bear on a rectangular or trapezoidal shaped footing.

*Unsymmetrical footing* – A footing with a shape that does not evenly distribute bearing pressure from column loads and moments. It typically involves a hole or a non-rectangular shape influenced by a boundary or property line.

*Strap footing* – A combined footing consisting of two spread footings with a beam or strap connecting the slabs. The purpose of this is to limit differential settlements.

*Mat foundation* – A slab that supports multiple columns. The mat can be stiffened with a grid or grade beams. It is typically used when the soil capacity is very low.

Deep Foundation Types

Considerable material and excavation is required, increasing cost and effort.

*Retaining Walls* – A wall that retains soil or other materials, and must resist sliding and overturning. Can have counterforts, buttresses or keys.

*Basement Walls* – A wall that encloses a basement space, typically next to a floor slab, and that may be restrained at the top by a floor slab.

*Piles* – Next choice when spread footings or mats won’t work, piles are used to distribute loads by end bearing to strong soil or friction to low strength soils. Can be used to resist uplift, a moment causing overturning, or to compact soils. Also useful when used in combination to control settlements of mats or slabs.

*Drilled Piers* – Soil is removed to the shape of the pier and concrete is added.

*Caissons* – Water and possibly wet soil is held back or excavated while the footing is constructed or dropped into place.
Loads and Stresses

Bearing loads must be distributed to the soil materials, but because of their variability and the stiffness of the footing pad, the resulting stress, or soil pressure, is not necessarily uniform. But we assume it is for design because dealing with the complexity isn’t worth the time or effort.

The increase in weight when replacing soil with concrete is called the overburden. Overburden may also be the result of adding additional soil to the top of the excavation for a retaining wall. It is extra uniformly distributed load that is considered by reducing the allowable soil pressure (instead of increasing the loads), resulting in a net allowable soil pressure, $q_{net}$:

$$q_{net} = q_{allowable} - hf(\gamma_c - \gamma_s)$$

In order to design the footing size, the actual stress $P/A$ must be less than or equal to the allowable pressure:

$$\frac{P}{A} \leq q_{net}$$

Design Stresses

The result of a uniform pressure on the underside of a footing is identical to a distributed load on a slab over a column when looked at upside down. The footing slab must resist bending, one-way shear and two-way shear (punching).

Stresses with Eccentric Loading

Combined axial and bending stresses increase the pressure on one edge or corner of a footing. We assume again a linear distribution based on a constant relationship to settling. If the pressure combination is in tension, this effectively means the contact is gone between soil and footing and the pressure is really zero. To avoid zero pressure, the eccentricity must stay within the kern. The maximum pressure must not exceed the net allowable soil pressure.

If the contact is gone, the maximum pressure can be determined knowing that the volume of the pressure wedge has to equal the column load, and the centroid of the pressure wedge coincides with the effective eccentricity.

Wedge volume is $V = \frac{wx}{2}$ where $w$ is the width, $p$ is the soil pressure, and $x$ is the wedge length ($3a$), so $p = \frac{2P}{wx}$ or $\frac{2N}{wx}$ (and $e = \frac{M}{P}$ or $\frac{M}{N}$ and $a = \frac{1}{2}$ width - $e$).
Overturning is considered in design such that the resisting moment from the soil pressure (equivalent force at load centroid) is greater than the overturning moment, \( M \), by a factor of safety of at least 1.5

\[
SF = \frac{M_{\text{resist}}}{M_{\text{overturning}}} \geq 1.5
\]

where

\( M_{\text{resist}} = \) average resultant soil pressure \( \times \) width \( \times \) location of load centroid with respect to column centroid

\( M_{\text{overturning}} = P \times e \)

**Combined Footings**

The design of combined footing requires that the centroid of the area be as close as possible to the resultant of the two column loads for uniform pressure and settling.

**Retaining Walls**

The design of retaining walls must consider overturning, settlement, sliding and bearing pressure. The water in the retained soil can significantly affect the loading and the active pressure of the soil. The lateral force acting at a height of \( H/3 \) is determined from the active pressure, \( p_A \) (in force/cubic area) as:

\[
H_A = \frac{p_A H^2}{2}
\]

Overturning is considered the same as for eccentric footings:

\[
SF = \frac{M_{\text{resist}}}{M_{\text{overturning}}} \geq 1.5 - 2
\]

where

\( M_{\text{resist}} = \) summation of moments about “\( o \)” to resist rotation, typically including the moment due to the weight of the stem and base and the moment due to the passive pressure.

\( M_{\text{overturning}} = \) moment due to the active pressure about “\( o \)”.

Sliding must also be avoided:

\[
SF = \frac{F_{\text{horizontal resist}}}{F_{\text{sliding}}} \geq 1.25 - 2
\]

where:

\( F_{\text{horizontal resist}} = \) summation of forces to resist sliding, typically including the force from the passive pressure and friction (\( F = \mu \cdot N \) where \( \mu \) is a constant for the materials in contact and \( N \) is the normal force to the ground acting down and shown as \( R \)).

\( F_{\text{sliding}} = \) sliding force as a result of active pressure.
For sizing, some rules of thumbs are:

- footing size, $B$
- reinforced concrete, $B \approx 2/5 - 2/3$ wall height ($H$)
- footing thickness, $h_f \approx 1/12 - 1/8$ footing size ($B$)
- base of stem, $b \approx 1/10 - 1/12$ wall height ($H+h_f$)
- top of stem, $t \geq 12$ inches

**Example 1 (page 533)**

*Assume the soil has a density of 90 lb/ft$^3$.**

**Example 2.** Design a square column footing for the following data:

- Column load = 200 kips [890 kN] dead load and 300 kips [1334 kN] live load
- Column size = 15 in. [380 mm] square
- Maximum allowable soil pressure = 4000 psf [200 kPa]
- Concrete design strength = 3000 psi [21 MPa]
- Yield stress of steel reinforcement = 40 ksi [280 MPa]

*Assume the soil has a density of 90 lb/ft$^3$.**
(b) For punching shear

Shear section

4(41.9" + 26.9")

Force due to soil pressure

(c) For beam-type shear

Shear section

141" + 26.9"

Force due to soil pressure

Critical section for bending & development

141" + 26.9"

Average d

26.9"

Force due to soil pressure

11'-8" or 141"
Example 2
For the 16 in. thick 8.5 ft. square reinforced concrete footing carrying 150 kips dead load and 100 kips live load on a 24 in. square column, determine if the footing thickness is adequate for 4000 psi. A 3 in. cover is required with concrete in contact with soil.
Also determine the moment for reinforced concrete design.

SOLUTION:
1. Find design soil pressure: 
   \[ q_u = \frac{P_u}{A} \]
   \[ P_u = 1.2D + 1.6L = 1.2(150 \text{ k}) + 1.6(100 \text{ k}) = 340 \text{ k} \]
   \[ q_u = \frac{340k}{(8.5 \text{ ft})^2} = 4.71 \text{ k/ft}^2 \]

2. Evaluate one-way shear at \( d \) away from column face (Is \( V_u < \phi V_c \)?)
   \[ d = h - \text{c.c. - distance to bar intersection} \]
   presuming #8 bars:
   \[ d = 16 \text{ in. - 3 in. (soil exposure) - 1 in. x (1 layer of #8’s) = 12 in.} \]
   \[ V_u = \text{total shear} = q_u \text{ (edge area)} \]
   \[ V_u \text{ on a 1 ft strip} = q_u \text{ (edge distance) (1 ft)} \]
   \[ \phi V_u = \text{one-way shear resistance} = \phi 2 \sqrt{\frac{f_c'}{f_c}} bd \]
   for a one foot strip, \( b = 12 \text{ in.} \)
   \[ \phi V_u = 0.75(2)(1) \sqrt{4000 \text{ psi}}(12 \text{ in.})(12 \text{ in.}) = 13.7 \text{ k} > 10.6 \text{ k OK} \]

3. Evaluate two-way shear at \( d/2 \) away from column face (Is \( V_u < \phi V_c \)?)
   \[ b_o = \text{perimeter} = 4 \text{ (24 in. + 12 in.) = 4 (36 in.) = 144 in} \]
   \[ V_u = \text{total shear on area outside perimeter} = P_u - q_u \text{ (punch area)} \]
   \[ V_u = 340 \text{ k} - (4.71 \text{ k/ft}^2)(36 \text{ in.})(1 \text{ ft/12 in.})^2 = 297.6 \text{ kips} \]
   \[ \phi V_u = \text{two-way shear resistance} = \phi 4 \sqrt{f_c'} b_o \text{d} = 0.75(4)(1) \sqrt{4000 \text{ psi}}(144 \text{ in.})(12 \text{ in.}) = 327.9 \text{ k} > 297.6 \text{ k OK} \]

4. Design for bending at column face
   \[ M_u = w_u L^2/2 \text{ for a cantilever.} \quad L = (8.5 \text{ ft - 2 ft})/2 = 3.25 \text{ ft, and} \quad w_u \text{ for a 1 ft strip} = q_u \text{ (1 ft)} \]
   \[ M_u = 4.71 \text{ k/ft}^2(1 \text{ ft})(3.25 \text{ ft})^2/2 = 24.9 \text{ k-ft (per ft of width)} \]
   To complete the reinforcement design, use \( b = 12 \text{ in.} \) and trial \( d = 12 \text{ in.} \), choose \( \rho \), determine \( A_s \), find if \( \phi M_n > M_u \ldots \)

5. Check transfer of load from column to footing:
   \[ \phi P_n = \phi 0.85\sqrt{f_c'} A_t \leq \phi 0.85 \sqrt{f_c'} A_t = 0.65(0.85)(4000 \text{ psi})(2)(12 \text{ in.})(12 \text{ in.}) = 636.5 \text{ k} > 340 \text{ k OK} \]
Example 3

Determine the depth required for the group of 4 friction piles having 12” diameters if the column load is 100 kips and the frictional resistance is 400 lbs/ft².

SOLUTION:
The downward load is resisted by a friction force. Friction is determined by multiplying the friction resistance (a stress) by the area: $F = fA_{SKIN}$
The area of $n$ cylinders is: $A_{SKIN} = n(2\pi \frac{d}{2} L)$

Our solution is to set $P \leq F$ and solve for length:

$$100k \leq 400 \sqrt{\frac{lb}{f}} \left(4\frac{\text{pile}}{\text{ft}^3}\right) \left(2\pi \right) \left(\frac{12\text{in}}{2}\right) L \left(\frac{1\text{ft}}{12\text{in}}\right) \left(\frac{1k}{1000\text{lb}}\right)$$

$$L \geq 19.9 \sqrt{\frac{ft}{\text{pile}}}$$

Example 4

Determine the depth required for the friction & bearing pile having a 36” diameter if the column load is 300 kips, the frictional resistance is 600 lbs/ft² and the end bearing pressure allowed is 8000 psf.

SOLUTION:
The downward load is resisted by a friction force and a bearing force, which can be determined from multiplying the bearing pressure by the area in contact: $F = fA_{SKIN} + qA_{TIP}$
The area of a circle is: $A_{TIP} = \pi \frac{d^2}{4}$

Our solution is to set $P \leq F$ and solve for length:

$$300k \leq 600 \sqrt{\frac{lb}{f}} \left(2\pi \right) \left(\frac{36\text{in}}{2}\right) L \left(\frac{1\text{ft}}{12\text{in}}\right) \left(\frac{1k}{1000\text{lb}}\right) + 8000 \sqrt{\frac{lb}{f}} \pi \left(\frac{36\text{in}}{4}\right) \left(\frac{1\text{ft}}{12\text{in}}\right) \left(\frac{1k}{1000\text{lb}}\right)$$

$$L \geq 43.1\text{ft}$$

Example 5

Determine the factor of safety for overturning and sliding on the 15’ retaining wall, 16” wide stem, 10’ base, 16” heigh base, when the equivalent fluid pressure is 30 pcf, the weight of the stem of the footing is 4 kips, the weight of the pad is 5 kips, the passive pressure is ignored for this design, and the friction coefficient for sliding is 0.58. The center of the stem is located 3’ from the toe.
Design of Isolated Square and Rectangular Footings (ACI 318-14)

Notation:

\[ a = \text{equivalent square column size in spread footing design} \]
\[ \bar{a} = \text{depth of the effective compression block in a concrete beam} \]
\[ A_g = \text{gross area, equal to the total area ignoring any reinforcement} \]
\[ A_{req} = \text{area required to satisfy allowable stress} \]
\[ A_s = \text{area of steel reinforcement in concrete design} \]
\[ A_1 = \text{area of column in spread footing design} \]
\[ A_2 = \text{projected bearing area of column load in spread footing design} \]
\[ b = \text{rectangular column dimension in concrete footing design} \]
\[ b_f = \text{width, often cross-sectional} \]
\[ b_o = \text{perimeter length for two-way shear in concrete footing design} \]
\[ B = \text{spread footing dimension in concrete design} \]
\[ B_s = \text{width within the longer dimension of a rectangular spread footing that reinforcement must be concentrated within for concrete design} \]
\[ c = \text{rectangular column dimension in concrete footing design} \]
\[ C = \text{dimension of a steel base plate for concrete footing design} \]
\[ d = \text{effective depth from the top of a reinforced concrete member to the centroid of the tensile steel} \]
\[ d_b = \text{bar diameter of a reinforcing bar} \]
\[ d_f = \text{depth of a steel column flange (wide flange section)} \]
\[ f_c' = \text{concrete design compressive stress} \]
\[ f_y = \text{yield stress or strength} \]
\[ h_f = \text{height of a concrete spread footing} \]
\[ l_a = \text{development length for reinforcing steel} \]
\[ l_{dc} = \text{development length for column} \]
\[ l_{sc} = \text{lap splice length in compression for reinforcement} \]
\[ L = \text{name for length or span length} \]
\[ L_m = \text{projected length for bending in concrete footing design} \]
\[ L' = \text{length of the one-way shear area in concrete footing design} \]
\[ M_n = \text{nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete flexure design} \]
\[ M_u = \text{maximum moment from factored loads for LRFD beam design} \]
\[ P = \text{name for axial force vector} \]
\[ P_{dowel} = \text{nominal capacity of dowels from concrete column to footing in concrete design} \]
\[ P_D = \text{dead load axial force} \]
\[ P_L = \text{live load axial force} \]
\[ P_n = \text{nominal column or bearing load capacity in concrete design} \]
\[ P_u = \text{factored axial force} \]
\[ q_{allowable} = \text{allowable soil bearing stress in allowable stress design} \]
\[ q_{net} = \text{net allowed soil bearing pressure} \]
\[ q_u = \text{factored soil bearing capacity in concrete footing design from load factors} \]
\[ V_c = \text{shear force capacity in concrete} \]
\[ V_n = \text{nominal shear force capacity} \]
\[ V_{u1} = \text{maximum one-way shear from factored loads for LRFD beam design} \]
\[ V_{u2} = \text{maximum two-way shear from factored loads for LRFD beam design} \]
\[ \beta_c = \text{ratio of long side to short side of the column in concrete footing design} \]
\[ \phi = \text{resistance factor} \]
\[ \gamma_c = \text{density or unit weight of concrete} \]
\[ \gamma_s = \text{density or unit weight of soil} \]
\[ \rho = \text{reinforcement ratio in concrete beam design} = \frac{A_s}{bd} \]
\[ \nu_c = \text{shear strength in concrete design} \]
NOTE: This procedure assumes that the footing is concentrically loaded and carries no moment so that the soil pressure may be assumed to be uniformly distributed on the base.

1) Find service dead and live column loads:
   \[ P_D = \text{Service dead load from column} \]
   \[ P_L = \text{Service live load from column} \]
   \[ P = P_D + P_L \quad \text{(typically see ACI 5.3)} \]

2) Find design (factored) column load, \( P_U \):
   \[ P_U = 1.2P_D + 1.6P_L \]

3) Find an approximate footing depth, \( h_f \):
   \[ h_f = d + 4'' \] and is usually in multiples of 2, 4 or 6 inches.
   
   a) For rectangular columns
   \[ 4d^2 + 2(b + c)d = \frac{P_u}{\phi \nu_c} \]

   b) For round columns
   \[ d^2 + ad = \frac{P_u}{\phi \nu_c} \quad a = \sqrt{\frac{\pi d^2}{4}} \]

   where: \( a \) is the equivalent square column size
   \[ \nu_c = 4\lambda \sqrt{f_c'} \] for two-way shear
   \[ \phi = 0.75 \] for shear
   \[ \lambda = 1.0 \] for normalweight concrete

4) Find net allowable soil pressure, \( q_{net} \):

   By neglecting the weight of any additional top soil added, the net allowable soil pressure takes into account the change in weight when soil is removed and replaced by concrete:
   \[ q_{net} = q_{allowable} - h_f (\gamma_c - \gamma_s) \]
   where \( \gamma_c \) is the unit weight of concrete (typically 150 lb/ft\(^3\)) and \( \gamma_s \) is the unit weight of the displaced soil

5) Find required area of footing base and establish length and width:

   \[ A_{req} \geq \frac{P}{q_{net}} \]

   For square footings choose \( B \geq \sqrt{A_{req}} \)

   For rectangular footings choose \( B \times L \geq A_{req} \)
6) Check transfer of load from column to footing: **ACI 16.3**
   
a) Find load transferred by bearing on concrete in column: **ACI 22.8**
   
   \[ P_{n} = 0.85 f'_{c} A_{1} \]
   
   where \( f'_{c} \) is the concrete strength and \( A_{1} \) is the area of the column
   
   with confinement: \[ P_{n} = 0.85 f'_{c} A_{1} \sqrt{\frac{A_{2}}{A_{1}}} \]
   
   where \( \frac{A_{2}}{A_{1}} \) cannot exceed 2.
   
   IF the column concrete strength is lower than the footing, calculate \( P_{n} \) for the column too.
   
   b) Find load to be transferred by dowels:
   
   \[ \phi P_{dowels} = P_{u} - \phi P_{n} \]
   
   IF \( \phi P_{n} \geq P_{u} \) only nominal dowels are required.
   
   c) Find required area of dowels and choose bars
   
   \[ A_{d} = \frac{\phi P_{dowels}}{\phi f_{y}} \]
   
   where \( \phi = 0.65 \) and \( f_{y} \) is the reinforcement grade
   
   Choose dowels to satisfy the required area and nominal requirements:
   
   i) Minimum of 4 bars
   
   ii) Minimum \( A_{d} = 0.005 A_{g} \) **ACI 16.3.41**
   
   where \( A_{g} \) is the gross column area
   
   d) Check dowel embedment into footing for compression: **ACI 12.3**
   
   \[ l_{dc} = \frac{f_{y} d_{b}}{50 \lambda \sqrt{f_{c}}} \]
   
   but not less than 0.0003 \( f_{y} d_{b} \) or 8” where \( d_{b} \) is the bar diameter
   
   NOTE: The footing must be deep enough to accept \( l_{dc} \). Hooks are not considered effective in compression and are only used to support dowels during construction.
   
   e) Find length of lapped splices of dowels with column bars: **ACI 25.5.5**
   
   \( l_{sc} \) is the largest of:
   
   i) larger of \( l_{dc} \) or 0.0005 \( f_{y} d_{b} \) (\( f_{y} \) of grade 60 or less)
   
   of smaller bar \( (0.0009 f_{y} - 24) d_{b} \) (\( f_{y} \) over grade 60)
   
   ii) \( l_{dc} \) of larger bar
   
   iii) not less than 12”
   
   See **ACI 10.7.5** for possible reduction in \( l_{s} \)
7) Check two-way (slab) shear:

a) Find dimensions of loaded area:

i) For concrete columns, the area coincides with the column area, if rectangular, or equivalent square area if circular (see 3)(b))

ii) For steel columns an equivalent loaded area whose boundaries are halfway between the faces of the steel column and the edges of the steel base plate is used: ACI 13.2.7.1.

\[ b = b_f + \frac{(B-b_f)}{2} \] where \( b_f \) is the width of column flange and \( B \) is base plate side

\[ c = d_f + \frac{(C-d_f)}{2} \] where \( d_f \) is the depth of column flange and \( C \) is base plate side

b) Find shear perimeter: ACI 22.6.4

Shear perimeter is located at a distance of \( \frac{d}{2} \) outside boundaries of loaded area and length is \( b_o = 2(c+d) + 2(b+d) \)

(average \( d = h – 3 \) in. cover – 1 assumed bar diameter)

c) Find factored net soil pressure, \( q_u \):

\[ q_u = \frac{P_u}{B^2} \text{ or } \frac{P_u}{B \times L} \]

d) Find total shear force for two-way shear, \( V_{u2} \):

\[ V_{u2} = P_u - q_u (c+d) (b+d) \]

e) Compare \( V_{u2} \) to two-way capacity, \( \phi \) \( V_n \):

\[ V_{u2} \leq \phi \left( 2 + \frac{4}{\beta} \right) \lambda \sqrt{f_c} b_o d \leq \phi 4 \lambda \sqrt{f_c} b_o d \] ACI 22.6.5.1

where \( \phi = 0.75 \) and \( \beta \) is the ratio of long side to short side of the column

NOTE: This should be acceptable because the initial footing size was chosen on the basis of two-way shear limiting. If it is not acceptable, increase \( h_f \) and repeat steps starting at b).
8) **Check one-way (beam) shear:**

The critical section for one-way shear extends across the width of the footing at a distance \( d \) from the face of the loaded area (see 7)a for loaded area). The footing is treated as a cantilevered beam. **ACI 7.4.3.2**

a) Find projection, \( L' \):

i) For square footing:

\[
L' = \frac{B}{2} - \left( d + \frac{b}{2} \right) \text{ where } b \text{ is the smaller dim. of the loaded area}
\]

ii) For rectangular footings:

\[
L' = \frac{L}{2} - \left( d + \frac{\bullet}{2} \right) \text{ where } \bullet \text{ is the dim. parallel to the long side of the footing}
\]

b) Find total shear force on critical section, \( V_{u1} \):

\[
V_{u1} = B L' q_u
\]

c) Compare \( V_{u1} \) to one-way capacity, \( \phi V_n \):

\[
V_{u1} \leq \phi 2 \lambda \sqrt{f_{c}'} B d \quad V_{u1} \leq \phi 2 \sqrt{f_{c}'} B d \quad \text{ACI 22.5.5.1}
\]

where \( \phi = 0.75 \)

**NOTE:** If it is not acceptable, increase \( h_f \).

9) **Check for bending stress and design reinforcement:**

Square footings may be designed for moment in one direction and the same reinforcing used in the other direction. For rectangular footings the moment and reinforcing must be calculated separately in each direction. The critical section for moment extends across the width of the footing at the face of the loaded area. **ACI 13.2.7.1**

a) Find projection, \( L_m \):

\[
L_m = \frac{B}{2} - \frac{\bullet}{2} \text{ where } \bullet \text{ is the smaller dim. of column for a square footing. For a rectangular footing, use the value perpendicular to the critical section.}
\]

b) Find total moment, \( M_u \), on critical section:

\[
M_u = q_u \left( \frac{B L_m^2}{2} \right) \text{ (find both ways for a rectangular footing)}
\]
c) Find required $A_s$:

$$R_u = \frac{M_u}{bd^2} = \frac{M_u}{\phi bd^2},$$

where $\phi = 0.9$, and $\rho$ can be found from Figure 3.8.1 of Wang & Salmon.

or:

i) guess $a$

ii) $A_s = \frac{0.85 f'y ba}{f_y}$

iii) solve for $a = 2 \left( d - \frac{M_u}{\phi A_s f_y} \right)$

iv) repeat from ii) until a converges, solve for $A_s$

Minimum $A_s$

$$= 0.0018 bh \quad \text{Grade 60 for temperature and shrinkage control}$$
$$= 0.002 bh \quad \text{Grade 40 or 50}$$

ACI 7.6.1.1 specifies the requirements of ACI 7.6.4 must be met, and max. spacing of 18”

d) Choose bars:

For square footings use the same size and number of bars uniformly spaced in each direction (ACI 13.3.3.2). Note that required $A_s$ must be furnished in each direction.

For rectangular footings bars in long direction should be uniformly spaced. In the short direction bars should be distributed as follows (ACI 13.3.3.3):

i) In a band of width $B_s$ centered on column:

$$\# \text{ bars} = \frac{2}{L/B + 1} \cdot (\# \text{bars in } B) \quad \text{(integer)}$$

ii) Remaining bars in short direction should be uniformly spaced in outer portions of footing.

e) Check development length:

Find required development length, $l_d$, in tension from handout or from equations in ACI 25.4. $l_d$ must be less than $(L_m - 2’’)$ (end cover). If not possible, use more bars of smaller diameter.
Masonry Design

Notation:

\( A \) = name for area
\( A_n \) = net area, equal to the gross area subtracting any reinforcement
\( A_{nv} \) = net shear area of masonry
\( A_s \) = area of steel reinforcement in masonry design
\( A_{st} \) = area of steel reinforcement in masonry column design

\( ACI \) = American Concrete Institute
\( ASCE \) = American Society of Civil Engineers

\( b' \) = width, often cross-sectional
\( C \) = name for a compression force
\( C_m \) = compression force in the masonry for masonry design

\( CMU \) = shorthand for concrete masonry unit
\( d \) = effective depth from the top of a reinforced masonry beam to the centroid of the tensile steel

\( e \) = eccentric distance of application of a force (\( P \)) from the centroid of a cross section

\( f_a \) = axial stress
\( f_b \) = bending stress
\( f_m \) = calculated compressive stress in masonry
\( f_m' \) = masonry design compressive stress
\( f_s \) = stress in the steel reinforcement for masonry design

\( f_v \) = shear stress
\( F_a \) = allowable axial stress
\( F_b \) = allowable bending stress
\( F_s \) = allowable tensile stress in reinforcement for masonry design
\( F_t \) = allowable tensile stress
\( F_v \) = allowable shear stress
\( F_{vm} \) = allowable shear stress of the masonry
\( F_{vs} \) = allowable shear stress of the shear reinforcement

\( h \) = name for height
\( I_x \) = moment of inertia with respect to an \( x \)-axis

\( j \) = multiplier by effective depth of masonry section for moment arm, \( j_d \)
\( k \) = multiplier by effective depth of masonry section for neutral axis, \( k_d \)
\( L \) = name for length or span length
\( M \) = internal bending moment

\( M_m \) = moment capacity of a reinforced masonry beam governed by steel stress
\( M_s \) = moment capacity of a reinforced masonry beam governed by masonry stress

\( MSJC \) = Masonry Structural Joint Council
\( n \) = modulus of elasticity transformation coefficient for steel to masonry
\( n.a. \) = shorthand for neutral axis (N.A.)
\( N \) = type of masonry mortar

\( NCMA \) = National Concrete Masonry Association

\( O \) = type of masonry mortar
\( P \) = name for axial force vector
\( P_a \) = allowable axial load in columns
\( r \) = radius of gyration
\( S \) = section modulus

\( S_x \) = section modulus with respect to an \( x \)-axis
\( t \) = name for thickness
\( T \) = name for a tension force
\( T_s \) = tension force in the steel reinforcement for masonry design

\( TMS \) = The Masonry Society
\( w \) = name for distributed load
\( \beta_1 \) = coefficient for determining stress block height, \( c \), in masonry LRFD design

\( \varepsilon_m \) = strain in the masonry
\( \varepsilon_s \) = strain in the steel
\( \rho \) = reinforcement ratio in masonry design
Reinforced Masonry Design

Structural design standards for reinforced masonry are established by the Masonry Standards Joint Committee consisting of ACI, ASCE and The Masonry Society (TMS), and presents allowable stress design as well as limit state (strength) design.

Materials

\( f'_m \) = masonry prism compressive strength from testing

Reinforcing steel grades are the same as those used for reinforced concrete beams.

Units can be brick, concrete or stone.

Mortar consists of masonry cement, lime, sand, and water. Grades are named from the word MASONWORK, with average strengths of 2500psi, 1800 psi, 750 psi, 350 psi, and 75 psi, respectively.

Grout is a flowable mortar, usually with a high amount of water to cement material. It is used to fill voids and bond reinforcement.

Clay and concrete masonry units are porous, and their durability with respect to weathering is an important consideration. The amount of water in the mortar is important as well as the absorption capacity of the units for good bond; both for strength and for weatherproofing. Because of the moisture and tendency for shrinkage and swelling, it is critical to provide control joints for expansion and contraction.

Sizes

Common sizes for clay brick and concrete masonry units (CMU) are shown in the figure, along with definitions.

Typical section properties for CMU’s are provided for reference at the end of the document.

Allowable Stress Design

For unreinforced masonry, like masonry walls, tension stresses are allowed in flexure. Masonry walls typically see compression stresses too.
For reinforced masonry, the steel is presumed to resist all tensile stresses and the tension in the masonry is ignored.

Factors of Safety are applied to the limit stresses for allowable stress values:

- Bending (unreinforced) \( F_b = \frac{1}{3} f'_m \)
- Bending (reinforced) \( F_b = 0.45 f'_m \)
- Bending (tension/unreinforced) \( F_b \) per table 2.2.3.2
- Beam shear (unreinforced for flexure) \( F_v = 1.5 \sqrt{f'_m} \leq 120 \text{ psi} \)
- Beam shear (reinforced) \( - \frac{M}{V_d} \leq 0.25 F_v = 3.0 \sqrt{f'_m} \)
- Beam shear (reinforced) \( - \frac{M}{V_d} \geq 1.0 F_v = 2.0 \sqrt{f'_m} \)
- Grades 40 or 50 reinforcement \( F_s = 20 \text{ ksi} \)
- Grades 60 reinforcement \( F_s = 32 \text{ ksi} \)
- Wire joint reinforcement \( F_s = 30 \text{ ksi} \)

where \( f' \) = specified compressive strength of masonry

**Internal Equilibrium for Bending**

\[ C_m = \text{compression in the masonry} = \text{stress} \times \text{area} = f_m \frac{b(kd)}{2} \]

\[ T_s = \text{tension in steel} = \text{stress} \times \text{area} = A_s f_s \]

\[ C_m = T_s \text{ and } \cdot \]

\[ M_m = T_s(d-kd/3) = T_s(jd) \]

\[ M_s = C_m(jd) \]

\[ \rho = \frac{A_s}{bd} \]

\[ \Sigma F = 0: \quad A_s f_s = f_m b \frac{kd}{2} \]

where

- \( f_m \) = compressive stress in the masonry from flexure
- \( f_s \) = tensile stress in the steel reinforcement
- \( kd \) = the height to the neutral axis
- \( b \) = width of stress area
- \( d \) = effective depth of section = depth to n.a. of reinforcement
- \( jd \) = moment arm from tension force to compression force
- \( A_s \) = area of steel
- \( n = E_d/E_m \) used to transform steel to equivalent area of masonry for elastic stresses
- \( \rho \) = reinforcement ratio
Criteria for Beam Design

For flexure design:
\[ M_m = f_m b \frac{kd}{2} jd = 0.5 f_m bd^2 jk \quad \text{or} \quad M_s = A_s f_s jd = \rho bd^2 jf_s \]

The design is adequate when \( f_b \leq F_b \) in the masonry and \( f_s \leq F_s \) in the steel.

Shear stress is determined by \( f_v = V/A_{nv} \) where \( A_{nv} \) is net shear area. Shear strength is determined from the shear capacity of the masonry and the stirrups: \( F_v = F_{vm} + F_{vs} \). Stirrup spacings are limited to \( d/2 \) but not to exceed 48 in.

where:
\[ F_{vm} = \frac{1}{2} \left[ 4.0 - 1.75 \left( \frac{M}{Vd} \right) \sqrt{f_m'} \right] + 0.25 \frac{P}{A_n} \quad \text{where } M/(Vd) \text{ is positive and cannot exceed } 1.0 \]
\[ F_{vs} = 0.5 \left( \frac{A_s f_s d}{A_{nv}s} \right) \]

(F\(v = 3.0 \sqrt{f_m'} \) when \( M/(Vd) \geq 0.25 \))

(F\(v = 2.0 \sqrt{f_m'} \) when \( M/(Vd) \geq 1.0 \).) Values can be linearly interpolated.

Load and Resistance Factor Design

The design methodology is similar to reinforced concrete ultimate strength design. It is useful with high shear values and for seismic design. The limiting masonry strength is \( 0.80 f_m' \).

Criteria for Column Design

(Masonry Joint Code Committee) Building Code Requirements and Commentary for Masonry Structures define a column as having \( b/t < 3 \) and \( h/t > 4 \).

where
\( b = \) width of the “wall”
\( t = \) thickness of the “wall”
\( h = \) height of the “wall”

A slender column has a minimum dimension of 8” on one side and \( h/t \leq 25 \).

Columns must be reinforced, and have ties. A minimum eccentricity (causing bending) of 0.1 times the side dimension is required.

Allowable Axial Load for Reinforced Masonry

\[ P_a = \left[ 0.25 f_m' A_n + 0.65 A_n f_s \right] \left[ 1 - \left( \frac{h}{140r} \right)^2 \right] \quad \text{for } h/t \leq 99 \]

\[ P_a = \left[ 0.25 f_m' A_n + 0.65 A_n f_s \right] \left( \frac{70r}{h} \right)^2 \quad \text{for } h/t > 99 \]
Allowable Axial Stresses for Unreinforced Masonry

\[ F_a = 0.25 f_m' \left[ 1 - \left( \frac{h}{140r} \right)^2 \right] \]
for \( h/t \leq 99 \)

\[ F_a = 0.25 f_m' \left( \frac{70r}{h} \right)^2 \]
for \( h/t > 99 \)

where
- \( h \) = effective length
- \( r \) = radius of gyration
- \( A_n \) = effective (or net) area of masonry
- \( A_a \) = area of steel reinforcement
- \( f_m' \) = specified masonry compressive strength
- \( F_s \) = allowable compressive stress in column reinforcement with lateral confinement.

Combined Stresses
When maximum moment occurs somewhere other than at the end of the column or wall, a “virtual” eccentricity can be determined from \( e = M/P \).

Masonry Columns and Walls
There are no modification factors, but in addition to satisfying \( \frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \), the tensile stress cannot exceed the allowable: \( f_b - f_a \leq F_t \), or the compressive stress exceed allowable for reinforced masonry: \( f_a + f_b \leq F_b \) provided \( f_a \leq F_a \).
Example 1

Determine if the unreinforced CMU wall can sustain its loads with the wind. Specify a mortar type and unit strength per MSJC.

\[
\frac{f_a}{f_a} + \frac{f_b}{f_b} \leq 1.0 \\
F_a = \frac{h}{2r} f'_m \\
F_b = \frac{M}{S} \quad f_a = \frac{P}{A} \\
F_a = 0.25 f'_m \left[1 - \left(\frac{h}{140r}\right)^2\right] \text{ for } h \leq 99 \\
F_a = 0.25 f'_m \left(\frac{70r}{h}\right)^2 \text{ for } h > 99
\]

\[
h = \frac{12 \text{ ft}(12 \text{ in})}{3.21 \text{ in}} = 44.9 \text{ so } F_a = 0.25 f'_m \left[1 - \left(\frac{12 \cdot 12 \text{ in}}{140 \cdot 3.21 \text{ in}}\right)^2\right] = 0.224 f'_m \\
f_a = \frac{4k(1000 \%)}{30 \text{ in}^2} = 133 \text{ psi}
\]

Case “A” with wind

at midheight of wall:

\[
M = \frac{P_e + wh}{8} = \frac{4 \text{kip} \times 3''}{8} + \left[\frac{(0.030)(12)^2}{8}\right] \times 12 = 12.5 \text{ kip} - \text{in.}
\]

\[
f_s = \frac{12,500 \text{ lb} - \text{in}}{81.0 \text{ in}^3} = 154 \text{ psi} \\
f_b \leq 1/3 f'_m \\
tension criterion:

\[
f'_m \geq 154/(1/3) = 462 \text{ psi}
\]

\[
-f_s + f_c = F_t \\
-133 \text{ psi} + 154 \text{ psi} = 21 \text{ psi} \\
F_{t \text{ req}} = 21 \text{ psi}
\]

compression criterion:

\[
\frac{f_s + f_c}{F_s} < 1, \quad \frac{133}{0.174 f'_m} + \frac{154}{0.333 f'_m} = 1; \quad f'_m = 1056 \text{ psi}
\]

Case “B” without wind

at top of wall:

\[
M = P_e = 12.0 \text{ kip} - \text{in.}
\]

\[
f_s = 12,000 \text{ lb} - \text{in}/81 \text{ in}^3 = 148 \text{ psi} \\
tension criterion:

\[
-f_s + f_c = F_t \\
-133 \text{ psi} + 148 \text{ psi} = 15 \text{ psi} \\
F_{t \text{ req}} = 15 \text{ psi}
\]

compression criterion:

\[
\frac{f_s + f_c}{F_s} \leq 1.0 \\
\frac{133}{0.224 f'_m} + \frac{148}{0.333 f'_m} = 1.00 \\
f'_m = 1038 \text{ psi} \quad f'_m = 1056 \text{ psi (governs)}
\]
### Table 2.2.3.2 — Allowable flexural tensile stresses for clay and concrete masonry, psi (kPa)

<table>
<thead>
<tr>
<th>Direction of flexural tensile stress and masonry type</th>
<th>Mortar types</th>
<th>Portland cement/lime or mortar cement</th>
<th>Masonry cement or air entrained portland cement/lime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M or S</td>
<td>N</td>
<td>M or S</td>
</tr>
<tr>
<td>Normal to bed joints</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid units</td>
<td>53 (366)</td>
<td>40 (276)</td>
<td>32 (221)</td>
</tr>
<tr>
<td>Hollow units(^1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ungrounded</td>
<td>33 (228)</td>
<td>25 (172)</td>
<td>20 (138)</td>
</tr>
<tr>
<td>Fully grouted</td>
<td>86 (593)</td>
<td>84 (579)</td>
<td>81 (559)</td>
</tr>
<tr>
<td>Parallel to bed joints in running bond</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid units</td>
<td>106 (731)</td>
<td>80 (552)</td>
<td>64 (441)</td>
</tr>
<tr>
<td>Hollow units</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ungrounded and partially grouted</td>
<td>66 (455)</td>
<td>50 (345)</td>
<td>40 (276)</td>
</tr>
<tr>
<td>Fully grouted</td>
<td>106 (731)</td>
<td>80 (552)</td>
<td>64 (441)</td>
</tr>
<tr>
<td>Parallel to bed joints in masonry not laid in running bond</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continuous grout section parallel to bed joints</td>
<td>133 (917)</td>
<td>133 (917)</td>
<td>133 (917)</td>
</tr>
<tr>
<td>Other</td>
<td>0 (0)</td>
<td>0 (0)</td>
<td>0 (0)</td>
</tr>
</tbody>
</table>

\(^1\) For partially grouted masonry, allowable stresses shall be determined on the basis of linear interpolation between fully grouted hollow units and ungrouted hollow units based on amount (percentage) of grouting.
REINFORCED BRICK MASONRY BEAMS

Abstract: Reinforced brick masonry (RBM) beams are an efficient and attractive means of spanning building openings. The addition of steel reinforcement and grout permits brick masonry to span considerable distances while maintaining continuity of the building facade. Attractive brick soffits and elimination of steel support members are two of the advantages of reinforced brick masonry beams. This Technical Notes addresses the design of reinforced brick masonry beams. Building code requirements are reviewed and design aids are provided to simplify the design process. Illustrations indicate the proper detailing and typical construction of reinforced brick masonry beams.

Key Words: beam, deflection, girder, lintel, reinforced brick masonry, reinforcement.

INTRODUCTION

Reinforced brick masonry (RBM) beams are widely used as flexural members. Common applications of RBM beams include girders supporting floor and roof systems, and arches and lintels spanning openings for windows and doors. Girder is the term applied to a large beam with a long span that usually supports smaller framing members. A lintel is a beam over a wall opening, typically simply supported with no framing members. The main advantage of RBM beams is that the structural element and the architectural finish are one and the same. In some cases, however, they provide economical solutions without considering the savings due to a built-in finish. They are often built as an integral part of a masonry wall as illustrated in Figure 1. RBM beams are designed to carry all superimposed loads, including that portion of the wall weight above supported by the beam. While steel lintels are more common, RBM beams provide distinct advantages over steel lintels. Among the advantages are:

1. More efficient use of materials. The masonry serves as a structural element with a relatively small amount of steel reinforcement added.
2. Elimination of differential movement. This movement is often the cause of cracks in masonry.
3. Inherent fire resistance.
4. Reduced maintenance. Periodic painting of exposed steel is eliminated.
5. Lower cost.

This Technical Notes provides a review of the design of RBM beams. Factors influencing design and performance are reviewed. Design recommendations and aids are provided and their use illustrated with an example. For additional information about RBM beams and design calculations, refer to the Masonry Designers' Guide (MDG) [2]. The MDG also provides an extensive review of the requirements of the Building Code Requirements for Masonry Structures (ACI 530/ASCE 5/TMS 402-95)[1], hereafter termed the MSJC Code. Other Technical Notes in this series provide the history of RBM, material and construction requirements, and design of other RBM elements.

This Technical Notes does not address the design of deep beams (wall beams) or bond beams. A deep beam is one with a depth-to-span ratio exceeding 0.8. Assumptions made in this Technical Notes regarding the distribution of stress in beams under flexure and the loading conditions do not apply to deep beams. Bond beams are formed by placing horizontal reinforcement in a wall without an opening underneath.

NOTATION

Following are notations used in the text, figures, and table in this Technical Notes.
A, Area of shear reinforcement, in.² (mm²)
b Length of bearing plate, ft (m)
d Effective depth of beam, in. (mm)
d₀ Nominal diameter of reinforcement, in. (mm)
Fₐ Allowable steel stress, psi (MPa)
fₘ Specified compressive strength of masonry, psi (MPa)
H Height of beam, in. (mm)
l₀ Embedment length of reinforcement, in. (mm)
M₀ Design moment due to gravity loads, in.-lb (N-m)
Mₓ Design moment due to in-plane shear, in.-lb (N-m)
Mₛ Design moment due to out-of-plane wind or seismic load, in.-lb (N-m)
P Design concentrated load, lb (kg)
s Spacing of shear reinforcement, in. (mm)
V Design shear force, lb (kg)
W Width of beam, in. (mm)
w Design uniform distributed load, lb/ft (kg/m)
y Distance from top of beam to bearing plate, ft (m)

DETERMINATION OF LOADING

The basic concept of a beam is as a pure flexural member. A flexural member spans an opening and transfers vertical gravity loads to its supports, as illustrated in Fig. 2(a). RBM beams act in this manner to support their own weight and other applied gravity loads. However, it is also common for RBM beams to be part of a masonry wall. As such, RBM beams are often subjected to out-of-plane wind and seismic forces, as depicted in Fig. 2(b). This causes bending of the RBM beam in the out-of-plane direction, which is often about the weak axis of the beam. In addition, reinforced masonry walls may be shear-resisting members, or “shear walls”, which are part of the lateral load-resisting system of a building. In such a structural system, RBM beams may be used as connections between shear walls or piers, as illustrated in Fig. 2(c). Such beams are called coupling beams because they “couple” the shear walls or piers. If the relative sizes of the two piers being coupled are similar, the RBM beam is subject to considerable load when an in-plane shear force is applied to the wall. This is why damage to masonry shear walls is often concentrated at coupling beams following an earthquake or high-wind event.

The designer should consider all aspects of loading for an RBM beam. It is difficult to predict the loading condition that will produce the critical design condition. For example, a RBM beam that is part of a wall will be subject to a combination of gravity loads and out-of-plane wind or seismic loads. Many factors influence the loading conditions for RBM beams.

Arching Action

Arching action is a property of all masonry walls which are laid in an overlapping bond pattern. Brick masonry will span, in a step-like manner similar to a corbel, over a wall opening when laid in running bond pattern. Vertical gravity loads above the openings are transferred to the wall elements on each side. This is the reason why sizable holes can be created in masonry walls without causing collapse. Arching action will occur provided that the following conditions are met:
1. An overlapping bond pattern is used in the masonry surrounding the opening.
2. The masonry above the apex of a 45 degree isosceles triangle above the beam exceeds 12 in. (300 mm).
3. There are no movement joints or adjacent wall openings that hinder the load path of arching action.
4. The abutments are sufficiently strong and rigid to resist the horizontal thrust due to arching action.

These concepts are illustrated in Fig. 3.

Provided arching action occurs, the self weight of masonry wall carried by the beam may be safely as-
assumed as the weight within a triangular area above the beam formed by 45 degree angles, as shown in Fig. 3. The self weight of the wall must be added to the live and dead loads of floors and roofs which bear on the wall above the opening. If a stack bond pattern is used, the full area of brick masonry above the wall opening should be considered in the RBM beam design with no assumption of arching action.

**Concentrated Loads**

Loads from beams, girders, trusses and other concentrated loads that frame into the wall must be applied to the RBM beam in the appropriate manner. Concentrated loads may be assumed to be distributed over a wall length equal to the base of a trapezoid whose top is at the point of load application and whose sides make an angle of 60 degrees with the horizontal. In Fig. 4, the portion of the concentrated load carried by the beam is distributed over the length indicated as a uniform load.

The distributed load, \( w_c \), on the RBM beam is computed by the following equation:

\[
  w_c = \frac{P}{(b + 2y\tan 30)} \quad \text{Eq. 1}
\]

where:
- \( w_c \) = design uniform distributed load, lb/ft (kg/m)
- \( P \) = design concentrated load, lb (kg)

\( b \) = length of bearing plate, ft (m)

\( y \) = distance from top of beam to bearing plate, ft (m)

This is approximately 0.866 times \( P \) divided by \( y \).

Because the apex of the 45 degree triangle is above the top of the wall in this example, the RBM beam should be designed assuming no arching action occurs.

The designer should check the stress condition at bearing points for RBM beams. This applies to loads on the beam and to the beam's reaction on the wall. The MSJC Code limits the bearing stress to 0.25 \( f'_{m} \), where \( f'_{m} \) is the specified compressive strength of masonry. A rule-of-thumb recommended for many years is to provide a minimum of 4 in. (100 mm) of bearing length for masonry beams. The masonry directly beneath a bearing point should be constructed with solid brick or with solidly grouted hollow brick. Concentrated loads should not bear directly on ungrouted hollow brick masonry because of the potential for localized cracking or crushing of the face shells.

**Construction Loads**

When designing a RBM beam that is prefabricated or built on the ground and lifted into place, it is important to consider the loads during transport and handling. To address these loads, the beam may require reinforcement at both the top and bottom of the beam. Beams built in place are constructed on shores. These must be designed for the dead weight of the beam plus any superimposed load prior to adequate curing of the reinforced brickwork.

**Movement Joints**

Movement joints are a necessity in masonry walls to accommodate differential movement and avoid cracking. It is common to place vertical expansion joints at or near the jamb of wall openings. In RBM buildings there is a reduced need for expansion joints and such joints may be spaced farther apart. Refer to Technical Notes 18 Series for a discussion of the placement of movement joints. The presence of a movement joint

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**Figures**

- **Fig. 3**: Conditions for Arching Action
- **Fig. 4**: Loads on RBM Beam
near a RBM beam will influence the loads and support conditions for the beam. For example, a simple support condition should be assumed since arching action will not occur if a movement joint is at or near the jamb of the opening. Furthermore, the beam will not act as a coupling beam between shear walls. This is, in fact, one means of simplifying the design and function of a RBM beam by eliminating loads due to in-plane shear.

**DESIGN OF RBM BEAMS**

RBM beam design should not be relegated to “rule-of-thumb” methods or arbitrary selection of beam configuration and steel reinforcement. In any beam design, a careful analysis of the loads to be carried and a calculation of the resultant stresses should be incorporated to provide adequate strength and to prevent excessive cracking and deflection.

In addition to adequate strength, it is preferred that beams exhibit ductile behavior when overloaded. If the beam is overloaded, it should deform (deflect) a considerable amount prior to collapse. Deformation allows redistribution of loads to other members and provides visual indication that the beam is overloaded. Some building codes stipulate a maximum reinforcement ratio for RBM beams for this purpose.

Another aspect is the relation between the RBM beam's strength and its cracking moment. Failure of unreinforced masonry in flexure is brittle, exhibiting sudden cracking and often collapse. Consequently, a reinforced beam should provide a moment strength in excess of its cracking moment. The amount of this overstrength is somewhat arbitrary, but a factor of 1.3 is required by the Uniform Building Code[3]. This means that the moment strength of a cracked-section, RBM beam should exceed 1.3 times the cracking moment of the beam. This is not a requirement of the MSJC Code, but is considered good engineering practice.

**Beam Sizing**

In the design of an RBM beam, the required cross-sectional area of masonry is based primarily on the maximum bending moment. However, there are other factors to consider when sizing an RBM beam. For example, it is often desirable to have the width of the RBM beam coincide with the specified wall thickness. RBM beams are sometimes formed with special U-shaped, hollow brick for this reason. These brick may be manufactured specially for this purpose or they may be cut from full-size units at the site. Manufactured special shapes may not be readily available in many localities, so it is best to contact the brick manufacturer as early as possible before proceeding with a design based on their use. The beam’s depth will be determined by the appropriate number of courses of masonry units present. The beam’s depth should be taken as only those courses of solid brick or that are solidly grouted. The beam’s depth may be limited by the height of the wall above an opening. In such cases, compression steel may be necessary when sufficient masonry area is not provided.

**Lateral Bracing**

With short spans and relatively deep beams, there is little likelihood of excessive cracking, deflection or rotation. This may not be the case, however, for beams that are relatively long span, shallow or highly loaded. Such beams may be vulnerable to lateral torsional buckling. The designer should consider the lateral bracing conditions to ensure that the beam is laterally braced. The MSJC Code requires that the compression face of beams be laterally supported at a maximum spacing of 32 times the beam thickness. A brick veneer wall is laterally braced by wall ties to the backup system. A RBM beam that is part of a load-bearing wall system may not be laterally braced along its span length. In addition, movement joints at the jamb of a wall opening may result in a lack of lateral bracing for the beam at its supports. In such cases, attachment of the wall to the floor or roof diaphragm is the common means of providing lateral bracing for the beam.

**RBM Arches**

Design of RBM arches should begin with an analysis assuming the arch is un-reinforced, in accordance with Technical Notes 31A or the ARCH computer program available from the Brick Industry Association. Such an analysis will indicate the locations of highest moment and shear, and the horizontal thrust at the abutments. Should the analysis so indicate, the arch should be designed as a reinforced beam. Further, if the conditions shown in Fig. 3 are not met, or if movement joints are provided at the abutments so that the arch may spread under load, the arch should be designed as if it were a straight, simply supported beam as a conservative measure. Alternately, a finite element analysis of the arch may be conducted to determine design moment, shear, and thrust values.

RBM arches cause both a vertical bearing stress and a horizontal thrust on their abutments. The designer has the option of resisting the horizontal thrust of the arch by the abutments or providing room for movement as the RBM arch deforms under load. Judicious placement of vertical expansion joints and flashing will permit horizontal movement and simplify the arch design. This is recommended for longer span arches because providing adequate thrust resistance is difficult and movement joint spacing is limited. In this case, it is very important to provide adequate bearing at the abutments.

**STEEL REINFORCEMENT AND TIES**

The quantity of reinforcement required for an RBM beam is typically determined by the applied loads. However, the applicable building code may prescribe a minimum amount of reinforcement and this may dictate the amount of reinforcement required in a RBM beam. For example, all building codes now stipulate a minimum amount of reinforcement for masonry members in areas prone to earthquakes. Some building codes re-
quire that reinforcement in masonry coupling beams be uniformly distributed throughout the beam’s height. This may require additional reinforcement and grouting of the masonry above wall openings in RBM beams.

**Bond and Hooks**

Typically, reinforcement is inserted in masonry beams to resist tension. The tension must be transferred from the masonry to the reinforcement. This is achieved through adequate bond between the steel reinforcement and the masonry. The bond stress along the length of the reinforcement should not exceed an allowable bond stress of 160 psi (1.1 MPa), according to the MSJC Code Commentary. A minimum embedment length must be provided in order to not exceed this bond stress. Consequently, the MSJC Code stipulates a required bond length for reinforcement in tension, called the minimum embedment length. The minimum embedment length is computed by the following equation:

\[ l_e = 0.0015d_fF_s \]  
Eq. 2

where:
- \( l_e \) = embedment length of reinforcement, in. (mm)
- \( d_f \) = nominal diameter of reinforcement, in. (mm)
- \( F_s \) = allowable steel stress, psi (MPa)

Table 1 provides the minimum development lengths for various bar and wire sizes, based on Grade 60 ksi (414 MPa) reinforcing bars and 70 ksi (483 MPa) steel wire.

The ends of reinforcing bars and wires may require a standard hook to properly secure the reinforcement and to achieve its strength. In simply-supported beams, the peak moment is often at midspan. For this case, the reinforcement in RBM beams can likely be developed by the bond between the bar or wire and the surrounding masonry with no need for hooks at the ends of the beam. However, a cantilever RBM beam may require a hook at the support end. In addition, shear reinforcement should always be terminated with a hook. Standard hooks for principal reinforcement may be either a 90 degree or 180 degree turn. Often, the designated space for grout and reinforcement in RBM beams is very small. It can be difficult for a contractor to execute a reinforcement detail properly. Consider that a 180 degree hook doubles the number of bars at a given cross section. The designer should always consider the reinforcement placement, tolerances, and cover restrictions stated in the building codes. Technical Notes 17A Revised provides further information on bar sizes, placement requirements and construction tolerances.

**Shear Reinforcement**

Where shear reinforcement is required, it should be spaced so that every potential crack is crossed by shear reinforcement. Shear cracks are assumed to be oriented at a 45 degree angle to the longitudinal axis of the RBM beam. This restricts the spacing of shear reinforcement to one-half the beam’s effective depth, \( d \). The spacing of shear reinforcement may be computed by the following equation:

\[ s = A_sF_sd/V \]  
Eq. 3

where:
- \( s \) = spacing of shear reinforcement, in. (mm)
- \( A_s \) = area of shear reinforcement, in.\(^2\) (mm\(^2\))
- \( F_s \) = allowable stress for shear reinforcement, psi (MPa)
- \( d \) = effective depth of beam, in. (mm)
- \( V \) = design shear force, lb (kg)

When shear reinforcement is required, it should be designed to resist the entire shear force. Shear reinforcement should always be placed parallel to the shear force. For RBM beams the shear reinforcement should be placed vertically. It can be difficult to provide shear reinforcement in RBM beams due to the limited size of grout spaces. This is especially the case with hollow brick units 6 in. (150 mm) or less in thickness and grout spaces between wythes less than approximately 2 in. (50 mm) in width. Consequently, it may be advantageous to increase the beam’s depth so that shear reinforcement is not necessary. In fact, this is often the method used by designers to determine the minimum depth of a RBM beam required for a given loading.

**Ties**

There are two instances when it may be necessary to include ties in reinforced brick beams. These instances occur only when the beam is formed by grouting between wythes. If the beam has sufficient depth, ties may be required between the wythes. The grout exerts a hydrostatic pressure that must be resisted during construction. The MSJC requires wall ties between wythes as follows:

- Wire size W1.7 (3.8 mm), one tie per 2½ ft\(^2\) (0.25 m\(^2\))
- Wire size W2.8 (4.8 mm), one tie per 4½ ft\(^2\) (0.42 m\(^2\))

Maximum spacing of 36 in. (914 mm) horizontally and 24 in. (610 mm) vertically.

Rectangular or Z ties may be used.

In beams that form deep soffits (large beam widths) it may be advisable to tie the soffit brickwork to the grout. Although the grout does bond to the brick, the metal ties

---

**TABLE 1**

**Minimum Development Lengths**

<table>
<thead>
<tr>
<th>Reinforcement</th>
<th>Type</th>
<th>No., in. (mm)</th>
<th>Minimum Development Length, ( l_e ) in. (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bars</td>
<td>60 ksl (414 MPa)</td>
<td>3, 0.38 (9.5)</td>
<td>13.5 (343)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4, 0.50 (12.7)</td>
<td>18.0 (457)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5, 0.63 (15.9)</td>
<td>22.5 (572)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6, 0.75 (19.1)</td>
<td>27.0 (686)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7, 0.88 (22.2)</td>
<td>31.5 (800)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8, 1.00 (25.4)</td>
<td>36.0 (914)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9, 1.13 (28.7)</td>
<td>40.6 (1030)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10, 1.27 (32.3)</td>
<td>45.7 (1160)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>11, 1.41 (35.8)</td>
<td>50.8 (1290)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wires</th>
<th>70 ksl (483 MPa)</th>
<th>W1.1, 11 Gage (3.1)</th>
<th>min. 6 (152) governs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>W1.7, 9 Gage (3.8)</td>
<td></td>
<td>6.7 (170)</td>
</tr>
<tr>
<td></td>
<td>W2.1, 8 Gage (4.1)</td>
<td></td>
<td>7.3 (185)</td>
</tr>
<tr>
<td></td>
<td>W2.8, 0.188 (4.8)</td>
<td></td>
<td>8.3 (214)</td>
</tr>
<tr>
<td></td>
<td>W4.9, 0.256 (6.4)</td>
<td></td>
<td>11.3 (286)</td>
</tr>
</tbody>
</table>
should provide additional capacity and safety. Such ties are placed in the mortar joint and extend into the grout.

DEFLECTION

Deflection of RBM beams is considered a serviceability issue. Excessive deflection might cause damage to interior finishes, functional problems with doors or windows, and cracking of masonry supported by the beam. The MSJC Code requires that the deflection of RBM beams that support unreinforced or empirically-designed masonry should not exceed the lesser of 0.3 in. (7.6 mm) or span length divided by 600. Deflection of RBM beams may be computed based on uncracked or cracked section properties. Use of uncracked sections results in underestimating the deflection. Deflection based on cracked sections only are over-estimated and are more difficult to calculate. Use of uncracked section is recommended.

Creep is a time-dependent property of brick masonry that will cause the deflection of RBM beams to increase over time. An accurate formula for the estimation of long-term deflections of RBM beams due to creep, that is applicable for all cases and easy to use, does not currently exist. A rule-of-thumb is that the long-term deflection of RBM beams due to creep will be approximately 50 percent greater than their instantaneous deflection. This means that a beam that deflects 1.0 in. (25 mm) when it is fully loaded will creep over time such that its final deflection will be approximately 1.5 in. (38 mm).

DESIGN CURVES

Maximum efficiency and safety dictate the need for a rational design of all RBM beams according to the applicable building code. However, it is often helpful for the designer to have design aids that can be used to quickly develop a preliminary beam design. The design curves in Figs. 5-9 are provided for that purpose. The size and configuration of masonry and quantity of reinforcement can be quickly determined from these curves based on the span of the beam and the uniform gravity load supported by the beam, including the beam’s self-weight. The curves are based on the following assumptions:
1. Compressive strength of masonry is not less than 2000 psi (14 MPa). For most brick masonry, this value will be exceeded. This value was chosen so that beam capacity was not limited by the masonry’s compressive strength.
2. Elastic modulus of masonry is not less than 1600 ksi (11030 MPa).
3. The beam is simply supported and subject to uniform gravity loads only.
4. No compression or shear reinforcement is provided.
5. Deflection is calculated on uncracked section properties. The deflection limit of span length divided by 600 does not govern for span lengths less than 14 ft. (4.3 m).

The effective depth, d, reflected in the design curves is based on the beam height, H, minus a value for masonry cover. The cover value is based on a reasonable approximation of brick, mortar and grout cover on the underside of reinforcement for the beams shown. The actual effective depth should always be checked for each particular RBM beam configuration.

DESIGN EXAMPLE

To illustrate the use of the Design Curves, consider the following example. A RBM beam is to span over a garage door with a clear span of 9 ft (2.7 m). The beam supports its own weight and the weight of the brick masonry wall above the beam, so that the uniform load on the beam is 250 lbs/ft (372 kg/m) of span. The RBM beam and the wall above the beam are nominal 6 in. (150 mm) wide and constructed with hollow brick. Determine the beam depth and reinforcement required for these conditions. From Figs. 5(b) and 5(e), one concludes that a 4 in. (100 mm) or 8 in. (200 mm) high by 6 in. (150 mm) wide RBM beam is not adequate for the given span and loading. Therefore, the applicable Design Curve is Fig. 6(b), which is for a full unit depth, RBM beam. For the given conditions, a minimum depth of 12 in. (300 mm) and one No. 4 bar are required. At this point, any deflection criteria should be considered and may require a greater beam depth.

SUMMARY

RBM beams are an attractive and efficient means of spanning openings. Attention to detailing of reinforcement and proper design are the key aspects addressed in this Technical Notes. The most common RBM beam configurations are shown with consideration of the inter-connection of beam and wall elements. Design curves provided in this Technical Notes can be used to develop preliminary beam designs for many different applications and loading conditions.

The information and suggestions contained in this Technical Notes are based on the available data and the experience of the engineering staff of the Brick Industry Association. The information contained herein must be used in conjunction with good technical judgment and a basic understanding of the properties of brick masonry. Final decisions on the use of the information contained in this Technical Notes are not within the purview of the Brick Industry Association and must rest with the project architect, engineer and owner.

REFERENCES

Design Curves for Partial Soldier Course Beams

FIG. 6
Design Curves for Soldier Course Beams

FIG. 6
Design Curves for 12 in. (305 mm) Wide Beams

FIG. 7

ARCH 614 Note Set 27.2

S2008abn
Design Curves for 16 in. (406 mm) Wide Beams

FIG. 8

- a) $H = 8\text{ in.} (203\text{ mm})$
- b) $H = 12\text{ in.} (305\text{ mm})$
- c) $H = 16\text{ in.} (406\text{ mm})$
- d) $H = 24\text{ in.} (610\text{ mm})$
Design Curves for 24 in. (610 mm) Wide Beams

FIG. 9

385
Excerpts from NCMA TEK Manual for Concrete Masonry Design and Construction

Section Properties (14-1B 2007)

Table for Horizontal Cross Sections (net)

<table>
<thead>
<tr>
<th>Units Description</th>
<th>Grouted Spacing</th>
<th>Mortar Bedding</th>
<th>A in²/ft (10⁴mm²/m)</th>
<th>Iₘ in⁴/ft (10⁵mm⁴/m)</th>
<th>Sₘ in⁴/ft (10⁶mm⁶/m)</th>
<th>r in (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 Inch Single Wythe Walls, ¾ in. Face Shells (standard)</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>Hollow No grout</td>
<td>Faceshell</td>
<td>18.0 (38.1)</td>
<td>38.0 (51.9)</td>
<td>21.0 (1.13)</td>
<td>1.45 (36.9)</td>
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</tr>
<tr>
<td>Hollow No grout</td>
<td>Full</td>
<td>21.6 (45.7)</td>
<td>39.4 (53.8)</td>
<td>21.7 (1.17)</td>
<td>1.35 (34.3)</td>
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</tr>
<tr>
<td>100% solid/grouted</td>
<td>Full</td>
<td>43.5 (92.1)</td>
<td>47.4 (64.7)</td>
<td>26.3 (1.41)</td>
<td>1.04 (26.5)</td>
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<td>6 Inch Single Wythe Walls, 1 in. Face Shells (standard)</td>
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<tr>
<td>Hollow No grout</td>
<td>Faceshell</td>
<td>24.0 (50.8)</td>
<td>130.3 (178)</td>
<td>46.3 (2.49)</td>
<td>2.33 (59.2)</td>
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<tr>
<td>Hollow None</td>
<td>Full</td>
<td>32.2 (68.1)</td>
<td>159.3 (210)</td>
<td>49.5 (2.66)</td>
<td>2.08 (52.9)</td>
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</tr>
<tr>
<td>100% Solid/grouted</td>
<td>Full</td>
<td>67.5 (143)</td>
<td>176.9 (242)</td>
<td>63.3 (3.40)</td>
<td>1.62 (41.1)</td>
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<tr>
<td>Hollow 16” o. c.</td>
<td>Faceshell</td>
<td>46.6 (98.6)</td>
<td>158.1 (216)</td>
<td>55.1 (2.96)</td>
<td>1.79 (45.5)</td>
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<td>Hollow 24” o. c.</td>
<td>Faceshell</td>
<td>39.1 (82.7)</td>
<td>151.8 (207)</td>
<td>52.2 (2.81)</td>
<td>1.87 (47.4)</td>
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<td>Hollow 32” o. c.</td>
<td>Faceshell</td>
<td>35.3 (74.7)</td>
<td>148.7 (203)</td>
<td>50.7 (2.73)</td>
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<td>Faceshell</td>
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<td>1.94 (49.3)</td>
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<td>Faceshell</td>
<td>31.5 (66.7)</td>
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<td>2.00 (50.8)</td>
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<tr>
<td>Hollow 96” o. c.</td>
<td>Faceshell</td>
<td>27.8 (58.8)</td>
<td>142.4 (194)</td>
<td>50.6 (2.72)</td>
<td>2.02 (51.3)</td>
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<td>Hollow 122” o. c.</td>
<td>Faceshell</td>
<td>27.0 (57.1)</td>
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<td>50.4 (2.71)</td>
<td>2.03 (51.5)</td>
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<td></td>
</tr>
<tr>
<td>Hollow No grout</td>
<td>Faceshell</td>
<td>30.0 (63.5)</td>
<td>308.7 (422)</td>
<td>81.0 (4.35)</td>
<td>3.21 (81.5)</td>
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<tr>
<td>Hollow No grout</td>
<td>Full</td>
<td>41.5 (87.9)</td>
<td>334.0 (456)</td>
<td>87.6 (4.71)</td>
<td>2.84 (72.0)</td>
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<tr>
<td>100% solid/grouted</td>
<td>Full</td>
<td>91.5 (194)</td>
<td>440.2 (601)</td>
<td>116.3 (6.25)</td>
<td>2.19 (55.7)</td>
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<tr>
<td>Hollow 16” o. c.</td>
<td>Faceshell</td>
<td>62.0 (131)</td>
<td>387.1 (529)</td>
<td>99.3 (5.34)</td>
<td>2.43 (61.6)</td>
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<tr>
<td>Hollow 24” o. c.</td>
<td>Faceshell</td>
<td>51.3 (109)</td>
<td>369.4 (504)</td>
<td>93.2 (5.01)</td>
<td>2.53 (64.3)</td>
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<td>Faceshell</td>
<td>46.0 (97.3)</td>
<td>360.5 (492)</td>
<td>90.1 (4.85)</td>
<td>2.59 (65.8)</td>
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<td>Faceshell</td>
<td>42.8 (90.6)</td>
<td>355.2 (485)</td>
<td>88.3 (4.75)</td>
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<td>40.7 (86.0)</td>
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<td>2.66 (67.6)</td>
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<td>Faceshell</td>
<td>37.1 (78.5)</td>
<td>345.8 (472)</td>
<td>85.0 (4.57)</td>
<td>2.71 (69.0)</td>
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<td>Hollow 92” o. c.</td>
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<td>35.3 (74.7)</td>
<td>342.8 (468)</td>
<td>89.9 (4.83)</td>
<td>2.74 (69.6)</td>
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<tr>
<td>Hollow 120” o. c.</td>
<td>Faceshell</td>
<td>34.3 (72.6)</td>
<td>341.0 (466)</td>
<td>89.5 (4.81)</td>
<td>2.76 (70.1)</td>
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<tr>
<td>Units</td>
<td>Grouted Cores</td>
<td>Mortar Bedding</td>
<td>A</td>
<td>I_x</td>
<td>S_x</td>
<td>r</td>
</tr>
<tr>
<td>---------------------------</td>
<td>---------------</td>
<td>----------------</td>
<td>------</td>
<td>------</td>
<td>------</td>
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<td>10 Inch Single Wythe Walls, 1 ¼ in. Face Shells (standard)</td>
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<td>Faceshell</td>
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<td>530.0</td>
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<td>606.3</td>
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<td>891.7</td>
<td>185.3</td>
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<td>744.7</td>
<td>154.7</td>
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<td>140.4</td>
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<tr>
<td>12 Inch Single Wythe Walls, 1 ¼ in. Face Shells (standard)</td>
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<tr>
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<td>No grout</td>
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<td>811.2</td>
<td>139.6</td>
<td>5.20</td>
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<td>Full</td>
<td>53.1</td>
<td>971.5</td>
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<td>Full</td>
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<td>14 Inch Single Wythe Walls, 1 ¼ in. Face Shells (standard)</td>
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<td>1513.2</td>
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<td>No grout</td>
<td>Faceshell</td>
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<td>1553.7</td>
<td>198.9</td>
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<td>2030.6</td>
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<td>57.5</td>
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<td>Hollow 120” o. c.</td>
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<td>41.0</td>
<td>2146.0</td>
<td>274.7</td>
<td>5.49</td>
</tr>
</tbody>
</table>
Allowable Stresses for Unreinforced Concrete Masonry (14-7C 2012)

**Compression**

Axial ....................... \( F_a = \frac{1}{4} f'_m [1 - (h/140r)^2] \), where \( h/r \geq 99 \)

Flexural ..................... \( F_a = \frac{1}{4} f'_m (70r/h)^2 \), where \( h/r > 99 \)

Shear

where \( f_s = \frac{VQ}{I_n b} \)

\[ 1.5 \sqrt{f'_m} \leq 120 \text{ psi} \]

| Direction of flexural tensile stress and masonry type | Mortar types |  |
|------------------------------------------------------|--------------|
|                                                       | Portland cement/ lime or mortar cement | Masonry cement or air-entrained portland cement/lime |
|                                                       | M or S | N | M or S | N |
| Normal to bed joints: \( h/r \geq 99 \)             |       |   |        |    |
| Solid units                                          | 53 (366) | 40 (276) | 32 (221) | 20 (138) |
| Hollow units                                        |       |   |        |    |
| Ungrooved                                           | 33 (228) | 25 (172) | 20 (138) | 12 (83) |
| Fully grooved                                       | 86 (593) | 84 (579) | 81 (559) | 77 (531) |
| Parallel to bed joints in running bond: \( h/r > 99 \) |       |   |        |    |
| Solid units                                          | 106 (731) | 80 (552) | 64 (441) | 40 (276) |
| Hollow units                                        |       |   |        |    |
| Ungrooved & partially grouted                       | 66 (455) | 50 (345) | 40 (276) | 25 (172) |
| Fully grooved                                       | 106 (731) | 80 (552) | 64 (441) | 40 (276) |
| Parallel to bed joints in masonry not laid in running bond: |       |   |        |    |
| Continuous grout section parallel to bed joints     | 133 (917) | 133 (917) | 133 (917) | 133 (917) |
| Other                                                | 0 (0)  | 0 (0)  | 0 (0)  | 0 (0)  |

\( ^A \) For partially grouted masonry, allowable stresses are determined on the basis of linear interpolation between fully grooved hollow units and ungrooved hollow units based on amount (percentage) of grouting.

Allowable Stresses for Reinforced Concrete Masonry (14-7C 2012)

**Compression**

Axial .......................... \( P_a = \left[0.25 f'_m A_n + 0.65 A_s F_s \right] \left[1 - \left(\frac{h}{140r}\right)^2\right] \), where \( h/r \geq 99 \)

Flexural ....................... \( F_b = 0.45 f'_m \)
Shear

where \( f_v = \frac{V}{A_{nv}} \) and \( F_v = F_{vm} + F_{vs} \)

\[
M/Vd \leq 0.25 \quad \text{.................} \quad F_v = 3 \sqrt{f'_m} \\
M/Vd \geq 1.0 \quad \text{.................} \quad F_v = 2 \sqrt{f'_m} \\
M/Vd \text{ falls between .......} \text{ may be linearly interpolated}
\]

and

\[
F_{vm} = \frac{1}{2} \left[ 4.0 - 1.75 \left( \frac{M}{Vd} \right) \right] \sqrt{f'_m} + 0.25 \frac{P}{A_n} \\
F_{vs} = 0.5 \left( \frac{A_n F_d d}{A_n s} \right)
\]

Steel Reinforcement

Tension

Grade 40...................... \( F_s = 20,000 \text{ psi (137.9 MPa)} \)
Grade 60...................... \( F_s = 32,000 \text{ psi (220.7 MPa)} \)
Joint reinforcement..... \( F_s = 30,000 \text{ psi (206.9 MPa)} \)

NOTATIONS

| \( A_n \) | net cross-sectional area of masonry, in.\(^2\) (mm\(^2\)) |
| \( A_{nv} \) | net shear area, \( \text{in.}^2 \) (mm\(^2\)) |
| \( A_v \) | cross-sectional area of shear reinforcement, \( \text{in.}^2 \) (mm\(^2\)) |
| \( b \) | width of section, in. (mm) |
| \( d \) | distance from extreme compression fiber to centroid of tension reinforcement, in. (mm) |
| \( F_a \) | allowable compressive stress due to axial load only, psi (MPa) |
| \( F_b \) | allowable compressive stress due to flexure only, psi (MPa) |
| \( F_s \) | allowable tensile or compressive stress in reinforcement, psi (MPa) |
| \( F_v \) | allowable shear stress in masonry, psi (MPa) |
| \( F_{vm} \) | allowable shear resisted by the masonry, psi (MPa) |
| \( F_{vx} \) | allowable shear resisted by the shear reinforcement, psi (MPa) |
| \( f'_m \) | specified compressive strength of masonry, psi (MPa) |
| \( f_s \) | calculated shear stress in the masonry, psi (MPa) |
| \( h \) | effective height of column, wall, or pilaster, in. (mm) |
| \( I_n \) | moment of inertia of net cross-sectional area of a member, \( \text{in.}^4 \) (mm\(^4\)) |
| \( M \) | maximum moment occurring simultaneously with design shear force, \( V \), at section under consideration, \( \text{in.}^-\text{lb (N.m)} \) |
| \( P \) | axial compression load, lb (N) |
| \( P_a \) | allowable axial compressive force in a reinforced member, lb (N) |
| \( Q \) | first moment of inertia about the neutral axis of an area between the extreme fiber and the plane at which the shear stress is being calculated, \( \text{in.}^3\) (mm\(^3\)) |
| \( r \) | radius of gyration, in. (mm) |
| \( s \) | spacing of shear reinforcement, in. (mm) |
| \( V \) | design shear force, lb (N) |
System Selection and Design

from Architectural Structures, Wayne Place, Wiley, 2007:

1.1 Nature of the Process

Architects have a huge array of issues to address in architectural practice. Among these are the following: keeping rain out of a building, getting water off a site, thermal comfort, visual comfort, space planning, fire egress, fire resistance, corrosion and rot resistance, vermin resistance, marketing, client relations, the law, contracts, construction administration, the functional purposes of architecture, the role of the building in the larger cultural context, security, economy, resource management, codes and standards, and how to make a building withstand all the forces to which it will likely be subjected during its lifetime. This last subject area is referred to as architectural structures.

Because of the extraordinary range of demands on an architect's time and skills and the extraordinary number of subjects that architecture students must master, architectural structures are typically addressed in only two or three lecture courses in an accredited architectural curriculum in the United States. These two or three lecture courses must be contrasted with the ten or twelve courses that will normally be taken by a graduate of an accredited structural engineering curriculum. This contrast in level of focus makes it clear why a good structural engineering consultant is a very valuable asset to an architect.

However, having a good structural consultant does not relieve the architect of serious responsibility in the structural domain. All architects must be well versed in matters related to structures. The architect has the primary responsibility for establishing the structural concept for a building, as part of the overall design concept, and must be able to speak the language of the structural consultant with sufficient skill and understanding to take full advantage of the consultant's capabilities.

1.2 General Comments Regarding Architectural Education

Structural design is one of the more rigorous aspects of architectural design. Much knowledge has been generated and codified over the centuries that human beings have been practicing in and developing this field. This book gives primary attention to those things that are known, quantified, and codified.

However, very few things in the realm of architecture yield a single solution. To any given design problem, there are many possible solutions, and picking the best solution is often the subject of intense debate. Therefore, no one should come to this subject matter assuming that this text, or any text, is going to serve up a single, optimized solution to any design problem, unless that design problem has been so narrowly defined as to be artificial.

In design, there is always a great deal of latitude for personal expression. Design is purposeful action. The designer must have an attitude to act. Architecture students develop an attitude through a chaotic learning process involving a lot of trial and error. In going through this process, an architecture student must remain aware of a fundamental premise: the process is more important than the product; that is, the student's learning and development are more important than the output. The student has a license to make mistakes. It is actually more efficient to plow forward and make mistakes than to spend too much time trying to figure out how to do it perfectly the first time. To paraphrase the immortal words of Thomas Edison: To have good ideas, you should have many ideas and then throw out the bad ones. Of course, throwing out the bad ones requires a lot of rigorous and critical thinking. No one should ever fall in love with any idea that has not been subjected to intense and prolonged critical evaluation and withstood the test with flying colors. Furthermore, important ideas should be subjected to periodic reevaluation. Times and conditions change. Ideas that once seemed unassailable may outlive their usefulness or, at the very least, need updating in the light of new knowledge and insights.

In pursuing this subject matter, it is valuable to have a frame of reference regarding the roles of the architect, as the leader of the design team, and the structural engineer, as a crucial contributor of expertise and hard work needed to execute the project safely and effectively. The diagram in Figure 1.1 will help provide that frame of reference.

In contemplating the diagram in Figure 1.1, keep in mind that design and analysis are two sides of the same coin and that the skills and points of view of architects and engineers, although distinctive, also overlap and sometimes blur together. The most effective design teams consist of individuals with strong foci who can play their respective roles while having enough overlap in understanding and purpose that they can see each other's point of view and cooperate in working toward mutually understood and shared goals. The most harmful poison to a design team is to have such a separation in points of view and understanding that a rift develops between the members of the team. Cooperation is the watchword in this process, as in all other team efforts.
Design Criteria for the Behavior of the Overall System

Components of a system consist of vertical and horizontal elements. Connections of the vertical to horizontal elements are also necessary. For the structural elements to behave and respond as designed, the system must have the following qualities:

- the components stay together
- the system resists overturning, sliding, twisting and excessive distortion
- the system has internal stability
- the system has overall strength and stiffness

“Order” of Design

There is no set order to design of a structural system. But there are certain stages that can be recognized. These may be referred to as preliminary, revised and final, or more formally as:

First order: which can include determining structural type and organization, design intent, and contextual or programmatic emphasis. Preliminary member size charts are useful at this stage.

Second order: which can include evaluating structural strategies, choice of construction materials, and structural system options with those materials. System selection design aids are useful at this stage.

Third order: which, after the design has been narrowed down, is where analysis and design (shape and size) of individual structural elements (beams, columns, connections, etc.) is performed. The outcome here may direct further first order or second order investigations!!!
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<tbody>
<tr>
<td>Exposed, fire-resilient construction</td>
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<td>Irregular column placement</td>
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<td>Minimize floor thickness</td>
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<td>Allow for future renovations</td>
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<td>Permit construction in poor weather</td>
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<td>Minimize off-site fabrication time</td>
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<td>Minimize on-site erection time</td>
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<td>Minimize low-rise construction time</td>
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<td>Minimize medium-rise construction time</td>
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<td>Minimize high-rise construction time</td>
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<td>Minimize shear walls or diagonal bracing</td>
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<td>Minimize dead load on foundations</td>
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<td>Minimize damage due to foundation settlement</td>
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<td>Minimize the number of separate trades on job</td>
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<td>Provide concealed space for mech. services</td>
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<td>Minimize the number of supports</td>
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<td>Long spans</td>
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**RATIONALE**

- Inherently fire-resilient construction
- Simple, site-fabricated systems
- Systems without beams in roof or floors
- Precast-concrete systems without ribs
- Short-span, one-way, easily modified
- Quickly erected; avoid site-cast concrete
- Easily formed or built on site
- Highly prefabricated; modular components
- Lightweight, easily formed or prefabricated
- Precast, site-cast concrete; steel frames
- Strong; prefabricated; lightweight
- Capable of forming rigid joints
- Lightweight, short-span systems
- Systems without rigid joints
- Multipurpose components
- Systems that inherently provide voids
- Two-way, long-span systems
- Long-span systems

Figure 18.6: Framing system selection chart.

<table>
<thead>
<tr>
<th>DESIGN CRITERIA: SUMMARY CHART</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ARCH 614</strong> Note Set 28.1 S2009abn</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SYSTEM</th>
<th>CRITERIA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precast Concrete</td>
<td></td>
</tr>
<tr>
<td>Roof Deck</td>
<td></td>
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<tr>
<td>Floor Slab</td>
<td></td>
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<tr>
<td>Window</td>
<td></td>
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<tr>
<td>Door</td>
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<tr>
<td>Two-Way Floor Slab</td>
<td></td>
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<tr>
<td>One-Way Floor Slab</td>
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<tr>
<td>Fireproof</td>
<td></td>
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<tr>
<td>Stucco</td>
<td></td>
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<tr>
<td>Decorative</td>
<td></td>
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<tr>
<td>Steel</td>
<td></td>
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<tr>
<td>High Rise Connection</td>
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<tr>
<td>High Floor Connection</td>
<td></td>
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<tr>
<td>Column</td>
<td></td>
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<tr>
<td>Timber Frame</td>
<td></td>
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<tr>
<td>Truss</td>
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<tr>
<td>Panel Frame</td>
<td></td>
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</tbody>
</table>

**WOOD AND FRAME**

**STEEL**

**DESIGN CRITERIA:**

- **Give Special Consideration to the Systems Indicated if You Wish To:**
  - Create a highly irregular building form
  - Ensure the structure is strong enough to support a high load
  - Allow column placements that deviate from a regular grid
  - Minimize floor thickness
  - Minimize the area occupied by columns or bearing walls
  - Allow for changes in the building over time
  - Minimize construction under adverse weather conditions
  - Minimize on-site fabrication time
  - Minimize construction time for a one or two-story building
  - Minimize construction time for a four to ten-story building
  - Provide a foundation for the building
  - Avoid the need for dig-up beams or shear walls
  - Avoid the need for a second or third story
  - Provide dynamic support for ducts, pipes, etc.

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System Types by Material

from Structures, Schodek & Bechthold, 6th ed., Pearson, 2008:

Timber Systems

(a) Light frame construction.
(b) Stressed-skin panels.
(c) Box beams.
(b) Heavy timber construction: laminated beams.
(e) Heavy timber construction: knee-braced frame.
(f) Trusses: special designs.
(g) Trusses: mass-produced "trussed rafters" for housing.
(h) Trusses: mass-produced open-web joists.
(i) Arches laminated timber members.
(j) Folded plates.
(k) Arch panels.
(l) Lamella construction.

Reinforced Concrete Systems

(a) One-way flat plate (poured in place).
(b) One-way beam-and-slab system (poured in place).
(c) One-way panel system (poured in place).
Reinforced Concrete Systems (continued)

(d) Two-way flat plate (poured in place).
(e) Two-way flat slab (poured in place).
(f) Two-way beam-and-slab system (poured in place).
(g) Two-way waffle slab (poured in place).
(h) Prestressed long-span planks (precast).
(i) Prestressed channels (precast).
(j) Prestressed single tees (precast).
(k) Beam-and-column system (precast).
(l) Housing system (precast walls and planks post-tensioned together).

Steel Systems

(a) Steel deck and beam floor system.
(b) Steel deck and open-web bar joist system.
(c) Composite steel and concrete floor system.
(d) Plate girders.
(e) Welded trusses: double-angle members.
(f) Welded trusses: tube members.
Steel Systems (continued)

(g) Arches.
(h) Space frame.
(i) Stressed-skin space frame.

(j) Ribbed dome.
(k) Prestressed membrane structure.
(l) Folding roof cable structure.
Structural Planning

Design Issues


Lateral Stability: Wind forces and inertial forces due to ground acceleration are two types of lateral loads buildings must be designed to resist. Without resisting elements or systems, the buildings will move a little, a lot, or suddenly. Stability is the ability to flex and not suddenly “snap” or in other words, the ability to remain in the configuration intended to transfer load.

- Resisting systems include shear walls, braced frames and rigid frames:

- Configurations are important for the systems to be effective. Symmetrical or balanced arrangements are the most effective for resisting the lateral forces from all directions.
Vertical Load Resistance: Load bearing walls, columns and frames are examples of vertical load resisting elements. They can support a variety of horizontal spanning elements, such as beams and slabs. The order, or modular placement, becomes important, and uniform arrangements are economical. Load bearing walls can also function as shear walls to resist lateral loads. They are commonly constructed of reinforced concrete or masonry.

Horizontal Load Resistance: The combination of vertical and horizontal load resistance is dependant upon construction materials and size or utility of spaces. Slabs can act as diaphragms to transmit loads to the columns, shear wall or frames. They are commonly constructed of reinforced concrete. Rigid frames are commonly steel or monolithically cast reinforced concrete.
Multistory Design Issues: As a building gets taller, it is exposed to more wind load that it must resist laterally. It also increases in mass at each story, which makes the inertial forces from ground acceleration very complex. The behavior of a structure under these types of loads is dependant upon the arrangement of the masses and the stiffness and placement of the horizontal and vertical load resisting elements.

Cores are quite common to increase stiffness vertically. Unfortunately, they can’t provide effective horizontal load transfer, and should not be relied on as the sole lateral resistant mechanism! Exterior bracing or tube formations, such as the Sears Tower in Chicago, are other multistory configurations to resist lateral loads.

Vertical and horizontal “discontinuities” contribute to irregular or poor lateral response. Vertical discontinuities include “cut-outs” in stories, or changes in plan vertically, while horizontal continuities include problems such as “soft stories” which have different stiffness from the rest of the structure, and unbalanced placement of shear walls.
Structural Plans and Grids

Foundation

Figure 7.4 This foundation plan uses a grid referencing system, though not the one promoted by the National CAD Standard. Note the idiosyncrasies in this drawing: north is normally the top of the page. (From The Professional Practice of Architectural Working Drawings, 2nd edition, by Osamu Wakita and Linde, Richard, John Wiley & Sons, Inc., 1995. Used with permission of John Wiley & Sons, Inc.)

Footing Detail

Figure 8.5b Footings are often depicted in wall sections on subsequent sheets, but in this instance the engineer is showing just a footing section, denoted C S31 on the plan in 8.5a.
Steel

Figure 8.5a Drawings of structural steel framing systems begin with the foundation plan, which is where the columns and footings that carry the frame are described. (Drawing courtesy of Buehler and Buehler Structural Engineers.)

Figure 8.5c The first floor framing plan commonly shows column locations and lists girders and beams by size. The floor deck is also described on the plan. The girder designation W21 × 50 C = + ≮ (above gridline F) and 30-5-30 (below gridline F) is, respectively, the girder size and camber and number of headed stud anchors required in each third of the beam (left, center, right). The beam designation is slightly different (see lines perpendicular to girder lines). Above the beam line following the beam size is the number of headed stud anchors to be uniformly distributed between columns on the top of the beam, with the camber listed below the beam line. (Drawing courtesy of Buehler and Buehler Structural Engineers.)
Reinforced Masonry

Figure 8.6a In this partial floor plan for a reinforced masonry structure, the wall descriptions are very simple. Note the conservative use of the masonry symbol and the consequent uncluttered appearance of the drawing. The split-bubble referencing system used throughout these drawings directs the reader's attention to several details, depicted on other pages as well as the page on which they originate. Details 1 A-4/A-6 and 3 A-4/A-6 are building sections; details A and B A-4/A-4 are details of the connection to existing concrete columns; and detail E A-4/A-11 is a roof connection detail. In the upper right part of the drawing is the reference to an exterior elevation (A A-4/A-5).

Timber

Figure 8.7a This partial roof framing plan shows the glued-laminated girder and beam system. Note the weight of AC unit 1 and how the structural engineer has addressed the additional loading where mechanical equipment is supported by the roof. (Drawing courtesy of Buehler and Buehler Structural Engineers.)
Common Span Lengths and Depths:
from *Structures, 6th ed.*, Schodek & Bechthold, Pearson/Prentice Hall, 2007

**Span Range by System**

![Span Range Diagram](image)

**FIGURE 13.12** Approximate span ranges of different systems. (See also more detailed charts in Chapter 15.)
Timber

FIGURE 15.4  Approximate span ranges for timber systems. So that typical sizes of different timber members can be compared, the diagrams of the members are scaled to represent typical span lengths for each of the respective elements. The span lengths that are actually possible for each element are noted by the maximum and minimum span marks.
Reinforced Concrete

<table>
<thead>
<tr>
<th>Member Type</th>
<th>Span Lengths</th>
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<tbody>
<tr>
<td>Slabs (poured in place)</td>
<td>Simply supported: L/25, L/30, L/35</td>
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<tr>
<td></td>
<td>One end continuous: L/20</td>
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<td></td>
<td>Both ends continuous: L/23, L/26</td>
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<td></td>
<td>Cantilever: L/12</td>
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<tr>
<td>Beams (poured in place)</td>
<td>Simply supported: L/20</td>
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<tr>
<td></td>
<td>One end continuous: L/23</td>
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<td></td>
<td>Both ends continuous: L/26</td>
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<td></td>
<td>Cantilever: L/10</td>
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<tr>
<td>Pan joist system (poured in place)</td>
<td>L/20-L/25</td>
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<tr>
<td>Folded plate (poured in place)</td>
<td>L/8-L/15</td>
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<tr>
<td>Barrel shell (poured in place)</td>
<td>L/8-L/15</td>
</tr>
<tr>
<td>Planks (precast)</td>
<td>L/25-L/40</td>
</tr>
<tr>
<td>Channels (precast)</td>
<td>L/20-L/28</td>
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<tr>
<td>Tees (precast)</td>
<td>L/20-L/28</td>
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<tr>
<td>Flat plate (poured in place)</td>
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**FIGURE 15.6** Approximate span ranges for reinforced-concrete systems. So that typical sizes of different members can be compared, the diagrams of the members are scaled to represent typical span lengths for each of the respective elements. The span lengths that are actually possible for each element are noted by the maximum and minimum span marks.
Steel

FIGURE 15.9 Approximate span ranges for steel systems. So that typical sizes of different members can be compared, the diagrams of the members are scaled to represent typical span lengths for each of the respective elements. The span lengths that are actually possible for each element are noted by the maximum and minimum span marks.
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