other beams & pinned frames

Continental train platform, Grimshaw 1993
Continuous Beams

- statically indeterminate
- reduced moments than simple beam
Continuous Beams

- loading pattern affects
  - moments & deflection

\[
\Delta_{\text{max}} (0.446 \text{ from } A \text{ or } D) = 0.0069 \frac{w}{l^4} EI
\]
Continuous Beams

- unload end span
Continuous Beams

- **unload middle span**

![Diagram of continuous beam](image-url)
Moment Redistribution

- **continuous slabs & beams with uniform loading**
  - joints similar to fixed ends, but can rotate
- **change in moment to center** = \( \frac{wL^2}{8} \)
  - \( M_{\text{max}} \) for simply supported beam
Moment Distribution (a)

- no load

http://nisee.berkeley.edu/godden
Moment Distribution (b)

- add load
Moment Distribution Method (c)

- release joint 2
Moment Distribution Method (d)

- release joint 3

http://nisee.berkeley.edu/godden
Moment Distribution Method (e)

- exposure of final shape after cycles over initial shape

http://nisee.berkeley.edu/godden
Analysis Methods

• Approximate Methods
  – location of inflection points

• Force Method
  – forces are unknowns

• Displacement Method
  – displacements are unknowns
Theorem of Three Moments

- moments at three adjacent supports (2 spans)
- distributed load and same $I$:
  \[ M_1 L_1 + 2M_2 (L_1 + L_2) + M_3 L_2 = -\frac{w_1 L_1^3}{4} - \frac{w_2 L_2^3}{4} \]

- concentrated loads and same $I$:
  \[ M_1 L_1 + 2M_2 (L_1 + L_2) + M_3 L_2 = -\sum P_1 L_1^2 (n_1 - n_1^3) - \sum P_2 L_2^2 (n_2 - n_2^3) \]
Two Span Beams & Charts

- equal spans & symmetrical loading
- middle support as flat slope

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—
CONCENTRATED LOAD AT ANY POINT

\[
R_1 = V_1 = \frac{Pb}{2l^2} (a + 2l)
\]
\[
R_2 = V_2 = \frac{Pa}{2l^2} (3l^2 - a^2)
\]
\[
M_1 = \begin{cases} R_1 a & \text{at point of load} \\ R_1 x & \text{when } x < a \\ R_1 x - P (x - a) & \text{when } x > a \end{cases}
\]
\[
M_2 = \begin{cases} Pab & \text{at fixed end} \\ \frac{Pa}{2l^2} (a + l) & \text{when } x < a \\ \frac{Pa}{2l^2} (a + l) & \text{when } x > a \end{cases}
\]
\[
\Delta \text{max.} = \begin{cases} \frac{Pa}{3EI} \left( \frac{a^4 + a^2}{3l^2 - a^2} \right) & \text{at point of load} \\ \frac{Pa^2 b^2}{12EI l^3} & \text{at } x < a \\ \frac{Pa^2 b^2}{12EI l^3} & \text{at } x > a \end{cases}
\]
Pinned Frames

- structures with at least one 3 force body
- connected with pins
- reactions are equal and opposite
  - non-rigid
  - rigid
Rigid Frames

- **rigid frames have no pins**
- **frame is all one body**
- **typically statically indeterminate**
- **types**
  - portal
  - gable
Rigid Frames with PINS

- frame pieces with connecting pins
- not necessarily symmetrical
Internal Pin Connections

- statically determinant
  - 3 equations per body
  - 2 reactions per pin + support forces
Arches

- ancient
- traditional shape to span long distances
Arches

- primarily sees compression
- a brick “likes an arch”
Arches

• behavior
  – thrust related to height to width
Three-Hinged Arch

- **statically determinant**
  - 2 bodies, 6 equilibrium equations
  - 4 support, 2 pin reactions (= 6)
Beams with Internal Pins

- **statically determinant when**
  - 3 equilibrium equations per link =>
  - total of support & pin reactions (properly constrained)

- **zero moment at pins**
Procedure

• solve for all support forces you can
• draw a FBD of each member
  – pins are integral with member
  – pins with loads should belong to 3+ force bodies
  – pin forces are equal and opposite on connecting bodies
  – identify 2 force bodies vs. 3+ force bodies
  – use all equilibrium equations