Elements of Architectural Structures: Form, Behavior, and Design

ARCH 614

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Lecture 11

Rigid Frames: Compression & Buckling
Rigid Frames

- **rigid** frames have no pins
- frame is all one body
- joints transfer moments and shear
- typically statically indeterminate
- types
  - portal
  - gable
Rigid Frames

- behavior
Rigid Frames

- moments get redistributed
- deflections are smaller
- effective column lengths are shorter
- very sensitive to settling
Rigid Frames

• resists lateral loadings

• shape depends on stiffness of beams and columns

• 90° maintained
Rigid Frames

- staggered truss
  - rigidity
  - clear stories
Rigid Frames

- connections
  - steel
  - concrete
Braced Frames

- **pin connections**
- **bracing to prevent lateral movements**
Braced Frames

- types of bracing
  - knee-bracing
  - diagonal
  - X
  - K or chevron
  - shear walls
Shear Walls

- resist lateral load in plane with wall
Compression Members

- designed for strength & stresses
- designed for serviceability & deflection
- need to design for stability
  - ability to support a specified load without sudden or unacceptable deformations
Column Buckling

- axially loaded columns
- long & slender
  - unstable equilibrium = buckling
  - sudden and not good
Modeling

- can be modeled with a spring at mid-height
- when moment from deflection exceeds the spring capacity … “boing”
- critical load $P$
Effect of Length

- **long & slender**
- **short & stubby**
Buckling Load

- related to deflected shape ($P\Delta$)
- shape of sine wave
- Euler’s Formula
- smallest $I$ governs

$$P_{critical} = \frac{\pi^2 EI}{(L)^2}$$
Critical Stress

- **short columns**

\[ f_{\text{critical}} = \frac{P_{\text{actual}}}{A} < F_a \]

- **slenderness ratio** = \( L_e/r \) \((L/d)\)

- **radius of gyration** = \( r = \sqrt{\frac{I}{A}} \)

\[
f_{\text{critical}} = \frac{P_{\text{critical}}}{A} = \frac{\pi^2 EA r^2}{A (L_e)^2} = \frac{\pi^2 E}{(L_e/r)^2} \]

\[
P_{\text{critical}} = \frac{\pi^2 EA}{(L_e/r)^2}
\]
Critical Stresses

- when a column gets stubby, $F_y$ will limit the load for steel
- real world has loads with eccentricity
Effective Length

- **end conditions affect shape**
- **effective length factor, \( K \) \[ L_e = K \cdot L \]**

![Diagram showing buckled shape of column and theoretical \( K \) values](image)

<table>
<thead>
<tr>
<th>Buckled shape of column shown by dashed line</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical ( K ) value</td>
<td>0.5</td>
<td>0.7</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Recommended design values when ideal conditions are approximated</td>
<td>0.65</td>
<td>0.80</td>
<td>1.0</td>
<td>1.2</td>
<td>2.10</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Bracing

- bracing affects shape of buckle in one direction
- both should be checked!
Centric & Eccentric Loading

- **centric**
  - allowable stress from strength or buckling

- **eccentric**
  - combined stresses
Combined Stresses

- axial + bending

\[ f_{\text{max}} = \frac{P}{A} + \frac{Mc}{I} \]

\[ M = P \cdot e \]

- design

\[ f_{\text{max}} \leq F_{cr} = \frac{f_{cr}}{F.S.} \]
Stress Limit Conditions

– ASD interaction formula

\[ \frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \]

– with biaxial bending

\[ \frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \]

interaction diagram
Stress Limit Conditions

– in reality, as the column flexes, the moment increases

– $P-\Delta$ effect

\[
\frac{f_a}{F_a} + \frac{f_b \times (\text{Magnification factor})}{F_{bx}} \leq 1.0
\]
Rigid Frame Analysis

• members see
  – shear
  – axial force
  – bending

• V & M diagrams
  – plot on “outside”
Rigid Frame Analysis

- need support reactions
- free body diagram each member
- end reactions are **equal and opposite** on next member
- “turn” member like beam
- draw V & M
Rigid Frame Analysis

– FBD & M

• opposite end reactions at joints

\[ M_{BA} = \frac{Ph}{2} \]

\[ M_{BC} = \frac{Ph}{2} \]

\[ M_{CA} = \frac{Ph}{2} \]

\[ M_{CB} = \frac{Ph}{2} \]

\[ M_{CD} = \frac{Ph}{2} \]
Rigid Frame Design

• **columns in frames**
  - ends can be “flexible”
  - stiffness affected by beams and column = $EI/L$

- for the joint
  
  \[
  G = \Psi = \frac{\sum EI}{l_c} \div \frac{\sum EI}{l_b}
  \]

- $l_c$ is the column length of each column
- $l_b$ is the beam length of each beam
- measured center to center
Rigid Frame Design

- column effective length, $k$
Tools – Multiframe

- in computer lab
Tools – Multiframe

- **frame window**
  - define frame members
    - or pre-defined frame
  - select points, assign supports
  - select members, assign section
- **load window**
  - select point or member, add point or distributed loads
Tools – Multiframe

- to run analysis choose
  - Analyze menu
    - Linear

- plot
  - choose options

- results
  - choose options