Concrete construction: materials & beams
Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for maximum stresses
  - limit state design
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity
  - concrete strength = $f'_c$
Concrete Construction

- cast-in-place
- tilt-up
- prestressing
- post-tensioning
Concrete

- low strength to weight ratio
- relatively inexpensive
  - Portland cement
  - aggregate
  - water
- hydration
- fire resistant
- creep & shrink
Reinforcement

- **deformed steel bars (rebar)**
  - Grade 40, $F_y = 40 \text{ ksi}$
  - Grade 60, $F_y = 60 \text{ ksi}$ - most common
  - Grade 75, $F_y = 75 \text{ ksi}$
  - US customary in # of 1/8” $\phi$

- **longitudinally placed**
  - bottom
  - top for compression reinforcement
  - spliced, hooked, terminated...
Behavior of Composite Members

- plane sections remain plane
- stress distribution changes

\[ f_1 = E_1 \varepsilon = - \frac{E_1 y}{R} \]
\[ f_2 = E_2 \varepsilon = - \frac{E_2 y}{R} \]
Transformation of Material

• *n* is the ratio of *E*’s

\[ n = \frac{E_2}{E_1} \]

• effectively widens a material to get same stress distribution
Stresses in Composite Section

with a section transformed to one material, new I

- stresses in that material are determined as usual
- stresses in the other material need to be adjusted by $n$

$$n = \frac{E_2}{E_1} = \frac{E_{\text{steel}}}{E_{\text{concrete}}}$$

$$f_c = -\frac{M_y}{I_{\text{transformed}}}$$

$$f_s = -\frac{M_y}{I_{\text{transformed}}}$$
Reinforced Concrete - stress/strain

Stresses in the concrete above the neutral axis are compressive and nonlinearly distributed. In the tension zone below the neutral axis, the concrete is assumed to be cracked and the tensile force present to be taken up by reinforcing steel.

FIGURE 6-37  Reinforced concrete beams.

Working stress analysis. (Concrete stress distribution is assumed to be linear. Service loads are used in calculations.)

Actual stress distribution near ultimate strength (nonlinear).

Ultimate strength analysis. (A rectangular stress block is used to idealize the actual stress distribution. Calculations are based on ultimate loads and failure stresses.)
Reinforced Concrete Analysis

• for stress calculations
  – steel is transformed to concrete
  – concrete is in compression above n.a. and represented by an equivalent stress block
  – concrete takes no tension
  – steel takes tension
  – force ductile failure
Location of n.a.

- **ignore concrete below n.a.**
- **transform steel**
- **same area moments, solve for x**

\[
\frac{bx \cdot \frac{x}{2}}{2} - nA_s (d - x) = 0
\]
**T sections**

- **n.a. equation is different if n.a. below flange**

\[
b_f h_f \left( x - \frac{h_f}{2} \right) + (x - h_f) b_w \frac{(x - h_f)}{2} - nA_s (d - x) = 0
\]
ACI Load Combinations*

- $1.4D$
- $1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$
- $1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$
- $1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$
- $1.2D + 1.0E + 1.0L + 0.2S$
- $0.9D + 1.0W$
- $0.9D + 1.0E$

*can also use old ACI factors*
Reinforced Concrete Design

- stress distribution in bending

Wang & Salmon, Chapter 3
**Force Equations**

- \( C = 0.85 f'_c a \)
- \( T = A_s f_y \)
- where
  - \( f'_c = \text{concrete compressive strength} \)
  - \( a = \text{height of stress block} \)
  - \( \beta_1 = \text{factor based on } f'_c \)
  - \( x = \text{location to the n.a.} \)
  - \( b = \text{width of stress block} \)
  - \( f_y = \text{steel yield strength} \)
  - \( A_s = \text{area of steel reinforcement} \)
Equilibrium

- $T = C$
- $M_n = T(d-a/2)$
  - $d =$ depth to the steel n.a.
- with $A_s$
  - $a = \frac{A_s f_y}{0.85 f'_c b}$
- $M_u \leq \phi M_n \quad \phi = 0.9$ for flexure
- $\phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2)$
Over and Under-reinforcement

- **over-reinforced**
  - steel won’t yield

- **under-reinforced**
  - steel will yield

- **reinforcement ratio**
  - \( \rho = \frac{A_s}{bd} \)
  - use as a design estimate to find \( A_s, b, d \)
  - \( \text{max } \rho \) is found with \( \varepsilon_{\text{steel}} \geq 0.004 \) (not \( \rho_{\text{bal}} \))

http://people.bath.ac.uk/abstji/concrete_video/virtual_lab.htm
Accurate Section

- several methods
  - guess a and iterate
    1. guess a (less than n.a.)
    2. \[ A_s = \frac{0.85 f'c b a}{f_y} \]
    3. solve for a from \( M_u = \phi A_s f_y (d-a/2) \)
    \[ a = 2\left( d - \frac{M_u}{\phi A_s f_y} \right) \]
    4. repeat from 2. until a from 3. matches a in 2.
A_s for a Given Section (cont)

- chart method
  - Wang & Salmon Fig. 3.8.1 \( R_n \) vs. \( \rho \)

1. calculate \( R_n = \frac{M}{bd^2} \)

2. find curve for \( f'_c \) and \( f_y \) to get \( \rho \)

3. calculate \( A_s \) and \( a \)

- simplify by setting \( h = 1.1d \)
Reinforcement

- min for crack control
- required
  \[ A_s = \frac{3\sqrt{f'_c}}{f_y} (bd) \]
- not less than
  \[ A_s = \frac{200}{f_y} (bd) \]
- \( A_{s\text{-max}} : a = \beta_1 (0.375d) \)
- typical cover
  - 1.5 in, 3 in with soil
- bar spacing

\[ d \]
\[ b \]
\[ cover \]
\[ spacing \]
Approximate Depths