Concrete construction: shear & deflection
Shear in Concrete Beams

- flexure combines with shear to form diagonal cracks

- horizontal reinforcement doesn’t help
- stirrups = vertical reinforcement
ACI Shear Values

- $V_u$ is at distance $d$ from face of support
- shear capacity: $V_c = \frac{V_c}{n.a.} \times b_w d$

- where $b_w$ means thickness of web at n.a.
ACI Shear Values

• shear stress (beams)

\[ \nu_c = 2 \sqrt{f'_c} \]
\[ \phi V_c = \phi 2 \sqrt{f'_c} b_w d \]

\[ \phi = 0.75 \text{ for shear} \]
\[ f'_c \text{ is in psi} \]

• shear strength:

\[ V_u \leq \phi V_c + \phi V_s \]

- \( V_s \) is strength from stirrup reinforcement
Stirrup Reinforcement

- shear capacity:

\[ V_s = \frac{A_v f_y d}{s} \]

- \( A_v \) = area in all legs of stirrups
- \( s \) = spacing of stirrup

- may need stirrups when concrete has enough strength!
### Required Stirrup Reinforcement

- **Spacing limits**

<table>
<thead>
<tr>
<th>Stirrup spacing, s</th>
<th>$V_u \leq \frac{\phi V_c}{2}$</th>
<th>$\phi V_c \geq V_u &gt; \frac{\phi V_c}{2}$</th>
<th>$V_u &gt; \phi V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required area of stirrups, $A_v^{**}$</td>
<td>none</td>
<td>(\frac{50b_w s}{f_y})</td>
<td>(\frac{(V_u - \phi V_c)s}{\phi f_y d})</td>
</tr>
<tr>
<td>Required</td>
<td></td>
<td>(\frac{A_v f_y}{50b_w})</td>
<td>(\frac{\phi A_v f_y d}{V_u - \phi V_c})</td>
</tr>
<tr>
<td>Recommended Minimum$^\dagger$</td>
<td></td>
<td></td>
<td>4 in.</td>
</tr>
<tr>
<td>Maximum$^{\ddagger\ddagger}$ (ACI 11.5.4)</td>
<td>(\frac{d}{2}) or 24 in.</td>
<td>(\frac{d}{2}) or 24 in. for (V_u - \phi V_c) (\leq \phi 4\sqrt{f'_c} b_w d)</td>
<td>(\frac{d}{4}) or 12 in. for (V_u - \phi V_c) (&gt; \phi 4\sqrt{f'_c} b_w d)</td>
</tr>
</tbody>
</table>

| Table 3-8 ACI Provisions for Shear Design* |

*Members subjected to shear and flexure only; $\phi V_c = \phi 2 \sqrt{f'_c} b_w d$, $\phi = 0.65$ (ACI 11.3.1.1)

**$A_v = 2 \times A_b$ for U stirrups; $f_y \leq 60$ ksi (ACI 11.5.2)

$^\dagger$A practical limit for minimum spacing is $d/4$

$^{\ddagger\ddagger}$Maximum spacing based on minimum shear reinforcement ($= A_v f_y / 50b_w$) must also be considered (ACI 11.5.5.3).
Torsional Stress & Strain

• can see torsional stresses & twisting of axi-symmetrical cross sections
  – torque
  – remain plane
  – undistorted
  – rotates

• not true for square sections....
Shear Stress Distribution

• depend on the deformation

• $\phi =$ angle of twist
  – measure

• can prove planar section doesn’t distort
Shearing Strain

- related to $\phi$
  \[ \gamma = \frac{\rho \phi}{L} \]

- $\rho$ is the radial distance from the centroid to the point under strain

- shear strain varies linearly along the radius: $\gamma_{\text{max}}$ is at outer diameter
Torsional Stress - Strain

- know \( f_v = \tau = G \cdot \gamma \) and \( \gamma = \frac{\rho \phi}{L} \)

- so \( \tau = G \cdot \frac{\rho \phi}{L} \)

- where \( G \) is the Shear Modulus
Torsional Stress - Strain

- from

\[ T = \Sigma \tau(\rho) \Delta A \]

- can derive

\[ T = \frac{T\rho}{J} \]

- where \( J \) is the polar moment of inertia

- elastic range

\[ \tau = \frac{T\rho}{J} \]
Shear Stress

- $\tau_{\text{max}}$ happens at outer diameter

- combined shear and axial stresses
  - maximum shear stress at 45° “twisted” plane
Shear Strain

• knowing \( \tau = G \cdot \frac{\rho \phi}{L} \) and \( \tau = \frac{T \rho}{J} \)

• solve: \( \phi = \frac{TL}{JG} \)

• composite shafts: \( \phi = \sum_i \frac{T_i L_i}{J_i G_i} \)
Noncircular Shapes

- torsion depends on $J$
- plane sections don’t remain plane
- $\tau_{max}$ is still at outer diameter

$$\tau_{max} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}$$

- where $a$ is longer side ($> b$)
Open Thin-Walled Sections

- with very large $a/b$ ratios:

\[ \tau_{\text{max}} = \frac{T}{\frac{1}{3}ab^2} \]

\[ \phi = \frac{TL}{\frac{1}{3}ab^3G} \]
Shear Flow in Closed Sections

- \( q \) is the internal shear force/unit length

\[
\tau = \frac{T}{2ta}
\]

\[
\phi = \frac{TL}{4ta^2} \sum_i \frac{s_i}{t_i}
\]

- \( a \) is the area bounded by the centerline
- \( s_i \) is the length segment, \( t_i \) is the thickness
Shear Flow in Open Sections

- Each segment has proportion of $T$ with respect to torsional rigidity,

$$\tau_{\text{max}} = \frac{T t_{\text{max}}}{\frac{1}{3} \Sigma b_i t_i^3}$$

- Total angle of twist:

$$\phi = \frac{TL}{\frac{1}{3} G \Sigma b_i t_i^3}$$

- I beams - web is thicker, so $\tau_{\text{max}}$ is in web
Torsional Shear Stress

- twisting moment
- and beam shear

Design torque may not be reduced because moment redistribution is not possible

Fig. R11.6.3.1—Addition of torsional and shear stresses
Torsional Shear Reinforcement

- closed stirrups
- more longitudinal reinforcement
- area enclosed by shear flow
Development Lengths

- required to allow steel to yield ($f_y$)
- standard hooks
  - moment at beam end
- splices
  - lapped
  - mechanical connectors
Development Lengths

- $l_d$, embedment required both sides
- proper cover, spacing:
  - No. 6 or smaller
    \[ l_d = \frac{d_b F_y}{25 \sqrt{f'_c}} \quad \text{or 12 in. minimum} \]
  - No. 7 or larger
    \[ l_d = \frac{d_b F_y}{20 \sqrt{f'_c}} \quad \text{or 12 in. minimum} \]
Development Lengths

- **hooks**
  - **bend and extension**

\[ l_{dh} = \frac{1200d_b}{\sqrt{f'_c}} \]

*Figure 9-17: Minimum requirements for 90° bar hooks.*

*Figure 9-18: Minimum requirements for 180° bar hooks.*
Development Lengths

- **bars in compression**
  \[
  l_d = \frac{0.02 d_b F_y}{\sqrt{f'_c}} \leq 0.0003 d_b F_y
  \]

- **splices**
  - tension minimum is function of \( l_d \) and splice classification
  - compression minimum
  - is function of \( d_b \) and \( F_y \)
Concrete Deflections

- **elastic range**
  - I transformed
  - $E_c$ (with $f'_c$ in psi)
    - normal weight concrete ($\sim 145$ lb/ft$^3$)
      $$E_c = 57,000 \sqrt{f'_c}$$
    - concrete between 90 and 160 lb/ft$^3$
      $$E_c = w^{1.5} 33 \sqrt{f'_c}$$

- **cracked**
  - I cracked
  - $E$ adjusted
**Deflection Limits**

- relate to whether or not beam supports or is attached to a damageable non-structural element
- need to check *service* live load and long term deflection against these

<table>
<thead>
<tr>
<th>Deflection Limit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/180</td>
<td>roof systems (typical) – live</td>
</tr>
<tr>
<td>L/240</td>
<td>floor systems (typical) – live + long term</td>
</tr>
<tr>
<td>L/360</td>
<td>supporting plaster – live</td>
</tr>
<tr>
<td>L/480</td>
<td>supporting masonry – live + long term</td>
</tr>
</tbody>
</table>