concrete construction: shear & deflection
Shear in Concrete Beams

- *flexure combines with shear to form diagonal cracks*

- *horizontal reinforcement doesn’t help*

- *stirrups = vertical reinforcement*
ACI Shear Values

- $V_u$ is at distance $d$ from face of support
- shear capacity: $V_c = \nu_c \times b_w d$

- where $b_w$ means thickness of web at n.a.
ACI Shear Values

- **shear stress (beams)**
  \[ \nu_c = 2\sqrt{f'_c} \]
  \[ \phi V_c = \phi 2\sqrt{f'_c} b_w d \]
  \[ \phi = 0.75 \text{ for shear} \]
  \[ f'_c \text{ is in psi} \]

- **shear strength:**
  \[ V_u \leq \phi V_c + \phi V_s \]
  - \( V_s \) is strength from stirrup reinforcement
Stirrup Reinforcement

- **shear capacity:**

$$V_s = \frac{A_v f_y d}{s}$$

- $A_v =$ area in all legs of stirrups
- $s =$ spacing of stirrup

• may need stirrups when concrete has enough strength!
# Required Stirrup Reinforcement

- **spacing limits**

### Table 3-8 ACI Provisions for Shear Design

<table>
<thead>
<tr>
<th></th>
<th>$V_u \leq \phi V_c$</th>
<th>$\phi V_c \geq V_u &gt; \frac{\phi V_c}{2}$</th>
<th>$V_u &gt; \phi V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required area of stirrups, $A_v$ **</td>
<td>none</td>
<td>$\frac{50b_w s}{f_y}$</td>
<td>$\frac{(V_u - \phi V_c)s}{\phi f_y d}$</td>
</tr>
<tr>
<td>Stirrup spacing, $s$</td>
<td>$\frac{A_v f_y}{50b_w}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Required</td>
<td>—</td>
<td>$\frac{\phi A_v f_y d}{V_u - \phi V_c}$</td>
<td></td>
</tr>
<tr>
<td>Recommended Minimum†</td>
<td>—</td>
<td>$\frac{4\text{ in.}}{2}$</td>
<td></td>
</tr>
<tr>
<td>Maximum†† (ACI 11.5.4)</td>
<td>$\frac{d}{2}$ or 24 in.</td>
<td>$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \leq 4\sqrt{f_y} b_w d$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) &gt; 4\sqrt{f_y} b_w d$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Members subjected to shear and flexure only; $\phi V_c = \phi 2 \sqrt{f_y} b_w d$, $\phi = 0.65$ (ACI 11.3.1.1)  
**$A_v = 2 \times A_b$ for U stirrups; $f_y \leq 60$ ksi (ACI 11.5.2)  
†A practical limit for minimum spacing is $d/4$  
††Maximum spacing based on minimum shear reinforcement ($= A_v f_y / 50b_w$) must also be considered (ACI 11.5.5.3).
Torsional Stress & Strain

- can see torsional stresses & twisting of axi-symmetrical cross sections
  - torque
  - remain plane
  - undistorted
  - rotates
- not true for square sections....
Shear Stress Distribution

• depend on the deformation
• \( \phi = \text{angle of twist} \)
  – measure
• can prove planar section doesn’t distort
Shearing Strain

• related to \( \phi \)

\[
\gamma = \frac{\rho \phi}{L}
\]

• \( \rho \) is the radial distance from the centroid to the point under strain

• shear strain varies linearly along the radius: \( \gamma_{\text{max}} \) is at outer diameter
Torsional Stress - Strain

- know $f_v = \tau = G \cdot \gamma$ and $\gamma = \frac{\rho \phi}{L}$
- so $\tau = G \cdot \frac{\rho \phi}{L}$
- where $G$ is the Shear Modulus
Torsional Stress - Strain

- from
  \[ T = \Sigma \tau(\rho) \Delta A \]

- can derive
  \[ T = \frac{\tau J}{\rho} \]

- where \( J \) is the polar moment of inertia

- elastic range
  \[ \tau = \frac{TP}{J} \]
Shear Stress

• $\tau_{\text{max}}$ happens at outer diameter

• combined shear and axial stresses
  - maximum shear stress at 45° “twisted” plane
Shear Strain

- knowing \[ \tau = G \cdot \frac{\rho \phi}{L} \] and \[ \tau = \frac{T \rho}{J} \]
- solve: \[ \phi = \frac{TL}{JG} \]
- composite shafts: \[ \phi = \sum_i \frac{T_i L_i}{J_i G_i} \]
Noncircular Shapes

- torsion depends on $J$
- plane sections don’t remain plane
- $\tau_{\text{max}}$ is still at outer diameter

\[
\tau_{\text{max}} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}
\]

- where $a$ is longer side ($> b$)

<table>
<thead>
<tr>
<th>$a/b$</th>
<th>$c_1$</th>
<th>$c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.208</td>
<td>0.1406</td>
</tr>
<tr>
<td>1.2</td>
<td>0.219</td>
<td>0.1661</td>
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<tr>
<td>1.5</td>
<td>0.231</td>
<td>0.1958</td>
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<tr>
<td>2.0</td>
<td>0.246</td>
<td>0.229</td>
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<tr>
<td>2.5</td>
<td>0.258</td>
<td>0.249</td>
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<tr>
<td>3.0</td>
<td>0.267</td>
<td>0.263</td>
</tr>
<tr>
<td>4.0</td>
<td>0.282</td>
<td>0.281</td>
</tr>
<tr>
<td>5.0</td>
<td>0.291</td>
<td>0.291</td>
</tr>
<tr>
<td>10.0</td>
<td>0.312</td>
<td>0.312</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.333</td>
<td>0.333</td>
</tr>
</tbody>
</table>
Open Thin-Walled Sections

- with very large a/b ratios:

\[ \tau_{\text{max}} = \frac{T}{\frac{1}{3}ab^2} \]

\[ \phi = \frac{TL}{\frac{1}{3}ab^3G} \]
Shear Flow in Closed Sections

- $q$ is the internal shear force/unit length

$$
\tau = \frac{T}{2t\alpha}
$$

$$
\phi = \frac{TL}{4t\alpha^2} \sum_i \frac{s_i}{t_i}
$$

- $\alpha$ is the area bounded by the centerline
- $s_i$ is the length segment, $t_i$ is the thickness
Shear Flow in Open Sections

- each segment has proportion of $T$ with respect to torsional rigidity,

$$\tau_{\text{max}} = \frac{T t_{\text{max}}}{\frac{1}{3} \Sigma b_i t_i^3}$$

- total angle of twist:

$$\phi = \frac{TL}{\frac{1}{3} G \Sigma b_i t_i^3}$$

- I beams - web is thicker, so $\tau_{\text{max}}$ is in web
Torsional Shear Stress

• twisting moment
• and beam shear

Design torque may not be reduced because moment redistribution is not possible.

(a) Hollow section
(b) Solid section

Fig. R11.6.3.1—Addition of torsional and shear stresses
Torsional Shear Reinforcement

- closed stirrups
- more longitudinal reinforcement
- area enclosed by shear flow

Fig. R11.6.3.6(a)—Space truss analogy

Fig. R11.6.3.6(b)—Definition of $A_{oh}$
Development Lengths

- required to allow steel to yield ($f_y$)
- standard hooks
  - moment at beam end

- splices
  - lapped
  - mechanical connectors
Development Lengths

- $l_d$, embedment required both sides
- proper cover, spacing:
  - No. 6 or smaller
    \[ l_d = \frac{d_b F_y}{25 \sqrt{f'_c}} \]  or 12 in. minimum
  - No. 7 or larger
    \[ l_d = \frac{d_b F_y}{20 \sqrt{f'_c}} \]  or 12 in. minimum
Development Lengths

- **hooks**
  - *bend and extension*

  ![Diagram of 90° bar hook](image1)
  ![Diagram of 180° bar hook](image2)

  - **minimum**
    \[
    l_{dh} = \frac{1200d_b}{\sqrt{f'_c}}
    \]
Development Lengths

- **bars in compression**
  
  $$l_d = \frac{0.02d_b F_y}{\sqrt{f'_c}} \leq 0.0003d_b F_y$$

- **splices**
  
  - tension minimum is function of $l_d$ and splice classification
  - compression minimum
  - is function of $d_b$ and $F_y$
Concrete Deflections

- **elastic range**
  - $E_c$ (with $f'_c$ in psi)
  - $E_c = 57,000 \sqrt{f'_c}$
    - normal weight concrete ($\sim 145$ lb/ft$^3$)
    - concrete between 90 and 160 lb/ft$^3$
  - $E_c = w_c^{1.5} \cdot 33 \sqrt{f'_c}$

- **cracked**
  - $I$ cracked
  - $E$ adjusted
Deflection Limits

• relate to whether or not beam supports or is attached to a damageable non-structural element

• need to check service live load and long term deflection against these

<table>
<thead>
<tr>
<th>L/180</th>
<th>roof systems (typical) – live</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/240</td>
<td>floor systems (typical) – live + long term</td>
</tr>
<tr>
<td>L/360</td>
<td>supporting plaster – live</td>
</tr>
<tr>
<td>L/480</td>
<td>supporting masonry – live + long term</td>
</tr>
</tbody>
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