Elements of Architectural Structures: Form, Behavior, and Design
ARCH 614
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Lecture Three

Equilibrium

and planar trusses
Equilibrium

- balanced
- steady
- resultant of forces on a particle is 0
Equilibrium on a Point

- analytically

\[ R_x = \sum F_x = 0 \]
\[ R_y = \sum F_y = 0 \]

- Newton convinces us it will stay at rest
Equilibrium on a Point

• collinear force system
  – ex: cables

\[ \sum F_{\text{in-line}} = 0 \]

\[
\begin{align*}
R_x &= \sum F_x = 0 \\
R_y &= \sum F_y = 0
\end{align*}
\]
Equilibrium on a Point

- concurrent force system
  - ex: cables

\[
R_x = \sum F_x = 0
\]

\[
R_y = \sum F_y = 0
\]
Free Body Diagram

- **FBD (sketch)**
- **tool to see all forces on a body or a point including**
  - external forces
  - weights
  - force reactions
  - internal forces
Free Body Diagram

- determine point
- FREE it from:
  - ground
  - supports & connections
- draw all external forces acting ON the body
  - reactions (supporting forces)
  - applied forces
  - gravity

Sign suspended from a strut and cable.

FBD of concurrent point B.

\[ \text{FBD of concurrent point B.} \]

\[ \text{Sign suspended from a strut and cable.} \]
Free Body Diagram

• sketch FBD with relevant geometry
• resolve each force into components
  – known & unknown angles – name them
  – known & unknown forces – name them
• are any forces related to other forces?
• for the unknowns
• write only as many equilibrium equations as needed
• solve up to 2 equations
Free Body Diagram

• solve equations
  – most times 1 unknown easily solved
  – plug into other equation(s)

• common to have unknowns of
  – force magnitudes
  – force angles
Force Reactions

- result of applying force
- unknown size
- connection or support type
  - known direction
  - related to motion prevented

no vertical motion
no translation
Friction

- resistance to movement
- contact surfaces determine $\mu$
- proportion of normal force (⊥)
  - opposite to slide direction
  - static > kinetic

$$F = \mu N$$
Cable Reactions

- **equilibrium:**
  - more reactions (4) than equations
  - *but, we have slope relationships*
  - *x component the same everywhere*
Cable-Stayed Structures

- diagonal cables support horizontal spans
- typically symmetrical
- Patcenter, Rogers 1986
Patcenter, Rogers 1986

- column free space
- roof suspended
- solid steel ties
- steel frame supports masts
Patcenter, Rogers 1986

- dashes – cables pulling

Figure 3.5: Patcenter, load path diagram.
Truss Structures

• ancient (?) wood
  – Romans 500 B.C.
• Renaissance revival
• 1800’s analysis
• efficient
Truss Structures
– analogous to cables and struts

(a) STABLE: pinned supports resist thrust

(b) UNSTABLE: substitution of roller support eliminates thrust resistance

(c) STABLE: wood strut resists thrust internally to form simple truss
Truss Structures

- comprised of straight members
- geometry with triangles is stable
- loads applied only at pin joints

http://nisee.berkeley.edu/godden
Truss Structures

• 2 force members
  – forces in line, equal and opposite
  – compression
  – tension

• 3 members connected by 3 joints

• 2 more members need 1 more joint
  \[ b = 2n - 3 \]
Truss Structures

• compression and tension
Truss Structures

- statically determinate
- indeterminate
- unstable

\[ b = 21 \quad n = 12 \quad 2(n) - 3 = 2(12) - 3 = 21 \]
(a) Determinate.

\[ b = 16 \quad n = 10 \quad b = 16 < 2(10) - 3 = 17 \]
(Too few members—square panel is unstable)
(c) Unstable.

\[ b = 18 \quad n = 10 \quad b = 18 > 2(10) - 3 = 17 \]
(Too many members)
(b) Indeterminate.
Trusses

- common designs
Trusses

- common designs

- Bowstring truss
- Lenticular truss
- "Scissors" truss
- Cantilevered truss (funicularly shaped)
- Northlight trusses
- Monitors with clerestories
Trusses

- uses
  - roofs & canopies
  - long spans
  - lateral bracing
Truss Connections

• “pins”
Sainsbury Center, Foster 1978
Sainsbury Center, Foster 1978

two pin-connection supports (typical of all trusses)

see detail

third pin connection at end trusses only (makes truss and supporting columns behave as a rigid frame to minimize movement around end glazing)

tubular steel prism columns are cantilevered from foundation (rigid base connection)

prism (3-sided) roof trusses

tubular cross-bracing between columns
Truss Analysis

• visualize compression and tension from deformed shape

http://nisee.berkeley.edu/godden
Truss Analysis

- **Method of Joints**
- **Graphical Methods**
- **Method of Sections**

- all rely on equilibrium
  - of bodies
  - internal equilibrium
Method of Joints

- isolate each joint
- enforce equilibrium in $F_x$ and $F_y$
- can find all forces

- long
- easy to mess up
Joint Cases

- two bodies connected

A B C

or

A B C

equal

equal and 0
Joint Cases

- three bodies with two in line

![Diagram showing joint cases with three bodies and two in line]

or even
Joint Cases

- crossed
Tools – Multiframe

• in computer lab
Tools – Multiframe

• frame window
  – define truss members
    • or pre-defined truss
  – select points, assign supports
  – select members, assign section & assign pin ends

• load window
  – select points, add point load
Tools – Multiframe

• to run analysis choose
  – Analyze menu
    • Linear

• plot
  – choose options

• results
  – choose options