Elements of Architectural Structures: Form, Behavior, and Design
ARCH 614
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Lecture Three

Equilibrium and planar trusses
Equilibrium

- balanced
- steady
- resultant of forces on a particle is 0

\[ \mathbf{X} = \mathbf{X} \]
Equilibrium on a Point

• analytically

\[ R_x = \sum F_x = 0 \]
\[ R_y = \sum F_y = 0 \]

• Newton convinces us it will stay at rest
Equilibrium on a Point

- collinear force system
  - ex: cables

\[ \sum F_{\text{in-line}} = 0 \]

\[
\begin{align*}
R_x &= \sum F_x = 0 \\
R_y &= \sum F_y = 0
\end{align*}
\]
Equilibrium on a Point

- concurrent force system
  - ex: cables

\[
R_x = \sum F_x = 0
\]

\[
R_y = \sum F_y = 0
\]
Free Body Diagram

- **FBD (sketch)**
- **tool to see all forces on a body or a point including**
  - external forces
  - weights
  - force reactions
  - internal forces
Free Body Diagram

• **determine point**

• **FREE it from:**
  – ground
  – supports & connections

• **draw all external forces acting ON the body**
  – reactions (supporting forces)
  – applied forces
  – gravity
Free Body Diagram

• sketch FBD with relevant geometry
• resolve each force into components
  – known & unknown angles – name them
  – known & unknown forces – name them
• are any forces related to other forces?
• for the unknowns
• write only as many equilibrium equations as needed
• solve up to 2 equations
Free Body Diagram

• solve equations
  – most times 1 unknown easily solved
  – plug into other equation(s)

• common to have unknowns of
  – force magnitudes
  – force angles
Force Reactions

• result of applying force
• unknown size
• connection or support type
  – known direction
  – related to motion prevented

no vertical motion  no translation
Friction

- resistance to movement
- contact surfaces determine $\mu$
- proportion of normal force ($N$) – opposite to slide direction
  - static $>$ kinetic

$$F = \mu N$$
Cable Reactions

• equilibrium:
  – more reactions (4) than equations
  – but, we have slope relationships
  – x component the same everywhere
Cable-Stayed Structures

- diagonal cables support horizontal spans
- typically symmetrical
- Patcenter, Rogers 1986


Patcenter, Rogers 1986

- column free space
- roof suspended
- solid steel ties
- steel frame supports masts

![Diagram of architectural structure](image)

- tubular steel bipod masts
- primary rod stays
- secondary rod stays
- ring connector
- vertical tie prevents wind uplift

- laboratory
- mechanical equipment exposed
- circ.
Patcenter, Rogers 1986

- dashes – cables pulling

Figure 3.5: Patcenter, load path diagram.
Truss Structures

- ancient (?) wood
  - Romans 500 B.C.
- Renaissance revival
- 1800’s analysis
- efficient
Truss Structures

– analogous to cables and struts

STABLE:
- pinned supports
- resist thrust

UNSTABLE:
- substitution of roller support
- eliminates thrust resistance

STABLE:
- wood strut
- resists thrust internally to form simple truss
Truss Structures

- comprised of straight members
- geometry with triangles is stable
- loads applied only at pin joints
Truss Structures

- 2 force members
  - forces in line, equal and opposite
  - compression
  - tension

- 3 members connected by 3 joints

- 2 more members need 1 more joint \( b = 2n - 3 \)
Truss Structures

• compression and tension
Truss Structures

- statically determinate
- indeterminate
- unstable

\[ b = 21 \quad n = 12 \quad 2(n) - 3 = 2(12) - 3 = 21 \]

(a) Determinate.

\[ b = 16 \quad n = 10 \quad b = 16 < 2(10) - 3 = 17 \]

(Too few members—square panel is unstable)

(c) Unstable.

\[ b = 18 \quad n = 10 \quad b = 18 > 2(10) - 3 = 17 \]

(Too many members)

(b) Indeterminate.
Trusses

• common designs

(a) King post
   Inverted king post

(b) Queen post
   Inverted queen post

(c) Pitched Pratt truss

(d) Pitched Howe truss

(e) Constant forces in upper chords and no forces in diagonals (normally built with rigid joints if diagonals are omitted).

(f) Pitched Fink truss

(g) Parallel chord
   Pratt truss

(h) Parallel chord
   Howe truss

(i) Parallel chord
   Warren trusses

(j) Parallel chord
   Crossed-diagonal truss
Trusses

- common designs

(i) Bowstring truss

(m) Lenticular truss

(w) "Scissors" truss

(x) Cantilevered truss (funicularly shaped)

(y) Northlight trusses

(z) Monitors with clerestories
Trusses

- uses
  - roofs & canopies
  - long spans
  - lateral bracing
Truss Connections

• “pins”
Sainsbury Center, Foster 1978
two pin-connection supports (typical of all trusses)

see detail

third pin connection at end trusses only (makes truss and supporting columns behave as a rigid frame to minimize movement around end glazing)

tubular steel prism columns are cantilevered from foundation (rigid base connection)

prism (3-sided) roof trusses
tubular cross-bracing between columns
Truss Analysis

- visualize compression and tension from deformed shape

http://nisee.berkeley.edu/godden
Truss Analysis

- Method of Joints
- Graphical Methods
- Method of Sections

- all rely on equilibrium
  - of bodies
  - internal equilibrium
Method of Joints

- isolate each joint
- enforce equilibrium in $F_x$ and $F_y$
- can find all forces

- long
- easy to mess up
Joint Cases

- two bodies connected

![Diagram showing joint cases](image)
Joint Cases

- three bodies with two in line
Joint Cases

- crossed
Tools – Multiframe

- in classrooms and open access labs
Tools – Multiframe

- **frame window**
  - define truss members
    - or pre-defined truss
  - select points, assign supports
  - select members, assign section & assign pin ends

- **load window**
  - select points, add point load
Tools – Multiframe

- to run analysis choose
  - Analyze menu
    - Linear
- plot
  - choose options
- results
  - choose options