Elements of Architectural Structures: Form, Behavior, and Design
ARCH 614
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Lecture four

Mechanics of materials

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Mechanics of Materials

- external loads and their effect on deformable bodies
- use it to answer question if structure meets requirements of
  - stability and equilibrium
  - strength and stiffness
- other principle building requirements
  - economy, functionality and aesthetics
Knowledge Required

- material properties
- member cross sections
- ability of a material to resist breaking
- structural elements that resist excessive
  - deflection
  - deformation

Figure 2.34 An example of torsion on a cantilever beam.
Problem Solving

1. STATICS:
   equilibrium of external forces, internal forces, stresses

2. GEOMETRY:
   cross section properties, deformations and conditions of geometric fit, strains

3. MATERIAL PROPERTIES:
   stress-strain relationship for each material obtained from testing
Stress

• stress is a term for the intensity of a force, like a pressure
• internal or applied
• force per unit area

\[ \text{stress} = f = \frac{P}{A} \]
Design

- materials have a critical stress value where they could break or yield
  - ultimate stress
  - yield stress
  - compressive stress
  - fatigue strength
  - (creep & temperature)

Design acceptance vs. failure
Design (cont)

- we’d like
  \[ f_{\text{actual}} << F_{\text{allowable}} \]
- stress distribution may vary: average
- uniform distribution exists IF the member is loaded axially (concentric)
Scale Effect

- **model scale**
  - material weights by volume, small section areas

- **structural scale**
  - much more material weight, bigger section areas

- **scale for strength is not proportional:**
  \[ \frac{\gamma L^3}{L^2} = \gamma L \]
Normal Stress (direct)

- **normal stress is normal to the cross section**
  - stressed area is perpendicular to the load

\[
\sigma = \frac{P}{A}
\]

Figure 5.7  Two columns with the same load, different stress.
Shear Stress

• stress parallel to a surface

\[ f_v = \frac{P}{A} = \frac{P}{td} \]
Bearing Stress

- stress on a surface by contact in compression

\[ f_p = \frac{P}{A} = \frac{P}{td} \]
Bending Stress

- normal stress caused by bending

\[
f_b = \frac{Mc}{I} = \frac{M}{S}
\]

Figure 8.8 Bending stresses on section b-b.
Torsional Stress

- shear stress caused by twisting

\[ f_v(\tau) = \frac{T \rho}{J} \]
Structures and Shear

- what structural elements see shear?
  - beams
  - bolts
  - splices
  - slabs
  - footings
  - walls
    - wind
    - seismic loads
Bolts

- connected members in tension cause shear stress

- connected members in compression cause bearing stress
Single Shear

- seen when 2 members are connected

\[ f_v = \frac{P}{A} = \frac{P}{\pi \frac{d^2}{4}} \]
Double Shear

- seen when 3 members are connected
- two areas

$$F = \frac{P}{2A} = \frac{P}{2A} = \frac{P}{\pi d^2/4}$$

Free-body diagram of middle section of the bolt in shear.

Figure 5.12  A bolted connection in double shear.
Bolt Bearing Stress

- compression & contact
- projected area

\[ f_p = \frac{P}{A_{\text{projected}}} = \frac{P}{td} \]
Strain

- materials deform
- axially loaded materials change length
- bending materials deflect

\[
\text{strain} = \varepsilon = \frac{\Delta L}{L}
\]
Shearing Strain

- deformations with shear
- parallelogram
- change in angles
- stress: $\tau$
- strain: $\gamma$
  - unitless (radians)

\[
\gamma = \frac{\delta_s}{L} = \tan \phi \approx \phi
\]
Shearing Strain

- deformations with torsion
- twist
- change in angle of line
- stress: \( \tau \)
- strain: \( \gamma = \frac{\rho \phi}{L} \)
- unitless (radians)
Load and Deformation

- for stress, need $P$ & $A$
- for strain, need $\delta$ & $L$
  - how?
  - TEST with load and measure
  - plot $P/A$ vs. $\varepsilon$
Material Behavior

- every material has its own response
  - 10,000 psi
  - \( L = 10 \text{ in} \)
  - Douglas Fir vs. steel?

Figure 5.20 Stress-strain diagram for various materials.
Behavior Types

• ductile - “necking”
• true stress

\[ f = \frac{P}{A} \]

• engineering stress
  – (simplified)

\[ f = \frac{P}{A_o} \]
Behavior Types

- brittle

- semi-brittle
Stress to Strain

• important to us in $f-\varepsilon$ diagrams:
  – straight section
  – LINEAR-ELASTIC
  – recovers shape (no permanent deformation)
Hooke’s Law

- straight line has constant slope
- Hooke’s Law

\[ f = E \cdot \varepsilon \]

- \( E \)
  - Modulus of elasticity
  - Young’s modulus
  - units just like stress
Stiffness

- **ability to resist strain**

- **steels**
  - same $E$
  - different *yield points*
  - different *ultimate strength*

*Figure 5.20 Stress-strain diagram for various materials.*
Isotropy & Anisotropy

• **ISOTROPIC**
  
  – materials with $E$ same at any direction of loading
  
  – ex. steel

• **ANISOTROPIC**
  
  – materials with different $E$ at any direction of loading
  
  – ex. wood is orthotropic
Elastic, Plastic, Fatigue

- elastic springs back
- plastic has permanent deformation
- fatigue caused by reversed loading cycles
Plastic Behavior

- ductile

Figure 5.22  Stress-strain diagram for mild steel (A36) with key points highlighted.
Lateral Strain

- or “what happens to the cross section with axial stress”

\[ \varepsilon_x = \frac{f_x}{E} \]

\[ f_y = f_z = 0 \]

- strain in lateral direction
  - negative
  - equal for isometric materials

\[ \varepsilon_y = \varepsilon_z \]
Poisson’s Ratio

- constant relationship between longitudinal strain and lateral strain

\[ \mu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x} \]

\[ \varepsilon_y = \varepsilon_z = -\frac{\mu f_x}{E} \]

- sign! \[ 0 < \mu < 0.5 \]
Calculating Strain

• from Hooke’s law

\[ f = E \cdot \varepsilon \]

• substitute

\[ \frac{P}{A} = E \cdot \frac{\delta}{L} \]

• get \( \Rightarrow \)

\[ \delta = \frac{PL}{AE} \]
Orthotropic Materials

- non-isometric
- directional values of $E$ and $\mu$
- ex:
  - plywood
  - laminates
  - polymer composites
Stress Concentrations

- why we use $f_{ave}$
- increase in stress at changes in geometry
  - sharp notches
  - holes
  - corners
Maximum Stresses

- if we need to know where $max f$ and $f_v$ happen:

$$\theta = 0^\circ \rightarrow \cos \theta = 1 \quad f_{max} = \frac{P}{A_o}$$

$$\theta = 45^\circ \rightarrow \cos \theta = \sin \theta = \sqrt{0.5} \quad f_{v-max} = \frac{P \cos \theta}{2A_o} = \frac{f_{max}}{2}$$
Maximum Stresses

**FIG. 2-37** Shear failure along a 45° plane of a wood block loaded in compression

**FIG. 2-38** Slip bands (or Lüders' bands) in a polished steel specimen loaded in tension
Deformation Relationships

- physical movement
  - axially (same or zero)
  - rotations from axial changes

\[ \delta = \frac{PL}{AE} \]

relates \( \delta \) to \( P \)
Deformations from Temperature

- atomic chemistry reacts to changes in energy
- solid materials
  - can contract with decrease in temperature
  - can expand with increase in temperature
- linear change can be measured per degree
Thermal Deformation

- $\alpha$ - the rate of strain per degree
- **UNITS**: $^\circ\text{F}$, $^\circ\text{C}$
- **length change**: $\delta_T = \alpha(\Delta T)L$
- **thermal strain**: $\varepsilon_T = \alpha(\Delta T)$
  
  – no stress when movement allowed
## Coefficients of Thermal Expansion

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficients ($\alpha$) [in./in./°F]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood</td>
<td>$3.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>Glass</td>
<td>$4.4 \times 10^{-6}$</td>
</tr>
<tr>
<td>Concrete</td>
<td>$5.5 \times 10^{-6}$</td>
</tr>
<tr>
<td>Cast Iron</td>
<td>$5.9 \times 10^{-6}$</td>
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<tr>
<td>Steel</td>
<td>$6.5 \times 10^{-6}$</td>
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<tr>
<td>Wrought Iron</td>
<td>$6.7 \times 10^{-6}$</td>
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<tr>
<td>Copper</td>
<td>$9.3 \times 10^{-6}$</td>
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<tr>
<td>Bronze</td>
<td>$10.1 \times 10^{-6}$</td>
</tr>
<tr>
<td>Brass</td>
<td>$10.4 \times 10^{-6}$</td>
</tr>
<tr>
<td>Aluminum</td>
<td>$12.8 \times 10^{-6}$</td>
</tr>
</tbody>
</table>
Stresses and Thermal Strains

- if thermal movement is restrained stresses are induced

1. bar pushes on supports
2. support pushes back
3. reaction causes internal stress

\[
f = \frac{P}{A} = \frac{\delta}{L} E
\]
Superposition Method

- can remove a support to make it look determinant
- replace the support with a reaction
- enforce the geometry constraint
Superposition Method

- total length change restrained to zero

constraint: $\delta_p + \delta_T = 0$

$$\delta_p = -\frac{PL}{AE}$$

$$\delta_T = \alpha(\Delta T)L$$

sub:

$$-\frac{PL}{AE} + \alpha(\Delta T)L = 0$$

$$f = -\frac{P}{A} = -\alpha(\Delta T)E$$
Dynamics

- **kinematics**
  - time, velocity, acceleration
  - linear motion \( s(t) = v(0)t + \frac{1}{2}at^2 \)
  - angular rotation

- **kinetics**
  - forces causing motion \( W = m \cdot g \)
  - work
  - conservation of energy
Dynamic Response

Lateral ground motions associated with earthquakes cause inertial forces to develop that are dependent on the weight of the structure. Sliding failures can occur.

The lateral ground motions can also cause a sculpture to overturn. The magnitude of the overturning effect depends on the weight of the sculpture and its height above the ground.

Back and forth ground motions can cause different parts of the sculpture to move in different directions. Overturning or cracking of elements can consequently occur.

Statue in front of the cathedral of Palermo, Sicily
Dynamic Response

- **period of vibration or frequency**
  - wave
  - sway/time period
- **damping**
  - reduction in sway
- **resonance**
  - amplification of sway

Properties of a sine wave:

- $y = A \sin(2\pi ft)$
- Frequency, $f = \frac{1}{T}$
- Wavelength, $\lambda$ (or Period, $T$)
- Amplitude, $A$
- Peak
- Distance, $x$ (or Time, $t$)
- Trough
Frequency and Period

- **natural period of vibration**
  - avoid resonance
  - hard to predict seismic period
  - affected by soil
  - short period
    - high stiffness
  - long period
    - low stiffness

“To ring the bell, the sexton must pull on the downswing of the bell in time with the natural frequency of the bell.”
Design of Members

• beyond allowable stress…

• materials aren’t uniform 100% of the time
  – ultimate strength or capacity to failure may be different and some strengths hard to test for

• RISK & UNCERTAINTY

\[ f_u = \frac{P_u}{A} \]
Factor of Safety

- accommodate uncertainty with a safety factor:

\[ \text{allowable load} = \frac{\text{ultimate load}}{F.S} \]

- with linear relation between load and stress:

\[ F.S = \frac{\text{ultimate load}}{\text{allowable load}} = \frac{\text{ultimate stress}}{\text{allowable stress}} \]
Load and Resistance Factor Design

- **loads on structures are**
  - not constant
  - can be more influential on failure
  - happen more or less often
  - UNCERTAINTY

\[ R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n \]

- \(\phi\) - resistance factor
- \(\gamma\) - load factor for (D)ead & (L)ive load