Elements of Architectural Structures:
Form, Behavior, and Design
ARCH 614
Dr. Anne Nichols
Spring 2014

Lecture 18

Steel Columns

Elements of Architectural Structures
ARCH 614

Steel Columns: Column Design

Structural Steel
- standard rolled shapes (W, C, L, T)
- tubing
- pipe
- built-up

Steel Columns 2

Design Methods (revisited)
- know
  - loads or lengths
- select
  - section or load
  - adequate for strength and no buckling

Steel Columns 3

Allowable Stress Design (ASD)
- AICS 9th ed

\[ F_a = \frac{f_{\text{critical}}}{F.S.} = \frac{12\pi^2 E}{23(\frac{Kl}{r})^2} \]

- slenderness ratio \( \frac{Kl}{r} \)
  - for \( kl/r \geq C_c \)
    - \( F_y = 36 \) ksi \( \Rightarrow 126.1 \)
    - \( F_y = 50 \) ksi \( \Rightarrow 107.0 \)
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C_c and Euler's Formula

• K_l/r < C_c
  – short and stubby
  – parabolic transition

• K_l/r > C_c
  – Euler's relationship
  – < 200 preferred

\[ C_c = \sqrt{\frac{2\pi^2 E}{F_y}} \]

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Short / Intermediate

• L_e/r < C_c

\[ F_a = 1 - \left( \frac{K_l/r}{2C_c^2} \right) \frac{F_y}{F.S.} \]

– where

\[ F.S. = \frac{5}{3} + \frac{3(K_l/r)}{8C_c} - \frac{(K_l/r)^3}{8C_c^3} \]

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Unified Design

• limit states for failure

\[ P_a \leq \frac{P_n}{\phi_c} \]

\[ P_u \leq \phi_c P_n \]

1. yielding

\[ \frac{K L}{r} \leq 4.71 \sqrt{\frac{E}{F_y}} \quad \text{or} \quad F_c \geq 0.44F_y \]

2. buckling

\[ \frac{K L}{r} > 4.71 \sqrt{\frac{E}{F_y}} \quad \text{or} \quad F_c < 0.44F_y \]

\[ F_e \quad \text{– elastic buckling stress (Euler)} \]
Unified Design

- \( P_n = F_{cr} A_g \)
  - for \( \frac{KL}{r} \leq 4.71 \frac{E}{F_y} \)
  - for \( \frac{KL}{r} > 4.71 \frac{E}{F_y} \)
  - where \( F_e = \frac{\pi^2 E}{(KL/r)^2} \)

Procedure for Analysis

1. calculate KL/r
   - biggest of KL/r with respect to x axes and y axis
2. find \( F_{cr} \) (see Note) from appropriate equation
   - tables are available
3. compute \( P_n = F_{cr} A_g \)
4. is \( P_a \leq P_n/\Omega \) or is \( P_u \leq \phi P_n \)?
   - yes: ok
   - no: insufficient capacity and no good

Procedure for Design

1. guess a size (pick a section)
2. calculate KL/r
   - biggest of KL/r with respect to x axes and y axis
3. find \( F_a \) or \( F_{cr} \) (see Note) from appropriate equations
   - or find a chart
4. compute \( P_n = F_{cr} A_g \)

Procedure for Design (cont’d)

5. is \( P_a \leq P_n/\Omega \) or is \( P_u \leq \phi P_n \)?
   - yes: ok
   - no: pick a bigger section and go back to step 2.
6. check design efficiency
   - percentage of stress = \( \frac{P_r}{P_c} \cdot 100\% \)
   - if between 90-100%: good
   - if < 90%: pick a smaller section and go back to step 2.
Column Charts, $\phi F_{cr}$

Available Critical Stress, $\phi F_{cr}$, for Compression Members, ksf ($F_y = 50$ ksf and $\phi_c = 0.90$)

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Beam-Column Design

- moment magnification (P-$\Delta$)

$$M_u = B_1 M_{max-factored} \quad B_1 = \frac{C_m}{1 - (P_u/P_{e1})}$$

$C_m$ – modification factor for end conditions

= 0.6 – 0.4($M_y/M_2$) or

0.85 restrained, 1.00 unrestrained

$P_{e1}$ – Euler buckling strength

$P_{e1} = \frac{\pi^2 E A}{(K/\ell)^2}$

Beam-Column Design

- LRFD Steel

$$\frac{P_r}{P_c} \geq 0.2 : \quad \frac{P_r}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0$$

$$\frac{P_r}{P_c} < 0.2 : \quad \frac{P_r}{2\phi_c P_n} + \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0$$

$P_r$ is required, $P_c$ is capaci

$\phi_c$ - resistance factor for compression = 0.9

$\phi_b$ - resistance factor for bending = 0.9
Design Steps Knowing Loads (revisited)

1. assume limiting stress
   • buckling, axial stress, combined stress
2. solve for $r, A$ or $S$
3. pick trial section
4. analyze stresses
5. section ok?
6. stop when section is ok

Rigid Frame Design (revisited)

• columns in frames
  – ends can be “flexible”
  – stiffness affected by beams and column $= EI/L$

$$G = \Psi = \frac{\sum EI}{l_c}$$

– for the joint
  • $l_c$ is the column length of each column
  • $l_b$ is the beam length of each beam
  • measured center to center

Rigid Frame Design (revisited)

• column effective length, $k$