Concrete Construction

- cast-in-place
- tilt-up
- prestressing
- post-tensioning

Concrete

- low strength to weight ratio
- relatively inexpensive
  - Portland cement
  - aggregate
  - water
- hydration
- fire resistant
- creep & shrink

Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for maximum stresses
  - limit state design
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity
  - concrete strength = $f'_c$

Concrete Construction: materials & beams

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**Reinforcement**

- deformed steel bars (rebar)
  - Grade 40, $F_y = 40$ ksi
  - Grade 60, $F_y = 60$ ksi - most common
  - Grade 75, $F_y = 75$ ksi
  - US customary in # of 1/8” φ
- longitudinally placed
  - bottom
  - top for compression reinforcement
  - spliced, hooked, terminated...

**Behavior of Composite Members**

- plane sections remain plane
- stress distribution changes

\[
f_1 = E_1 \varepsilon = - \frac{E_1 y}{R}
\]
\[
f_2 = E_2 \varepsilon = - \frac{E_2 y}{R}
\]

**Transformation of Material**

- $n$ is the ratio of $E$’s
  \[
n = \frac{E_2}{E_1}
\]
  - effectively widens a material to get same stress distribution

**Stresses in Composite Section**

- with a section transformed to one material, new $I$
  - stresses in that material are determined as usual
  - stresses in the other material need to be adjusted by $n$

\[
n = \frac{E_2}{E_1} = \frac{E_{steel}}{E_{concrete}}
\]
\[
f_c = - \frac{M_y}{I_{transformed}}
\]
\[
f_s = - \frac{Myn}{I_{transformed}}
\]
**Reinforced Concrete - stress/strain**

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent stress block
  - concrete takes no tension
  - steel takes tension
  - force ductile failure

**Location of n.a.**

- ignore concrete below n.a.
- transform steel
- same area moments, solve for $x$

$$bx \cdot \frac{x}{2} - nA_s (d - x) = 0$$

**T sections**

- n.a. equation is different if n.a. below flange

$$b_f h_f \left( x - \frac{h_f}{2} \right) + (x - h_f) b_w \frac{(x - h_f)}{2} - nA_s (d - x) = 0$$
**ACI Load Combinations**

- **1.4D**
- **1.2D + 1.6L + 0.5(L_r or S or R)**
- **1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)**
- **1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)**
- **1.2D + 1.0E + 1.0L + 0.5(L_r or S or R)**
- **0.9D + 1.0W**
- **0.9D + 1.0E**

*can also use old ACI factors*

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**Reinforced Concrete Design**

- **stress distribution in bending**

![Diagram](image)

- **Wang & Salmon, Chapter 3**

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**Force Equations**

- **C = 0.85 f’_c ba**
- **T = A_s f_y**
- **where**
  - **f’_c** = concrete compressive strength
  - **a** = height of stress block
  - **β_1** = factor based on **f’_c**
  - **x** = location to the n.a.
  - **b** = width of stress block
  - **f_y** = steel yield strength
  - **A_s** = area of steel reinforcement

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**Equilibrium**

- **T = C**
- **M_n = T(d-a/2)**
  - **d** = depth to the steel n.a.
- **with** **A_s**
  - **a = A_s f_y / 0.85 f’_c b**
  - **M_u ≤ φ M_n**
  - **M_u = φ T(d-a/2) = φ A_s f_y (d-a/2)**
**Over and Under-reinforcement**

- **over-reinforced**
  - steel won’t yield
- **under-reinforced**
  - steel will yield
- **reinforcement ratio**
  - \( \rho = \frac{A_s}{bd} \)
  - use as a design estimate to find \( A_s, b, d \)
  - \( \text{max } \rho \) is found with \( \varepsilon_{\text{steel}} \geq 0.004 \) (not \( \rho_{\text{bal}} \))

**A_s for a Given Section**

- **several methods**
  - guess a and iterate
    1. guess a (less than n.a.)
    2. \( A_s = \frac{0.85 f'_c b a}{f_y} \)
    3. solve for a from \( M_n = \phi A_s f_y (d-a/2) \)
    \[ a = 2 \left( d - \frac{M_n}{\phi A_s f_y} \right) \]
    4. repeat from 2. until a from 3. matches a in 2.

**A_s for a Given Section (cont)**

- **chart method**
  - Wang & Salmon Fig. 3.8.1 \( R_n \) vs. \( \rho \)
    1. calculate \( R_n = \frac{M_n}{bd^2} \)
    2. find curve for \( f'_c \) and \( f_y \) to get \( \rho \)
    3. calculate \( A_s \) and a
  - simplify by setting \( h = 1.1d \)

**Reinforcement**

- **min for crack control**
- **required**
  \[ A_s = \frac{3\sqrt{f'_c}}{f_y} (bd) \]
- **not less than**
  \[ A_s = \frac{200}{f_y} (bd) \]
- **\( A_{s-\text{max}} \)**
  \[ a = \beta_1 (0.375d) \]
- **typical cover**
  - 1.5 in, 3 in with soil
- **bar spacing**

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http://people.bath.ac.uk/abstji/concrete_video/virtual_lab.htm
Approximate Depths

Concrete Beams  Lecture 21
Elements of Architectural Structures
ARCH 614 S2009abn

[Diagram showing various beam depths and spans]