Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for maximum stresses
  - limit state design
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity
  - concrete strength = f'_c

Concrete Construction

- cast-in-place
- tilt-up
- prestressing
- post-tensioning

Concrete

- low strength to weight ratio
- relatively inexpensive
  - Portland cement
  - aggregate
  - water
- hydration
- fire resistant
- creep & shrink
Reinforcement

- deformed steel bars (rebar)
  - Grade 40, $F_y = 40$ ksi
  - Grade 60, $F_y = 60$ ksi - most common
  - Grade 75, $F_y = 75$ ksi
  - US customary in # of 1/8” φ
- longitudinally placed
  - bottom
  - top for compression reinforcement
  - spliced, hooked, terminated...

Behavior of Composite Members

- plane sections remain plane
- stress distribution changes

\[ f_1 = E_1 \epsilon = -\frac{E_1 y}{R} \]
\[ f_2 = E_2 \epsilon = -\frac{E_2 y}{R} \]

Transformation of Material

- $n$ is the ratio of $E$’s
  \[ n = \frac{E_2}{E_1} \]
- effectively widens a material to get same stress distribution

Stresses in Composite Section

- with a section transformed to one material, new $I$
  - stresses in that material are determined as usual
  - stresses in the other material need to be adjusted by $n$

\[ f_c = -\frac{M y}{I_{\text{transformed}}} \]
\[ f_s = -\frac{M y n}{I_{\text{transformed}}} \]
Reinforced Concrete - stress/strain

Reinforced Concrete Analysis

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent stress block
  - concrete takes no tension
  - steel takes tension
  - force ductile failure

Location of n.a.

- ignore concrete below n.a.
- transform steel
- same area moments, solve for $x$

\[ b x \cdot \frac{x}{2} - nA_s (d - x) = 0 \]

$T$ sections

- n.a. equation is different if n.a. below flange

\[ b_w h_f \left( x - \frac{h_f}{2} \right) + (x - h_f) b_w \frac{(x - h_f)}{2} - nA_s (d - x) = 0 \]
**ACI Load Combinations**

- **1.4D**
- **1.2D + 1.6L + 0.5(L_r or S or R)**
- **1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)**
- **1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)**
- **1.2D + 1.0E + 1.0L + 0.2S**
- **0.9D + 1.0W**
- **0.9D + 1.0E**

*can also use old ACI factors

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**Reinforced Concrete Design**

- **stress distribution in bending**

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**Force Equations**

- **C = 0.85 f'_c ba**
- **T = A_s f_y**
- **where**
  - **f'_c** = concrete compressive strength
  - **a** = height of stress block
  - **β_1** = factor based on f'_c
  - **x** = location to the n.a.
  - **b** = width of stress block
  - **f_y** = steel yield strength
  - **A_s** = area of steel reinforcement

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**Equilibrium**

- **T = C**
- **M_n = T(d - a/2)**
  - **d** = depth to the steel n.a.
  - **with A_s**
    - **a =** \( \frac{A_s f_y}{0.85 f'_c b} \)
    - **M_u \leq \phi M_n**
    - **M_u = \phi T(d - a/2) = \phi A_s f_y (d - a/2)**
Over and Under-reinforcement

- over-reinforced
  - steel won’t yield
- under-reinforced
  - steel will yield
- reinforcement ratio
  - \( \rho = \frac{A_s}{bd} \)
  - use as a design estimate to find \( A_s, b, d \)
  - max \( \rho \) is found with \( \varepsilon_{\text{steel}} \geq 0.004 \) (not \( \rho_{\text{bal}} \))

A_s for a Given Section

- several methods
  - guess a and iterate
  1. guess a (less than n.a.)
  2. \( A_s = \frac{0.85 f'_c b a}{f_y} \)
  3. solve for a from \( M_u = \phi A_s f_y (d-a/2) \)
    \[ a = 2 \left( d - \frac{M_u}{\phi A_s f_y} \right) \]
  4. repeat from 2. until a from 3. matches a in 2.

A_s for a Given Section (cont)

- chart method
  - Wang & Salmon Fig. 3.8.1 \( R_n \) vs. \( \rho \)
    1. calculate \( R_n = \frac{M_n}{bd^2} \)
    2. find curve for \( f'_c \) and \( f_y \) to get \( \rho \)
    3. calculate \( A_s \) and a
  - simplify by setting \( h = 1.1d \)

Reinforcement

- min for crack control
- required
  \[ A_s = \frac{3\sqrt{f'_c}}{f_y} (bd) \]
- not less than
  \[ A_s = \frac{200}{f_y} (bd) \]
- \( A_{s-\text{max}} \): \( a = \beta_1(0.375d) \)
- typical cover
  - 1.5 in, 3 in with soil
- bar spacing
Approximate Depths