concrete construction: shear & deflection

ACI Shear Values

• $V_u$ is at distance $d$ from face of support
• shear capacity: $V_c = \phi \times b_w d$
  
  – where $b_w$ means thickness of web at n.a.

Shear in Concrete Beams

• flexure combines with shear to form diagonal cracks

• horizontal reinforcement doesn’t help
• stirrups = vertical reinforcement

ACI Shear Values

• shear stress (beams)
  
  – $\phi V_c = 2\sqrt{f_c'} b_w d$
  
  – $f_c'$ is in psi

• shear strength:
  
  $V_u \leq \phi V_c + \phi V_s$

  – $V_s$ is strength from stirrup reinforcement
Stirrup Reinforcement

- shear capacity:
  \[ V_s = \frac{A_v f_v d}{s} \]
  - \( A_v \) = area in all legs of stirrups
  - \( s \) = spacing of stirrup

- may need stirrups when concrete has enough strength!

Table 3-8 ACI Provisions for Shear Design*

<table>
<thead>
<tr>
<th>( V_s &lt; \frac{f_v d_y}{2} )</th>
<th>( \phi V_s &gt; V_s &gt; \frac{f_v d_y}{2} )</th>
<th>( V_s &gt; \phi V_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required area of stirrups, ( A_v ) **</td>
<td>none</td>
<td>( 500 b d )</td>
</tr>
<tr>
<td>Stirrup spacing, ( s )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recommended Minimum†</td>
<td>--</td>
<td>( \frac{V_s}{f_y d} )</td>
</tr>
<tr>
<td>Maximum†† (ACI 11.5.4)</td>
<td>--</td>
<td>( \frac{d}{2} ) or 24 in.</td>
</tr>
</tbody>
</table>

Note: Members subjected to shear and tension only: \( V_s = \frac{f_v d_y}{2} \) with \( \phi = \frac{0.75}{(ACI 11.3.1.1)} \)

**\( A_v = 2 \times A_y \) for U stirrups; \( f_y \leq 50,000 \) (ACI 11.5.2)

††A practical limit for minimum spacing is \( d/4 \)

††Maximum spacing based on minimum shear reinforcement \( (A_v f_y/500b) \) must also be considered (ACI 11.5.5.3).

Required Stirrup Reinforcement

- spacing limits

Torsional Stress & Strain

- can see torsional stresses & twisting of axi-symmetrical cross sections
  - torque
  - remain plane
  - undistorted
  - rotates

- not true for square sections....

Shear Stress Distribution

- depend on the deformation

- \( \phi = \) angle of twist
  - measure

- can prove planar section doesn’t distort
**Shearing Strain**

- related to $\phi$
  \[ \gamma = \frac{\rho \phi}{L} \]
- $\rho$ is the radial distance from the centroid to the point under strain
- shear strain varies linearly along the radius: $\gamma_{\text{max}}$ is at outer diameter

**Torsional Stress - Strain**

- know $f_v = \tau = G \cdot \gamma$ and $\gamma = \frac{\rho \phi}{L}$
- so $\tau = G \cdot \frac{\rho \phi}{L}$
- where $G$ is the **Shear Modulus**

**Shear Stress**

- $\tau_{\text{max}}$ happens at outer diameter

- combined shear and axial stresses
  - maximum shear stress at 45° “twisted” plane
Shear Strain

• knowing \( \tau = G \cdot \frac{p \phi}{L} \) and \( \tau = \frac{T \rho}{J} \)

• solve: \( \phi = \frac{T L}{J G} \)

• composite shafts: \( \phi = \sum_i \frac{T_i L_i}{J_i G_i} \)

Noncircular Shapes

• torsion depends on \( J \)

• plane sections don’t remain plane

• \( \tau_{\text{max}} \) is still at outer diameter

\[
\tau_{\text{max}} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}
\]

– where \( a \) is longer side (> \( b \))

Open Thin-Walled Sections

• with very large \( a/b \) ratios:

\[
\tau_{\text{max}} = \frac{T}{\frac{1}{3} ab^2} \quad \phi = \frac{TL}{\frac{1}{3} ab^3 G}
\]

Shear Flow in Closed Sections

• \( q \) is the internal shear force/unit length

\[
\tau = \frac{T}{2 ta} \quad \phi = \frac{TL}{4 ta^2} \sum_i \frac{s_i}{t_i}
\]

• \( a \) is the area bounded by the centerline

• \( s_i \) is the length segment, \( t_i \) is the thickness
Shear Flow in Open Sections

- each segment has proportion of $T$ with respect to torsional rigidity,
  \[ \tau_{\text{max}} = \frac{Tt_{\text{max}}}{\frac{1}{3} \sum b_i t_i^3} \]
- total angle of twist:
  \[ \phi = \frac{TL}{\frac{1}{3} G \sum b_i t_i^3} \]
- I beams - web is thicker, so $\tau_{\text{max}}$ is in web

Torsional Shear Stress

- twisting moment
- and beam shear

Torsional Shear Reinforcement

- closed stirrups
- more longitudinal reinforcement
- area enclosed by shear flow

Development Lengths

- required to allow steel to yield ($f_y$)
- standard hooks
  - moment at beam end
- splices
  - lapped
  - mechanical connectors
Development Lengths

• $l_d$, embedment required both sides
• proper cover, spacing:
  – No. 6 or smaller
    \[ l_d = \frac{d_b F_y}{25 \sqrt{f'_c}} \text{ or 12 in. minimum} \]
  – No. 7 or larger
    \[ l_d = \frac{d_b F_y}{20 \sqrt{f'_c}} \text{ or 12 in. minimum} \]

Development Lengths

• hooks
  – bend and extension

Concrete Deflections

• elastic range
  – $I$ transformed
  – $E_c$ (with $f'_c$ in psi)
    – normal weight concrete ($\sim 145 \text{ lb/ft}^3$)
      \[ E_c = 57,000 \sqrt{f'_c} \]
      – concrete between 90 and 160 lb/ft$^3$
        \[ E_c = w_c^{1.5} 33 \sqrt{f'_c} \]
  – cracked
    – $I$ cracked
    – $E$ adjusted
**Deflection Limits**

- relate to whether or not beam supports or is attached to a damageable non-structural element
- need to check *service* live load and long term deflection against these

<table>
<thead>
<tr>
<th>Limit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/180</td>
<td>roof systems (typical) – live</td>
</tr>
<tr>
<td>L/240</td>
<td>floor systems (typical) – live + long term</td>
</tr>
<tr>
<td>L/360</td>
<td>supporting plaster – live</td>
</tr>
<tr>
<td>L/480</td>
<td>supporting masonry – live + long term</td>
</tr>
</tbody>
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