concrete construction: shear & deflection

Shear in Concrete Beams
- flexure combines with shear to form diagonal cracks
- horizontal reinforcement doesn’t help
- stirrups = vertical reinforcement

ACI Shear Values
- $V_u$ is at distance $d$ from face of support
- shear capacity: $V_c = \nu_c \times b_w d$
  - where $b_w$ means thickness of web at n.a.
- shear stress (beams)
  - $\nu_c = 2\sqrt{f'_c}$
  - $\phi = 0.75$ for shear
  - $\phi V_c = \phi 2\sqrt{f'_c} b_w d$ $f'_c$ is in psi
- shear strength:
  - $V_u \leq \phi V_c + \phi V_s$
  - $V_s$ is strength from stirrup reinforcement
**Stirrup Reinforcement**

- **shear capacity:**
  \[ V_s = \frac{A_v f_y d}{s} \]
  
  - \( A_v \) = area in all legs of stirrups
  - \( s \) = spacing of stirrup

- **may need stirrups when concrete has enough strength!**

**Required Stirrup Reinforcement**

- **spacing limits**

<table>
<thead>
<tr>
<th>( V_s )</th>
<th>( \frac{V_s}{f_y} )</th>
<th>( \frac{V_s}{f_y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>( 500d )</td>
<td>( 500d )</td>
</tr>
<tr>
<td>( A_v )</td>
<td>( \frac{50d}{s} )</td>
<td>( \frac{50d}{s} )</td>
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<tr>
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<td>( \frac{50d}{s} )</td>
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<td>( 50d )</td>
<td>( \leq 4 )</td>
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</tbody>
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**Torsional Stress & Strain**

- **can see torsional stresses & twisting of axi-symmetrical cross sections**
  
  - torque
  - remain plane
  - undistorted
  - rotates

- **not true for square sections...**

**Shear Stress Distribution**

- **depend on the deformation**

  - \( \phi \) = angle of twist
    
    - measure

  - can prove planar section doesn’t distort
Shearing Strain

- related to $\phi$

$$\gamma = \frac{\rho \phi}{L}$$

- $\rho$ is the radial distance from the centroid to the point under strain

- shear strain varies linearly along the radius: $\gamma_{\text{max}}$ is at outer diameter

Torsional Stress - Strain

- know $f_v = \tau = G \cdot \gamma$ and $\gamma = \frac{\rho \phi}{L}$

- so

$$\tau = G \cdot \frac{\rho \phi}{L}$$

- where $G$ is the Shear Modulus

Torsional Stress - Strain

- from

$$T = \sum \tau(\rho) \Delta A$$

- can derive

$$T = \frac{\tau J}{\rho}$$

- where $J$ is the polar moment of inertia

- elastic range

$$\tau = \frac{T \rho}{J}$$

Shear Stress

- $\tau_{\text{max}}$ happens at outer diameter

- combined shear and axial stresses

- maximum shear stress at 45° “twisted” plane
Shear Strain

- knowing \( \tau = G \cdot \frac{\rho \phi}{L} \) and \( \tau = \frac{T \rho}{J} \)
- solve: \( \phi = \frac{T L}{J G} \)
- composite shafts: \( \phi = \sum \frac{T_i L_i}{J_i G_i} \)

Noncircular Shapes

- torsion depends on \( J \)
- plane sections don’t remain plane
- \( \tau_{\text{max}} \) is still at outer diameter

\[
\tau_{\text{max}} = \frac{T}{c_1 ab^2} \quad \phi = \frac{T L}{c_2 ab^3 G}
\]

- where \( a \) is longer side (> \( b \))

Open Thin-Walled Sections

- with very large \( a/b \) ratios:

\[
\tau_{\text{max}} = \frac{T}{\frac{1}{3} ab^2} \quad \phi = \frac{T L}{\frac{1}{3} ab^3 G}
\]

Shear Flow in Closed Sections

- \( q \) is the internal shear force/unit length

\[
\tau = \frac{T}{2 t a} \quad \phi = \frac{T L}{4 t a^2} \sum \frac{s_i}{t_i}
\]

- \( a \) is the area bounded by the centerline
- \( s_i \) is the length segment, \( t_i \) is the thickness
Shear Flow in Open Sections
- each segment has proportion of $T$ with respect to torsional rigidity,
  \[ \tau_{\text{max}} = \frac{T_{\text{max}} t}{\frac{1}{3} \Sigma b_i t_i^3} \]
- total angle of twist:
  \[ \phi = \frac{T_L}{\frac{1}{3} G \Sigma b_i t_i^3} \]
- I beams - web is thicker, so $\tau_{\text{max}}$ is in web

Torsional Shear Stress
- twisting moment
- **and** beam shear

Torsional Shear Reinforcement
- closed stirrups
- more longitudinal reinforcement
- area enclosed by shear flow

Development Lengths
- required to allow steel to yield ($f_y$)
- standard hooks
  – moment at beam end
- splices
  – lapped
  – mechanical connectors
Development Lengths

- $l_d$, embedment required both sides
- proper cover, spacing:
  - No. 6 or smaller
    \[ l_d = \frac{d_b F_y}{25 \sqrt{f'_c}} \text{ or 12 in. minimum} \]
  - No. 7 or larger
    \[ l_d = \frac{d_b F_y}{20 \sqrt{f'_c}} \text{ or 12 in. minimum} \]

Concrete Deflections

- elastic range
  - $I$ transformed
  - $E_c$ (with $f'_c$ in psi)
    - normal weight concrete (~ 145 lb/ft$^3$)
      \[ E_c = 57,000 \sqrt{f'_c} \]
    - concrete between 90 and 160 lb/ft$^3$
      \[ E_c = w_{c1.5} 33 \sqrt{f'_c} \]
  - cracked
    - $I$ cracked
    - $E$ adjusted
Deflection Limits

- relate to whether or not beam supports or is attached to a damageable non-structural element
- need to check service live load and long term deflection against these

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<table>
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<tr>
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<tbody>
<tr>
<td>L/180</td>
<td>roof systems (typical) – live</td>
</tr>
<tr>
<td>L/240</td>
<td>floor systems (typical) – live + long term</td>
</tr>
<tr>
<td>L/360</td>
<td>supporting plaster – live</td>
</tr>
<tr>
<td>L/480</td>
<td>supporting masonry – live + long term</td>
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