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concrete construction: shear & deflection

ACI Shear Values

- V_u is at distance d from face of support
- shear capacity: $V_c = v_c \times b_w d$

– where b_w means
thickness of
web at n.a.

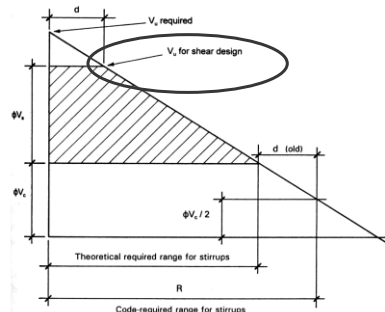
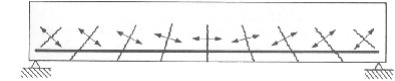


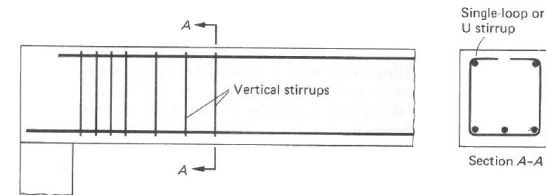
Figure 13.16 Layout for shear stress analysis: ACI Code requirements.

Shear in Concrete Beams

- flexure combines with shear to form diagonal cracks



- horizontal reinforcement doesn't help
- stirrups = vertical reinforcement



ACI Shear Values

- shear stress (beams)
 - $v_c = 2\lambda\sqrt{f'_c}$ $\phi = 0.75$ for shear
 - $\phi V_c = \phi 2\lambda\sqrt{f'_c} b_w d$ f'_c is in psi

– λ for lightweight mat'ls

- shear strength:

$$V_u \leq \phi V_c + \phi V_s$$

– V_s is strength from
stirrup reinforcement

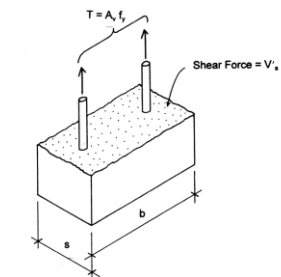


Figure 13.17 Consideration for spacing of a single stirrup.

Stirrup Reinforcement

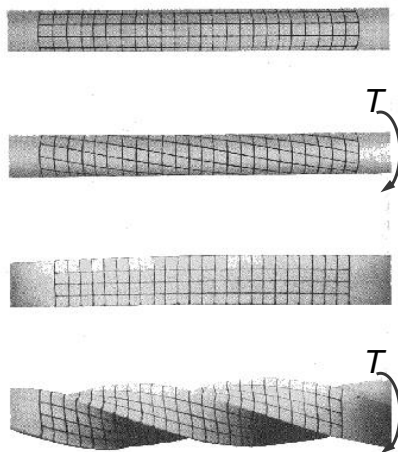
- shear capacity:

$$V_s = \frac{A_v f_{yt} d}{s} \leq 8 \sqrt{f'_c} b_w d$$

- A_v = area in all legs of stirrups
- s = spacing of stirrup
- may need stirrups when concrete has enough strength!

Torsional Stress & Strain

- can see torsional stresses & twisting of axi-symmetrical cross sections
 - torque
 - remain plane
 - undistorted
 - rotates
- not true for square sections....



Required Stirrup Reinforcement

Table 3-8 ACI Provisions for Shear Design*

		$V_u \leq \frac{\phi V_c}{2}$	$\phi V_c \geq V_u > \frac{\phi V_c}{2}$	$V_u > \phi V_c$
Required area of stirrups, A_v **		none	greater of $\frac{50 b_w s}{f_{yt}}$ and $\frac{0.75 \sqrt{f'_c} b_w s}{f_{yt}}$	$\frac{(V_u - \phi V_c) s}{\phi A_v f_{yt}}$
Stirrup spacing, s	Required	—	smaller of $\frac{A_v f_{yt}}{50 b_w}$ and $\frac{A_v f_{yt}}{0.75 \sqrt{f'_c} b_w}$	$\frac{\phi A_v f_{yt} d}{V_u - \phi V_c}$
	Recommended Minimum†	—	—	4 in.
	Maximum†† (ACI 9.7.6.22)	—	$\frac{d}{2}$ or 24 in.	$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \leq \phi 4 \sqrt{f'_c} b_w d$ $\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > \phi 4 \sqrt{f'_c} b_w d$

*Members subjected to shear and flexure only; $\phi V_c = \phi 2 \sqrt{f'_c} b_w d$ $\phi = 0.75$ (ACI 22.5.5.1)

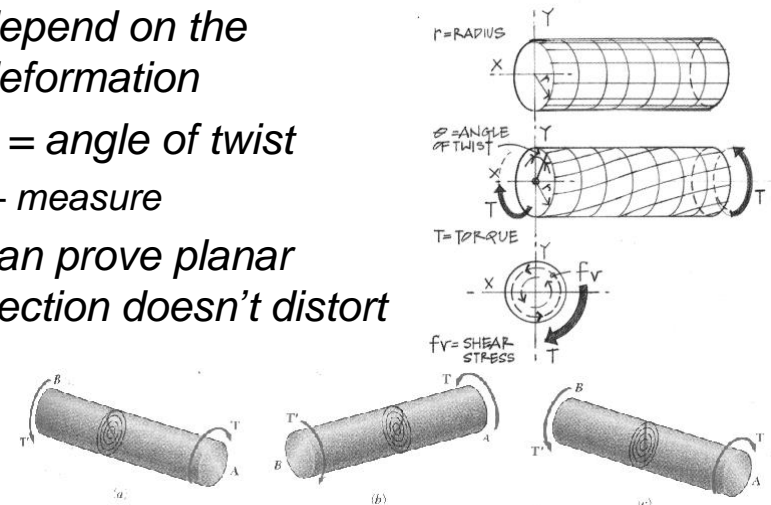
** $A_v = 2 \times A_b$ for U stirrups; $f_y \leq 60$ ksi (ACI 20.2.2.4)

†A practical limit for minimum spacing is $d/4$

††Maximum spacing based on minimum shear reinforcement ($= A_v f_{yt} / 50 b_w$ or $A_v f_{yt} / 0.75 \sqrt{f'_c} b_w$) must also be considered (ACI 9.6.3.3)

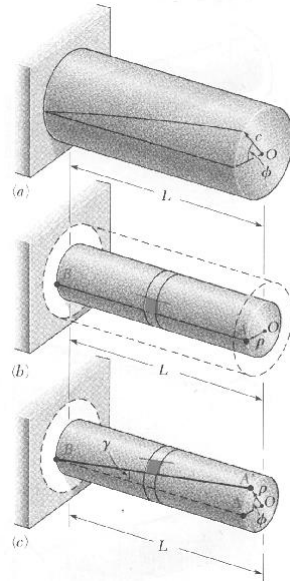
Shear Stress Distribution

- depend on the deformation
- ϕ = angle of twist
 - measure
- can prove planar section doesn't distort



Shearing Strain

- related to ϕ
$$\gamma = \frac{\rho\phi}{L}$$
- ρ is the radial distance from the centroid to the point under strain
- shear strain varies linearly along the radius: γ_{max} is at outer diameter



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Torsional Stress - Strain

- know $f_v = \tau = G \cdot \gamma$ and $\gamma = \frac{\rho\phi}{L}$
- so
$$\tau = G \cdot \frac{\rho\phi}{L}$$
- where G is the Shear Modulus

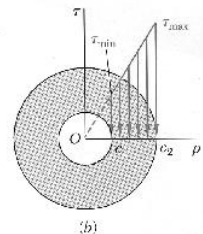
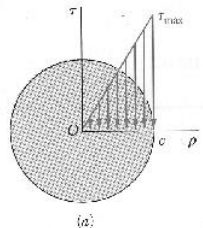
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Torsional Stress - Strain

- from
$$T = \sum \tau(\rho) \Delta A$$
- can derive
$$T = \frac{\tau J}{\rho}$$
 - where J is the polar moment of inertia
 - elastic range
$$\tau = \frac{T\rho}{J}$$



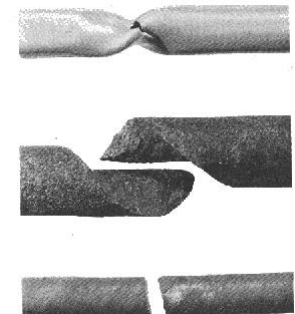
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Shear Stress

- τ_{max} happens at outer diameter
- combined shear and axial stresses
 - maximum shear stress at 45° "twisted" plane



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Shear Strain

- knowing $\tau = G \cdot \frac{\rho\phi}{L}$ and $\tau = \frac{T\rho}{J}$
- solve: $\phi = \frac{TL}{JG}$
- composite shafts: $\phi = \sum_i \frac{T_i L_i}{J_i G_i}$

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Noncircular Shapes

- torsion depends on J
- plane sections don't remain plane
- τ_{max} is still at outer diameter

$$\tau_{max} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}$$

– where a is longer side ($> b$)

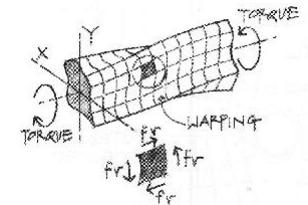


TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

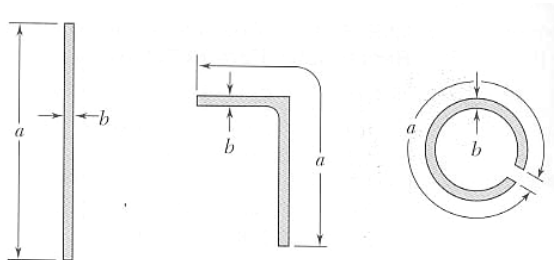
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Open Thin-Walled Sections

- with very large a/b ratios:



$$\tau_{max} = \frac{T}{\frac{1}{3} ab^2} \quad \phi = \frac{TL}{\frac{1}{3} ab^3 G}$$

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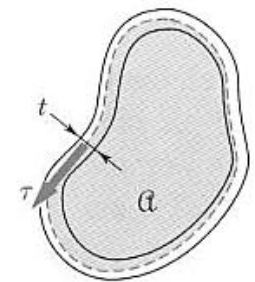
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Shear Flow in Closed Sections

- q is the internal shear force/unit length

$$\tau = \frac{T}{2t\mathcal{A}}$$

$$\phi = \frac{TL}{4t\mathcal{A}^2} \sum_i \frac{s_i}{t_i}$$



- \mathcal{A} is the area bounded by the centerline
- s_i is the length segment, t_i is the thickness

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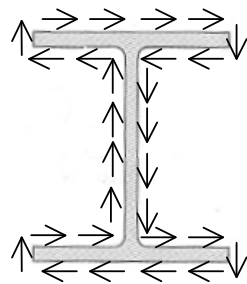
Shear Flow in Open Sections

- each segment has proportion of T with respect to torsional rigidity,

$$\tau_{\max} = \frac{Tt_{\max}}{\frac{1}{3} \sum b_i t_i^3}$$

- total angle of twist:

$$\phi = \frac{TL}{\frac{1}{3} G \sum b_i t_i^3}$$



- I beams - web is thicker, so τ_{\max} is in web

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Torsional Shear Reinforcement

- closed stirrups
- more longitudinal reinforcement

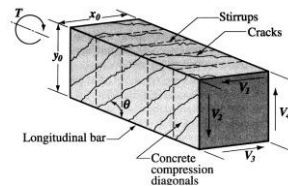
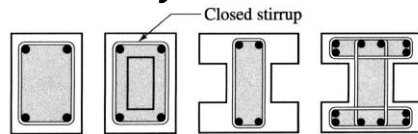


Fig. R11.6.3.6(a)—Space truss analogy

- area enclosed by shear flow



A_{oh} = shaded area

Fig. R11.6.3.6(b)—Definition of A_{oh}

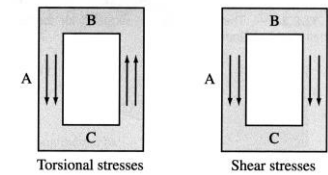
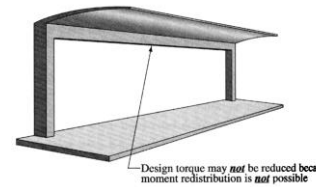
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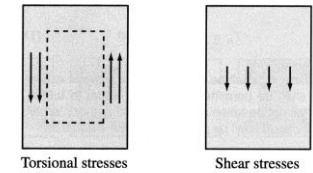
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Torsional Shear Stress

- twisting moment
- and beam shear



(a) Hollow section



(b) Solid section

Fig. R11.6.3.1—Addition of torsional and shear stresses

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Development Lengths

- required to allow steel to yield (f_y)
- standard hooks
- moment at beam end

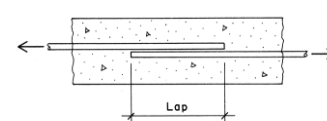
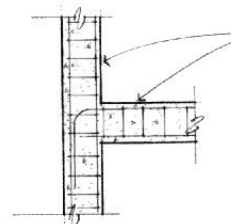
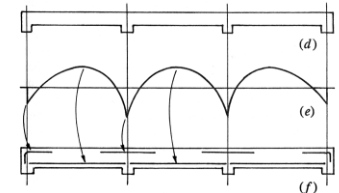


Figure 13.24 The lapped splice for steel reinforcing bars.

- splices
- lapped
- mechanical connectors



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Development Lengths

- l_d , embedment required both sides
- proper cover, spacing:

- No. 6 or smaller

$$l_d = \frac{d_b f_y}{25 \lambda \sqrt{f'_c}} \quad \text{or 12 in. minimum}$$

- No. 7 or larger

$$l_d = \frac{d_b f_y}{20 \lambda \sqrt{f'_c}} \quad \text{or 12 in. minimum}$$

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Development Lengths

- bars in compression

$$l_d = \frac{d_b f_y}{50 \lambda \sqrt{f'_c}} \leq 0.0003 f_y d_b$$

- splices

- tension minimum is function of l_d and splice classification
- compression minimum
- is function of d_b and F_y

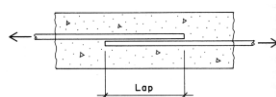


Figure 13.24 The lapped splice for steel reinforcing bars.

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Development Lengths

- hooks
 - bend and extension

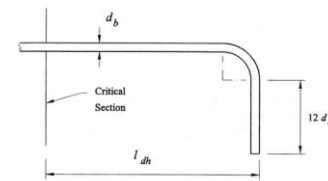


Figure 9-17: Minimum requirements for 90° bar hooks.

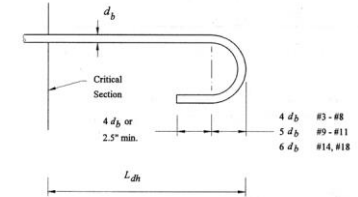


Figure 9-18: Minimum requirements for 180° bar hooks.

- minimum

$$l_{dh} = \frac{d_b f_y}{50 \lambda \sqrt{f'_c}}$$

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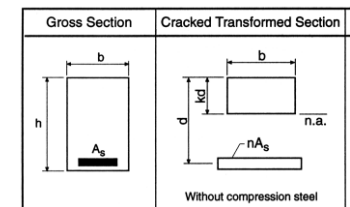
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Concrete Deflections

- elastic range

- I transformed
- E_c (with f'_c in psi)
 - normal weight concrete (~ 145 lb/ft³)

$$E_c = 57,000 \sqrt{f'_c}$$
 - concrete between 90 and 160 lb/ft³



- cracked

$$E_c = w_c^{1.5} 33 \sqrt{f'_c}$$
 - I cracked
 - E adjusted

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Deflection Limits

- relate to whether or not beam supports or is attached to a damageable non-structural element
- need to check service live load and long term deflection against these

<i>L/180</i>	<i>roof systems (typical) – live</i>
<i>L/240</i>	<i>floor systems (typical) – live + long term</i>
<i>L/360</i>	<i>supporting plaster – live</i>
<i>L/480</i>	<i>supporting masonry – live + long term</i>