

lecture
five



moments and rigid body equilibrium

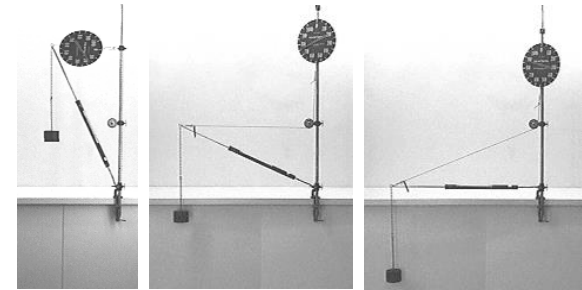
Moments 1
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Moments

- forces have the tendency to make a body rotate about an axis



<http://www.physics.umd.edu>

– same translation but different rotation

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Moments

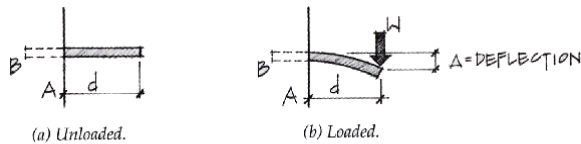


Figure 2.33 Moment on a cantilever beam.

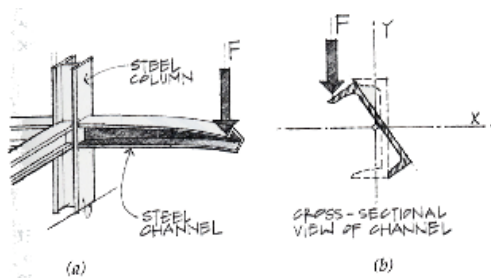


Figure 2.34 An example of torsion on a cantilever beam.

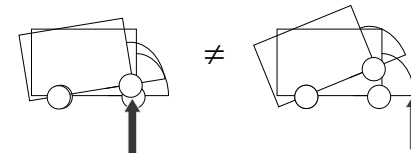
Moments 7

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Moments

- a force acting at a different point causes a different moment:



Moments 8

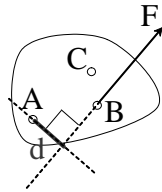
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Moments

- defined by magnitude and direction
- units: N·m, k·ft
- direction:
 - + CW (!)
 - CCW
- value found from F and \perp distance

$$M = F \cdot d$$
- d also called “lever” or “moment” arm



Moments 9
Lecture 4

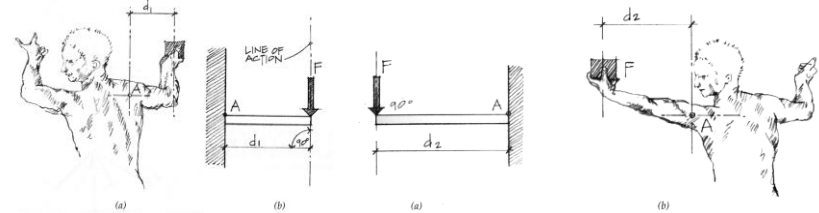
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Moments

- with same F :

$$M_A = F \cdot d_1 < M_A = F \cdot d_2 \quad (\text{bigger})$$



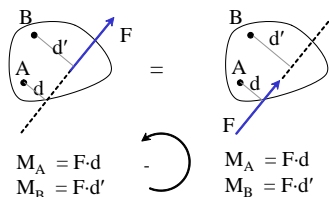
Moments 10
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Moments

- additive with sign convention
- can still move the force along the line of action
- location of moment independent



Moments 11

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Moments

- Varignon's Theorem
 - resolve a force into components at a point and finding perpendicular distances
 - calculate sum of moments
 - equivalent to original moment
- makes life easier!
 - geometry
 - when component runs through point, $d=0$

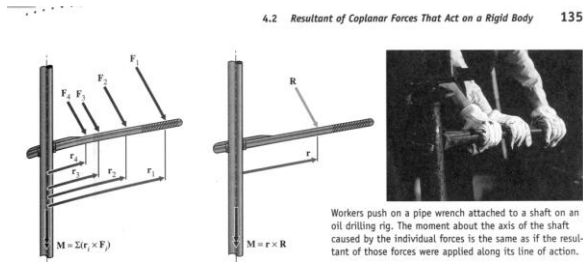
Moments 12

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Moments of a Force

- moments of a force
 - introduced in Physics as “Torque Acting on a Particle”
 - and used to satisfy rotational equilibrium



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Physics and Moments of a Force

- my Physics book (right hand rule):

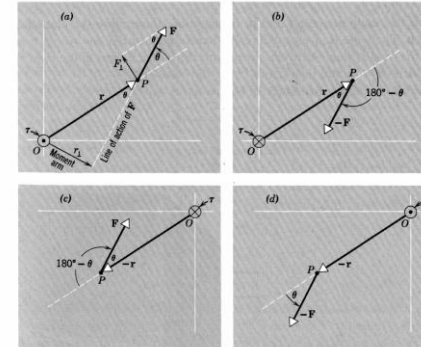


FIGURE 11-2 The plane shown is that defined by r and F in Fig. 11-1. (a) The magnitude of τ is given by $F r_{\perp}$ (Eq. 11-2b) or by $r F_{\perp}$ (Eq. 11-2c). (b) Reversing F reverses the direction of τ . (c) Reversing r reverses the direction of τ . (d) Reversing F and r leaves the direction of τ unchanged. The directions of τ are represented by \odot (perpendicularly out of the figure, the symbol representing the tip of an arrow) and by \otimes (perpendicularly into the figure, the symbol representing the tail of an arrow).

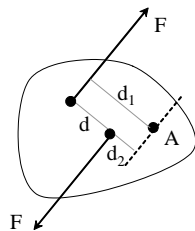
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Moment Couples

- 2 forces
 - same size
 - opposite direction
 - distance d apart
 - cw or ccw



$$M = F \cdot d$$

- not dependant on point of application

$$M = F \cdot d_1 - F \cdot d_2$$

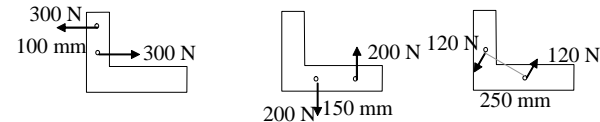
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Moment Couples

- equivalent couples
 - same magnitude and direction
 - F & d may be different



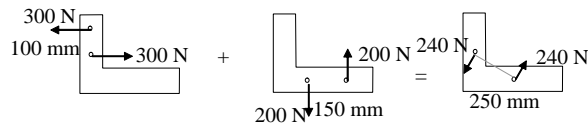
Moments 14

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Moment Couples

- added just like moments caused by one force
- can replace two couples with a single couple



Moments 15

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Equivalent Force Systems

- two forces at a point is equivalent to the resultant at a point
- resultant is equivalent to two components at a point
- resultant of equal & opposite forces at a point is zero
- put equal & opposite forces at a point (sum to 0)
- transmission of a force along action line

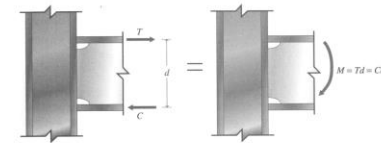
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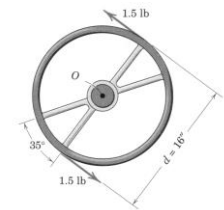
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Moment Couples

- moment couples in structures



The flanges of a steel beam are welded to the flange of a column. Equal and opposite forces F and C in the beam flanges form a couple with moment M that is transferred into the column.



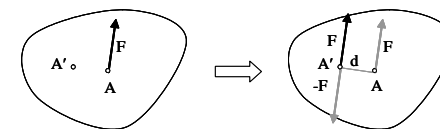
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Force-Moment Systems

- single force causing a moment can be replaced by the same force at a different point by providing the moment that force caused



- moments are shown as arched arrows



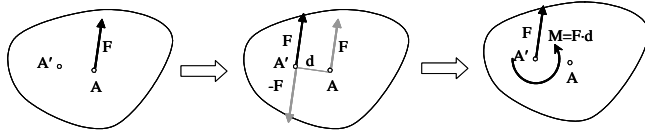
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Force-Moment Systems

- a force-moment pair can be replaced by a force at another point causing the original moment



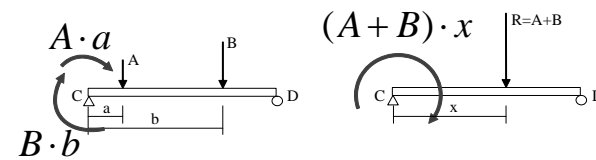
Moments 17

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Parallel Force Systems

- forces are in the same direction
- can find resultant force
- need to find location for equivalent moments



Moments 18

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Equilibrium

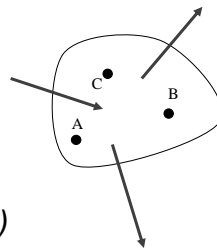
- rigid body
 - doesn't deform
 - coplanar force systems

- static:

$$R_x = \sum F_x = 0 \quad (\Sigma H)$$

$$R_y = \sum F_y = 0 \quad (\Sigma V)$$

$$M = \sum M = 0$$



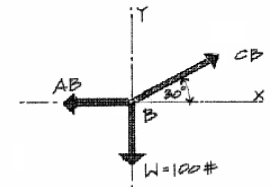
Equilibrium 3
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Free Body Diagram

- FBD (sketch)
- tool to see all forces on a body or a point including
 - external forces
 - weights
 - force reactions
 - external moments
 - moment reactions
 - internal forces



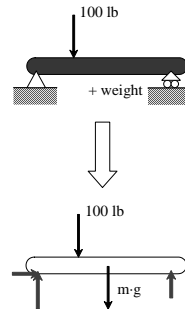
Equilibrium 10

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Free Body Diagram

- determine body
- FREE it from:
 - ground
 - supports & connections
- draw all external forces acting ON the body
 - reactions
 - applied forces
 - gravity



Equilibrium 11

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Free Body Diagram

- solve equations
 - most times 1 unknown easily solved
 - plug into other equation(s)
- common to have unknowns of
 - force magnitudes
 - force angles
 - moment magnitudes

Equilibrium 10
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Free Body Diagram

- sketch FBD with relevant geometry
- resolve each force into components
 - known & unknown angles – name them
 - known & unknown forces – name them
 - known & unknown moments – name them
- are any forces related to other forces?
- for the unknowns
- write only as many equilibrium equations as needed
- solve up to 3 equations

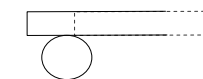
Equilibrium 12

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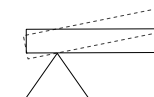
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Reactions on Rigid Bodies

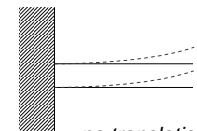
- result of applying force
- unknown size
- connection or support type
 - known direction
 - related to motion prevented



no vertical motion



no translation



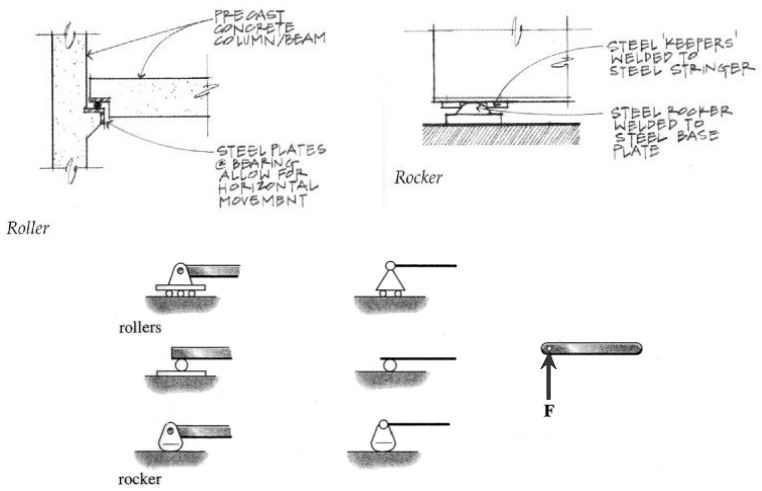
no translation
no rotation

Equilibrium 19

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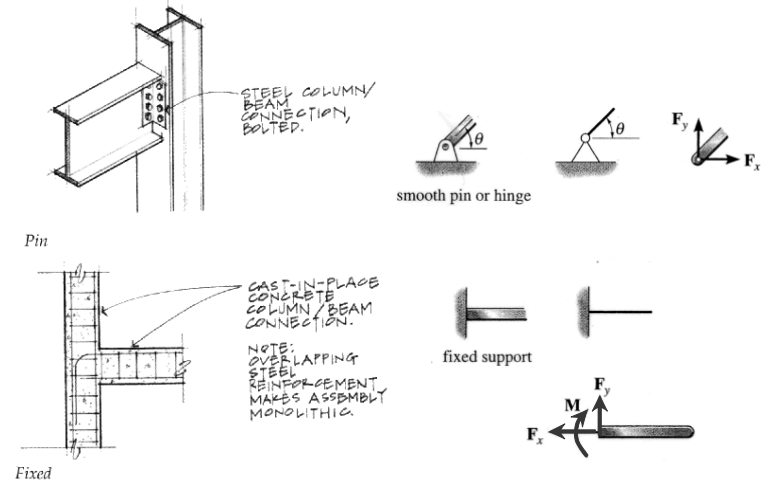
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Supports and Connections



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Supports and Connections



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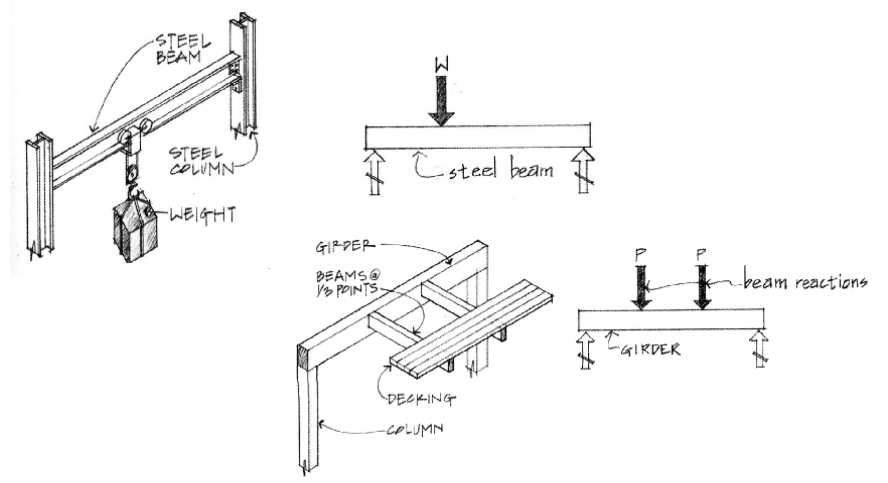
Moment Equations

- sum moments at intersection where the most forces intersect
- multiple moment equations may not be useful
- combos:

$$\begin{matrix} \sum F_x = 0 & \sum F = 0 & \sum M_1 = 0 \\ \sum F_y = 0 & \sum M_1 = 0 & \sum M_2 = 0 \\ \sum M_1 = 0 & \sum M_2 = 0 & \sum M_3 = 0 \end{matrix}$$

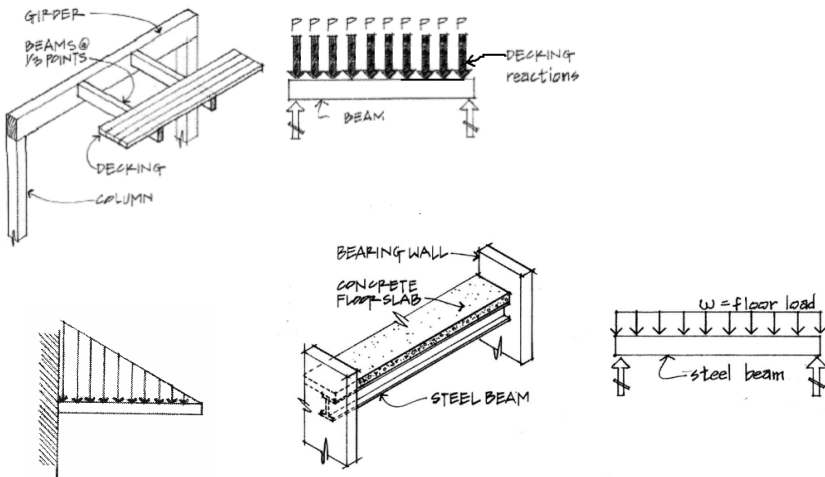
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Concentrated Loads



Loads 15 Elements of Architectural Structures ARCH 614 S2004abn

Distributed Loads



Loads 16

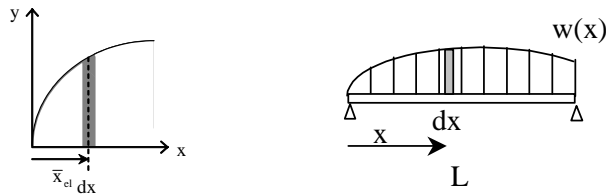
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Equivalent Force Systems

- replace forces by resultant
- place resultant where $M = 0$
- using calculus and area centroids

$$W = \int_0^L w dx = \int dA_{\text{loading}} = A_{\text{loading}}$$



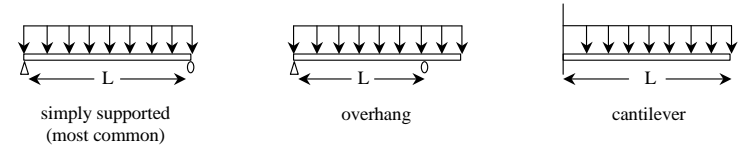
Loads 17
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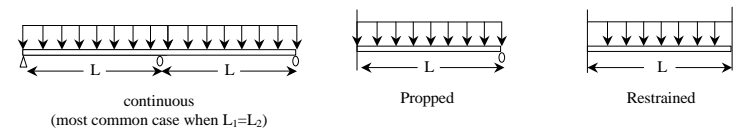
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Beam Supports

- *statically determinate*



- *statically indeterminate*



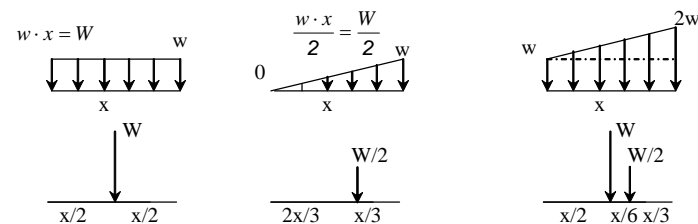
Internal Beam Forces 20
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Load Areas

- area is width \times "height" of load
- w is load per unit length
- W is total load



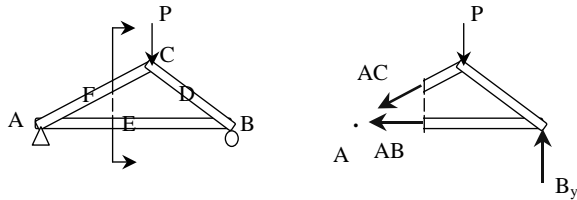
Loads 19
Lecture 9

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Method of Sections

- relies on internal forces being in equilibrium on a section
- cut to expose 3 or less members
- coplanar forces $\rightarrow \sum M = 0$ too



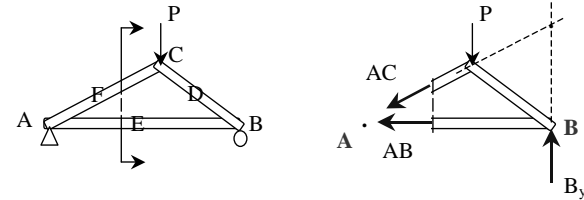
Steel Trusses 15
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Method of Sections

- joints on or off the section are good to sum moments
- quick for few members
- not always obvious where to cut or sum



Steel Trusses 16
Lecture 17

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