ARCH 331: Practice Quiz 5

Note: No aids are allowed for part 1. One side of a letter sized paper with notes is allowed during part 2, along with a silent, non-programmable calculator. There are reference charts for part 2, shown on pages 2-6.

Clearly show your work and answer.

Part 1)  Worth 5 points  (conceptual questions)

Part 2)  Worth 45 points

(NOTE: The loading type [ex, live, dead, wind...] and sizes can and will be changed for the quiz with respect to the beam diagrams and formula provided. The support condition, section, and bracing for the column can and will be changed.)

A wide flange beam of A992 steel (F_y = 50 ksi, E = 30 x 10^3 ksi) is needed to span 32 ft and support uniformly distributed loads of 980 lb/ft of dead load (from materials), the self weight, and 1150 lb/ft of live load over a length of 11 feet as shown. The beam is simply supported with a maximum unbraced length of 15 ft.

a) Select the most economical beam adequate for flexural strength using LRFD design and the chart provided (including self weight). Assume that the dead load determines the location of the maximum moment and superimpose the live load moment there.

b) Determine the minimum moment of inertia required such that the dead load deflection does not exceed 1.25 inches assuming a self weight of 60 lb/ft. [or live load deflection – using \( \Delta_{\text{max}} = \frac{wL^4}{(152EI)} \) because there is no equation – does not exceed 0.8 in; or total deflection assuming that the dead load determines the location of the maximum moment – using
\[
\Delta_t = 5wx(l^4 - 3lx^2 + 2x^3)/(192EI) \text{ because there is no equation for the partial distributed load – does not exceed 1.75 in.}]

A W 250 x 49 metric column is 5.75 m tall of A36 steel (F_y = 250 MPa, E = 200 x 10^3 MPa). The base is fixed and the top is pinned in the weak axis, while the strong axis is considered pinned at the top and bottom (no picture and approximated conditions). The section properties are:

\[
A = 6260 \text{ mm}^2, I_x = 70.7 \times 10^6 \text{ mm}^4, r_x = 106 \text{ mm}, I_y = 15.2 \times 10^6 \text{ mm}^4, r_y = 49.3 \text{ mm}
\]

c) If the column is to support 200 kN of dead load and 600 kN of live load, is it adequate for design using LRFD?

Answers – Not provided on actual quiz!

- a) \( M_u = 279.4 \text{ k-ft}, \text{ use W18x55 (} M_{u}^* < 294 \text{ k-ft)} \)
- b) \( I_{\text{req'd}} = 654.3 \text{ in}^4 \) (dead only) [\( I_{\text{req'd-live only}} = 571.2 \text{ in}^4; I_{\text{req'd-total}} = 725.8 \text{ in}^4 ]
- c) \( \phi P_n = 886 \text{ kN} \) . Not OK (weak axis governs because \( \phi P_{n\text{-strong}} = 1199 \text{ kN} \))

Disclaimer: Answers have NOT been painstakingly researched.
1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD

\[ W = \frac{wl}{2} \]

\[ R = V = \frac{wl}{2} \]

\[ V_x = \frac{wl}{2} \left( \frac{1}{2} - x \right) \]

\[ M_{\text{max. (at center)}} = \frac{wl^2}{8} - \frac{wl}{2} \left( \frac{1}{2} - x \right) \]

\[ \Delta_{x\text{max. (at center)}} = \frac{wl^3}{384EI} \left( \frac{9}{8} - 2l^2 + x^2 \right) \]

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END

\[ W = \frac{wl^4}{9} \]

\[ R_1 = V_1 \text{ (max. when } a < 0 \text{)} \]

\[ V_x = \frac{wl^4}{9} \left( \frac{1}{2} - x \right) \]

\[ M_{\text{max. (at center)}} = \frac{wl^2}{3} \left( \frac{1}{2} - x \right) \]

\[ \Delta_{x\text{max. (at center)}} = \frac{wl^3}{180EI} \left( 3x^2 - 10l^2x^2 + l^4 \right) \]

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER

\[ W = \frac{wl^3}{2} \]

\[ R = V = \frac{wl^4}{2} \]

\[ V_x = \frac{wl^4}{2} \left( \frac{1}{2} - x \right) \]

\[ M_{\text{max. (at center)}} = \frac{wl^2}{6} \left( \frac{1}{2} - 2x + \frac{3l^2}{2} \right) \]

\[ \Delta_{x\text{max. (at center)}} = \frac{wl^3}{480EI} \left( 3l^2 - 4x^2 \right) \]

4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED

\[ R_1 = V_1 \text{ (max. when } a < 0 \text{)} \]

\[ R_2 = V_2 \text{ (max. when } a > 0 \text{)} \]

\[ V_x = \frac{wl^4}{2l} \left( \frac{1}{2} - x \right) \]

\[ M_{\text{max. (at } x = a + \frac{R_1}{w} \text{)}} = \frac{wl^2}{2l} \left( \frac{1}{2} - x \right) \]

\[ \Delta_{x\text{max. (at center)}} = \frac{wl^3}{24EI} \left( a^2 - \frac{a^3}{2} - 2sl^2 \left( 2l - a \right) + l^3 \right) \]

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END

\[ W = \frac{wl^4}{9} \]

\[ R_1 = V_1 \text{ (max. when } a < 0 \text{)} \]

\[ R_2 = V_2 \text{ (max. when } a > 0 \text{)} \]

\[ V_x = \frac{wl^4}{9} \left( \frac{1}{2} - x \right) \]

\[ M_{\text{max. (at } x = a + \frac{R_1}{w} \text{)}} = \frac{wl^2}{2l} \left( \frac{1}{2} - x \right) \]

\[ \Delta_{x\text{max. (at center)}} = \frac{wl^3}{24EI} \left( a^2 - \frac{a^3}{2} - 2sl^2 \left( 2l - a \right) + l^3 \right) \]

6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END

\[ R_1 = V_1 \text{ (max. when } a < 0 \text{)} \]

\[ R_2 = V_2 \text{ (max. when } a > 0 \text{)} \]

\[ V_x = \frac{wl^4}{2l} \left( \frac{1}{2} - x \right) \]

\[ M_{\text{max. (at } x = a + \frac{R_1}{w} \text{)}} = \frac{wl^2}{2l} \left( \frac{1}{2} - x \right) \]

\[ \Delta_{x\text{max. (at center)}} = \frac{wl^3}{24EI} \left( a^2 - \frac{a^3}{2} - 2sl^2 \left( 2l - a \right) + l^3 \right) \]
REFERENCE CHARTS FOR QUIZ 5

7. SIMPLE BEAM—CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load = 2P

\[ R = V = \frac{P}{2} \]

\[ M_{\text{max.}} \text{ (at point of load)} = \frac{P}{4} \]

\[ \Delta_{\text{max.}} \text{ (at point of load)} = \frac{P^3}{48EI} \]

\[ \Delta x \text{ (when } x < \frac{l}{2} \text{)} = \frac{P}{24EI} \left( \frac{3x^2}{2} - ax^2 \right) \]

\[ M_{\text{max.}} \]

\[ \frac{x}{2} \]

\[ \text{Moment} \]

8. SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT

Total Equiv. Uniform Load = \( \frac{8Pab}{l} \)

\[ R_1 = V_1 \text{ (max. when } a < b \text{)} = \frac{Pb}{l} \]

\[ R_2 = V_2 \text{ (max. when } a > b \text{)} = \frac{Pa}{l} \]

\[ M_{\text{max.}} \text{ (at point of load)} = \frac{Pbx}{l} \]

\[ \Delta_{\text{max.}} \text{ (at } x = a + \frac{2b}{3} \text{ when } a > b \text{)} = \frac{Pab(a + 2b)b}{27EI} \]

\[ \Delta a \text{ (at point of load)} = \frac{Pab}{3EI} \]

\[ \Delta x \text{ (when } x < a \text{)} = \frac{Pbx}{6EI} \left( \frac{3x^2}{2} - b^2 - x^2 \right) \]

\[ M_{\text{max.}} \]

\[ \frac{x}{2} \]

\[ \text{Moment} \]

9. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED

Total Equiv. Uniform Load = \( \frac{8Pa}{l} \)

\[ R = V = P \]

\[ M_{\text{max.}} \text{ (between loads)} = \frac{Pa}{l} \]

\[ M_x \text{ (when } x < a \text{)} = \frac{Px}{24EI} \left( \frac{3x^2}{2} - ax^2 \right) \]

\[ \Delta_{\text{max.}} \text{ (at center)} = \frac{Pa}{24EI} \left( \frac{3x^2}{2} - ax^2 \right) \]

\[ \Delta x \text{ (when } x < a \text{ and } < (l - a) \text{)} = \frac{Pa}{6EI} \left( \frac{3x^2}{2} - 3a^2 - x^2 \right) \]

\[ M_{\text{max.}} \]

\[ \frac{x}{2} \]

\[ \text{Moment} \]

10. SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED

\[ R_1 = V_1 \text{ (max. when } a < b \text{)} = \frac{P}{l} (l - a + b) \]

\[ R_2 = V_2 \text{ (max. when } a > b \text{)} = \frac{P}{l} (l - b + a) \]

\[ V_x \text{ (when } x > a \text{ and } < (l - b) \text{)} = \frac{P}{l} (b - a) \]

\[ M_1 \text{ (max. when } a > b \text{)} = R_1a \]

\[ M_2 \text{ (max. when } a < b \text{)} = R_2b \]

\[ M_x \text{ (when } x < a \text{)} = R_1x \]

\[ M_x \text{ (when } x > a \text{ and } < (l - b) \text{)} = R_1x - P(x - a) \]

\[ \Delta_{\text{max.}} \text{ (at point of load)} = \frac{P^3}{48EI} \]

\[ \Delta x \text{ (when } x < \frac{l}{2} \text{)} = \frac{P}{24EI} \left( \frac{3x^2}{2} - 4a^2 \right) \]

\[ \Delta x \text{ (when } x > a \text{ and } < (l - a) \text{)} = \frac{Pa}{6EI} \left( \frac{3x^2}{2} - 3a^2 - a^2 \right) \]

11. SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED

\[ R_1 = V_1 = \frac{P_1 (l - a) + P_2 (l - b)}{l} \]

\[ R_2 = V_2 = \frac{P_1a + P_2 (l - b)}{l} \]

\[ V_x \text{ (when } x > a \text{ and } < (l - b) \text{)} = R_1 - P_1 \]

\[ M_1 \text{ (max. when } R_1 < P_1 \text{)} = R_1a \]

\[ M_2 \text{ (max. when } R_2 < P_2 \text{)} = R_2b \]

\[ M_x \text{ (when } x < a \text{)} = R_1x \]

\[ M_x \text{ (when } x > a \text{ and } < (l - b) \text{)} = R_1x - P_1(x - a) \]

\[ \Delta_{\text{max.}} \text{ (at point of load)} = \frac{P^3}{48EI} \]

\[ \Delta x \text{ (when } x < \frac{l}{2} \text{)} = \frac{P}{24EI} \left( \frac{3x^2}{2} - 4a^2 \right) \]

\[ \Delta x \text{ (when } x > a \text{ and } < (l - a) \text{)} = \frac{Pa}{6EI} \left( \frac{3x^2}{2} - 3a^2 - a^2 \right) \]

12. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load = \( \frac{wxl}{2} \)

\[ R_1 = V_1 = \frac{2wl}{8} \]

\[ R_2 = V_2 \max. = \frac{5wl}{8} \]

\[ V_x = \frac{3w}{8} \]

\[ M_{\text{max.}} \text{ (at } x = \frac{3}{4} l \text{)} = \frac{9}{16} \left( \frac{wx}{l} \right)^2 \]

\[ M_x = \frac{3}{8} \left( \frac{wx}{l} \right)^2 \]

\[ \Delta_{\text{max.}} \text{ (at } x = \frac{1}{16} (1 + \sqrt{33}) \cdot 4215x^l \text{)} = \frac{wx^3}{185EI} \]

\[ \Delta x \text{ (when } x > \frac{l}{2} \text{)} = \frac{wx^3}{48EI} \left( \frac{3x^2}{2} - 3x^2 + 2x^3 \right) \]
REFERENCE CHARTS FOR QUIZ 5

13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT CENTER

\[
\begin{align*}
\text{Total Equiv. Uniform Load} & = \frac{3P}{2} \\
R_1 & = V_1 = \frac{5P}{16} \\
R_2 & = V_2 \text{ max.} = \frac{11P}{16} \\
M_{\text{max.}} \text{ (at fixed end)} & = \frac{3P}{16} \\
M_1 \text{ (at point of load)} & = \frac{5Px}{16} \\
M_x \text{ (when } x < \frac{1}{2} \text{)} & = P \left( \frac{1}{2} - \frac{11x}{16} \right) \\
\Delta_{\text{max.}} \text{ (at } x = -l \sqrt{\frac{1}{5}} = .4472l \text{)} & = -\frac{P/l^3}{48EI} \sqrt{\frac{1}{5}} = .0093l/\sqrt{5} \frac{P/l^3}{EI} \\
\Delta_x \text{ (at point of load)} & = \frac{7P/3}{768EI} \\
\Delta_x \text{ (when } x < \frac{1}{2} \text{)} & = \frac{Px}{96EI} \left( 3x^2 - 5x^4 \right) \\
\Delta_x \text{ (when } x > \frac{1}{2} \text{)} & = \frac{P}{96EI} \left( x - l \right)^2 (11x - 2l)
\end{align*}
\]

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT ANY POINT

\[
\begin{align*}
R_1 & = V_1 = \frac{Pb^2}{2l^2} (a + 2l) \\
R_2 & = V_2 = \frac{Pa}{2l^2} (3l^2 - a^2) \\
M_1 \text{ (at point of load)} & = Rla \\
M_2 \text{ (at fixed end)} & = \frac{Pab}{2l^2} (a + l) \\
M_x \text{ (when } x < a \text{)} & = Rlx \\
M_x \text{ (when } x > a \text{)} & = Rlx - P(x - a) \\
\Delta_{\text{max.}} \text{ (when } x < \frac{1}{2} \text{)} & = \frac{P/3}{3EI} \left( \frac{1}{2} - \frac{a^2}{3l^2} \right)^2 \\
\Delta_{\text{max.}} \text{ (when } x > \frac{1}{2} \text{)} & = \frac{Pa}{6EI} \sqrt{\frac{a}{2l^2 - a}} \\
\Delta_x \text{ (at point of load)} & = \frac{Pab^2}{12EI/l^2} \left( 3l^2 - a^2 \right) \\
\Delta_x \text{ (when } x < a \text{)} & = \frac{Pb^2}{12EI/l^2} \left( 3a^2 - 2\frac{x^2}{l^2} - ax \right) \\
\Delta_x \text{ (when } x > a \text{)} & = \frac{Pb^2}{12EI/l^2} (x - a) \left( 3l^2 - ax \right)
\end{align*}
\]

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS

\[
\begin{align*}
\text{Total Equiv. Uniform Load} & = \frac{2wl}{3} \\
R = V & = \frac{wl}{2} \\
V_x & = \frac{w}{2} \left( \frac{1}{2} - x \right) \\
M_{\text{max.}} \text{ (at ends)} & = \frac{w}{12} \\
M_1 \text{ (at center)} & = \frac{w}{24} \\
M_x \text{ (when } x = \frac{l}{2} \text{)} & = \frac{w}{12} \left( 6l - l^2 - 6x^2 \right) \\
\Delta_{\text{max.}} \text{ (at center)} & = \frac{384EI}{w} \\
\Delta_x \text{ (when } x = \frac{l}{2} \text{)} & = \frac{wx^2}{24EI} \left( l - x \right)^2
\end{align*}
\]

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER

\[
\begin{align*}
\text{Total Equiv. Uniform Load} & = P \\
R = V & = \frac{P}{2} \\
M_{\text{max.}} \text{ (at center and ends)} & = \frac{8P}{9} \left( 4x - l \right) \\
M_x \text{ (when } x < \frac{l}{2} \text{)} & = \frac{P}{3} \\
\Delta_{\text{max.}} \text{ (at center)} & = \frac{192EI}{P} \\
\Delta_x \text{ (when } x < \frac{l}{2} \text{)} & = \frac{Pa^2}{48EI} \left( 3l^2 - 4x \right)
\end{align*}
\]

17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT

\[
\begin{align*}
R_1 & = V_1 \text{ (max., when } a < b \text{)} = \frac{Pb^2}{l^2} (3a + b) \\
R_2 & = V_2 \text{ (max., when } a > b \text{)} = \frac{Pa^2}{l^2} (a + 3b) \\
M_1 & = \text{ (max., when } a < b \text{)} = \frac{Pab^2}{l^2} \\
M_2 & = \text{ (max., when } a > b \text{)} = \frac{Pab^2}{l^2} \\
M_a \text{ (at point of load)} & = \frac{2Pa^2b^2}{l^2} \\
M_x \text{ (when } x < a \text{)} & = Rlx - \frac{Pab^2}{l^2} \\
\Delta_{\text{max.}} \text{ (when } a > b \text{ at } x = \frac{2a}{3a + b} \text{)} & = \frac{3EI}{5} \left( \frac{3a + b}{2a} \right)^2 \\
\Delta_a \text{ (at point of load)} & = \frac{Pab^2}{3EI/l^2} \\
\Delta_x \text{ (when } x < a \text{)} & = \frac{Pb^2}{6EI/l^2} \left( 3a^2 - 3ax - bx \right)
\end{align*}
\]
### REFERENCE CHARTS FOR QUIZ 5

Available Critical Stress, $\phi_{F_{cr}}$, for Compression Members, MPa ($F_y = 250$ MPa and $\phi_k = 0.90$)

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## REFERENCE CHARTS FOR QUIZ 5

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