other beams & pinned frames

Continental train platform, Grimshaw 1993
Pinned Frames

- structures with at least one force body
- connected with pins
- reactions are equal and opposite
  - non-rigid
  - rigid
Rigid Frames

- **rigid frames have no pins**
- **frame is all one body**
- **typically statically indeterminate**
- **types**
  - portal
  - gable
Rigid Frames with PINS

- frame pieces with connecting pins
- not necessarily symmetrical
Internal Pin Connections

- *statically determinant*
  - 3 equations per body
  - 2 reactions per pin + support forces
Arches

- **ancient**
- **traditional shape to span long distances**

*Rainbow Bridge National Monument*

*Packhorse Bridge, UK*

*Roman Aqueducts*
Arches

- primarily sees compression
- a brick “likes an arch”
Arches

- **behavior**
  - **thrust related to height to width**
Three-Hinged Arch

- statically determinant
  - 2 bodies, 6 equilibrium equations
  - 4 support, 2 pin reactions (= 6)
Compound Beams

- **statically determinant when**
  - 3 equilibrium equations per link =>
  - *total of support & pin reactions* (properly constrained)

- **zero moment at pins**

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**Diagram:**

- **Compound Beams**
  - $F_1$ and $F_2$
  - (internal) pin

- **Pinned Frame**
  - $R_{1x}$, $R_{1y}$, $R_{2x}$, $R_{2y}$, $M_{R1}$
  - *not independent*
Procedure

- solve for all support forces you can
- draw a FBD of each member
  - pins are integral with member
  - pins with loads should belong to 3+ force bodies
  - pin forces are equal and opposite on connecting bodies
  - identify 2 force bodies vs. 3+ force bodies
  - use all equilibrium equations
Rigid Body Types

• **two force bodies**
  – forces in line, equal and opposite

• **three force bodies**
  – concurrent or parallel forces
Continuous Beams

- statically indeterminate
- reduced moments than simple beam
Continuous Beams

- loading pattern affects
  - moments & deflection
Continuous Beams

- **unload end span**
Continuous Beams

- unload middle span
Analysis Methods

- **Approximate Methods**
  - location of inflection points

- **Force Method**
  - forces are unknowns

- **Displacement Method**
  - displacements are unknowns
Two Span Beams & Charts

- equal spans & symmetrical loading
- middle support as flat slope

\[ \begin{align*}
R_1 &= V_1 = \frac{Pb^2}{2I} (a + 2l) \\
R_2 &= V_2 = \frac{Pa}{2I} (3l^2 - a^2) \\
M_1 (\text{at point of load}) &= R_1 a \\
M_2 (\text{at fixed end}) &= \frac{Pab}{2I} (a + l) \\
M_x (\text{when } x < a) &= R_1 x \\
M_x (\text{when } x > a) &= R_1 x - P (x - a) \\
\Delta_{\text{max.}} (\text{when } a < 0.414l) &= \frac{Pa}{3EI} \left( \frac{l^2}{3} + a^2 \right) \\
\Delta_{\text{max.}} (\text{when } a > 0.414l) &= \frac{Pa^2}{6EI} \sqrt{\frac{a}{2l + a}} \\
\Delta_a (\text{at point of load}) &= \frac{Pa^2b^2}{12EI^2} (3l + a) \\
\Delta_x (\text{when } x < a) &= \frac{Pb^4x}{12EI^3} (3a^2 - 2x^2 - ax^3) \\
\Delta_x (\text{when } x > a) &= \frac{Pa}{12EI^3} (l - x)^2 (3l^2 - a^2 - 2ax^2) 
\end{align*} \]