Forces and Moments

Architectural Structures: Form, Behavior, and Design

Arch 331
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Lecture Three

Forces and Moments
Structural Math

• quantify environmental loads
  – how big is it?

• evaluate geometry and angles
  – where is it?
  – what is the scale?
  – what is the size in a particular direction?

• quantify what happens in the structure
  – how big are the internal forces?
  – how big should the beam be?
Structural Math

- physics takes observable phenomena and relates the measurement with rules: mathematical relationships

- need
  - reference frame
  - measure of length, mass, time, direction, velocity, acceleration, work, heat, electricity, light
  - calculations & geometry
Physics for Structures

- **measures**
  - *US customary & SI*

<table>
<thead>
<tr>
<th>Units</th>
<th>US</th>
<th>SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>in, ft, mi</td>
<td>mm, cm, m</td>
</tr>
<tr>
<td>Volume</td>
<td>gallon</td>
<td>liter</td>
</tr>
<tr>
<td>Mass</td>
<td>lb mass</td>
<td>g, kg</td>
</tr>
<tr>
<td>Force</td>
<td>lb force</td>
<td>N, kN</td>
</tr>
<tr>
<td>Temperature</td>
<td>F</td>
<td>C</td>
</tr>
</tbody>
</table>
Physics for Structures

- **scalars** – any quantity
- **vectors** - quantities with direction
  - like displacements
  - summation results in the “straight line path” from start to end
  - *normal* vector is perpendicular to something
Language

- symbols for operations: +, -, /, x
- symbols for relationships: (), =, <, >
- algorithms
  - cancellation
  - factors
  - signs
  - ratios and proportions
  - power of a number
  - conversions, ex. $1X = 10Y$
  - operations on both sides of equality

\[
\frac{2 \times 5}{5 \times 6} = \frac{2}{6} = \frac{2}{2 \times 3} = \frac{1}{3}
\]

\[
x = \frac{1}{6} = \frac{3}{3}
\]

\[
10^3 = 10000
\]

\[
\frac{10Y}{1X} or \frac{1X}{10Y} = 1
\]
On-line Practice

• eCampus / Study Aids
Geometry

- **angles**
  - right $= 90^\circ$
  - acute $< 90^\circ$
  - obtuse $> 90^\circ$
  - $\pi = 180^\circ$

- **triangles**
  - area $= \frac{b \times h}{2}$
  - hypotenuse
  - total of angles $= 180^\circ$

\[ A B^2 + A C^2 = B C^2 \]
Geometry

- lines and relation to angles
  - parallel lines can’t intersect
  - perpendicular lines cross at 90°
  - intersection of two lines is a point
  - opposite angles are equal when two lines cross
Geometry

- intersection of a line with parallel lines results in identical angles

- two lines intersect in the same way, the angles are identical
Geometry

- sides of two angles are parallel and intersect opposite way, the angles are supplementary - the sum is 180°

\[ \alpha + \beta = 180° \]

- two angles that sum to 90° are said to be complimentary

\[ \beta + \gamma = 90° \]
Geometry

- sides of two angles bisect a right angle (90°), the angles are complimentary

\[
\alpha + \gamma = 90^\circ
\]

- right angle bisects a straight line, remaining angles are complimentary
Geometry

– similar triangles have proportional sides

\[
\frac{AB}{AD} = \frac{AC}{AE} = \frac{BC}{DE}
\]

\[
\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}
\]
Trigonometry

• for right triangles

\[
\sin \alpha = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{AB}{CB}
\]

\[
\cos \alpha = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{AC}{CB}
\]

\[
\tan \alpha = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{AB}{AC}
\]

SOHCAHTOA
Trigonometry

- cartesian coordinate system
  - origin at 0,0
  - coordinates in (x,y) pairs
  - x & y have signs

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Trigonometry

- for angles starting at positive $x$
  - $\sin$ is y side
  - $\cos$ is x side

$\sin<0$ for $180-360^\circ$
$\cos<0$ for $90-270^\circ$
$\tan<0$ for $90-180^\circ$
$\tan<0$ for $270-360^\circ$
Trigonometry

• for all triangles
  – sides A, B & C are opposite angles $\alpha$, $\beta$ & $\gamma$

  – LAW of SINES
    \[
    \frac{\sin \alpha}{A} = \frac{\sin \beta}{B} = \frac{\sin \gamma}{C}
    \]

  – LAW of COSINES
    \[
    A^2 = B^2 + C^2 - 2BC \cos \alpha
    \]
Algebra

- equations (something = something)
- constants
  - real numbers or shown with a, b, c...
- unknown terms, variables
  - names like R, F, x, y
- linear equations
  - unknown terms have no exponents
- simultaneous equations
  - variable set satisfies all equations
Algebra

• solving one equation
  – only works with one variable
  – ex:
    • add to both sides
      \[2x - 1 = 0\]
      \[2x - 1 + 1 = 0 + 1\]
      \[2x = 1\]
    • divide both sides
      \[\frac{2x}{2} = \frac{1}{2}\]
      \[x = \frac{1}{2}\]
    • get x by itself on a side
Algebra

• solving one equations
  – only works with one variable
  – ex:
    \[ 2x - 1 = 4x + 5 \]
    • subtract from both sides
      \[ 2x - 1 - 2x = 4x + 5 - 2x \]
    • subtract from both sides
      \[ -1 - 5 = 2x + 5 - 5 \]
    • divide both sides
      \[ \frac{-6}{2} = \frac{-3 \cdot 2}{2} = \frac{2x}{2} \]
    • get x by itself on a side
      \[ x = -3 \]
Algebra

- solving two equations
  - only works with two variables
  - ex:
    \[ 2x + 3y = 8 \]
    \[ 12x - 3y = 6 \]
    - look for term similarity
    - can we add or subtract to eliminate one term?

- add
  \[ 2x + 3y + 12x - 3y = 8 + 6 \]
  \[ 14x = 14 \]
  \[ \frac{14x}{14} = \frac{14}{14} = x = 1 \]
Forces

• statics
  – physics of forces and reactions on bodies and systems
  – equilibrium (bodies at rest)

• forces
  – something that exerts on an object:
    • motion
    • tension
    • compression
Force

• “action of one body on another that affects the state of motion or rest of the body”

• Newton’s 3rd law:
  – for every force of action there is an equal and opposite reaction along the same line
Force Characteristics

- applied at a point
- magnitude
  - Imperial units: lb, k (kips)
  - SI units: N (newtons), kN
- direction
Forces on Rigid Bodies

• for statics, the bodies are ideally rigid
• can translate and rotate
• internal forces are
  – in bodies
  – between bodies (connections)
• external forces act on bodies
Transmissibility

- the force stays on the same line of action
- truck can’t tell the difference

- only valid for EXTERNAL forces
**Force System Types**

- **collinear**

**Collinear**—All forces acting along the same straight line.

Figure 2.17(a)  Particle or rigid body.
Force System Types

- coplanar

**Coplanar**—All forces acting in the same plane.
- **Figure 2.17(b)** Rigid bodies.

**Coplanar, parallel**—All forces are parallel and act in the same plane.
- **Figure 2.17(c)** Rigid bodies.

**Coplanar, concurrent**—All forces intersect at a common point and lie in the same plane.
- **Figure 2.17(d)** Particle or rigid body.

Forces in a buttress system.

A beam supported by a series of columns.

Loads applied to a roof truss.
Force System Types

- space

- Column loads in a concrete building.
- Noncoplanar, parallel—All forces are parallel to each other, but not all lie in the same plane.
  - Figure 2.17(e) Rigid bodies.

- One component of a three-dimensional space frame.
- Noncoplanar, concurrent—All forces intersect at a common point but do not all lie in the same plane.
  - Figure 2.17(f) Particle or rigid bodies.

- Array of forces acting simultaneously on a house.
- Noncoplanar, nonconcurrent—All forces are skewed.
  - Figure 2.17(g) Rigid bodies.
Adding Vectors

- **graphically**
  - parallelogram law
    - diagonal
    - long for 3 or more vectors

- **tip-to-tail**
  - more convenient with lots of vectors
Force Components

- convenient to resolve into 2 vectors
- at right angles
- in a “nice” coordinate system
- $\theta$ is between $F_x$ and $F$ from $F_x$

\begin{align*}
F_x &= F \cos \theta \\
F_y &= F \sin \theta \\
F &= \sqrt{F_x^2 + F_y^2} \\
\tan \theta &= \frac{F_y}{F_x}
\end{align*}
Trigonometry

- $F_x$ is negative
  - $90^\circ$ to $270^\circ$
- $F_y$ is negative
  - $180^\circ$ to $360^\circ$
- $\tan$ is positive
  - quads I & III
- $\tan$ is negative
  - quads II & IV
Component Addition

- find all x components
- find all y components
- find sum of x components, $R_x$ (resultant)
- find sum of y components, $R_y$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\tan \theta = \frac{R_y}{R_x}$$
Alternative Trig for Components

- doesn’t relate angle to axis direction
- $\phi$ is “small” angle between $F$ and EITHER $F_x$ or $F_y$
- no sign out of calculator!
- have to choose RIGHT trig function, resulting direction (sign) and component axis
Friction

- resistance to movement
- contact surfaces determine $\mu$
- proportion of normal force ($\perp$)
  - opposite to slide direction
  - static > kinetic

$$F = \mu N$$
Cables

- simple
- uses
  - suspension bridges
  - roof structures
  - transmission lines
  - guy wires, etc.
- have same tension all along
- can’t stand compression
Cables Structures

- use high-strength steel
- need
  - towers
  - anchors
- don’t want movement
Cable Structures
Cable Loads

- **straight line between forces**
- **with one force**
  - concurrent
  - symmetric

(a) Simple concentrated load—triangle.

(b) Several concentrated loads—polygon.
Cable Loads

- shape directly related to the distributed load

(c) Uniform loads (horizontally)—parabola.

(d) Uniform loads (along the cable length)—catenary.

(e) Comparison of a parabolic and a catenary curve.
Cable-Stayed Structures

- diagonal cables support horizontal spans
- typically symmetrical
- Patcenter, Rogers 1986
Patcenter, Rogers 1986

- column free space
- roof suspended
- solid steel ties
- steel frame supports masts
Patcenter, Rogers 1986

- *dashes – cables pulling*

![Diagram](image)

Figure 3.5: Patcenter, load path diagram.
Moments

- forces have the tendency to make a body rotate about an axis

- same translation but different rotation

http://www.physics.umd.edu
Moments

Figure 2.33  Moment on a cantilever beam.

Figure 2.34  An example of torsion on a cantilever beam.
Moments

- A force acting at a different point causes a different moment:
Moments

- **defined by magnitude and direction**
- **units**: $N \cdot m$, $k \cdot ft$
- **direction**: 
  - $+ \text{ ccw (right hand rule)}$
  - $- \text{ cw}$
- **value found from $F$ and $\perp$ distance**
  \[ M = F \cdot d \]
- **$d$ also called “lever” or “moment” arm**
Moments

• with same $F$:

\[ M_A = F \cdot d_1 < M_A = F \cdot d_2 \]

(bigger)
Moments

- additive with sign convention
- can still move the force along the line of action

\[ M_A = F \cdot d \\
M_B = F \cdot d' \]
Moments

- **Varignon’s Theorem**
  - resolve a force into components at a point and finding perpendicular distances
  - calculate sum of moments
  - equivalent to original moment

- **makes life easier!**
  - geometry
  - when component runs through point, \( d=0 \)
Moments of a Force

- **moments of a force**
  - introduced in Physics as "Torque Acting on a Particle"
  - and used to satisfy rotational equilibrium
Physics and Moments of a Force

• my Physics book:

FIGURE 11–2 The plane shown is that defined by \( \mathbf{r} \) and \( \mathbf{F} \) in Fig. 11–1. (a) The magnitude of \( \tau \) is given by \( F r_\perp \) (Eq. 11–2b) or by \( r F_\perp \) (Eq. 11–2c). (b) Reversing \( \mathbf{F} \) reverses the direction of \( \tau \). (c) Reversing \( \mathbf{r} \) reverses the direction of \( \tau \). (d) Reversing \( \mathbf{F} \) and \( \mathbf{r} \) leaves the direction of \( \tau \) unchanged. The directions of \( \tau \) are represented by \( \bigcirc \) (perpendicularly out of the figure, the symbol representing the tip of an arrow) and by \( \otimes \) (perpendicularly into the figure, the symbol representing the tail of an arrow).
Moment Couples

• 2 forces
  – same size
  – opposite direction
  – distance d apart
  – cw or ccw

\[ M = F \cdot d \]

– not dependant on point of application

\[ M = F \cdot d_1 - F \cdot d_2 \]
Moment Couples

- equivalent couples
  - same magnitude and direction
  - $F$ & $d$ may be different

\[
\begin{align*}
\text{300 N} & \quad 100 \text{ mm} & \quad 300 \text{ N} \\
\text{200 N} & \quad 150 \text{ mm} & \quad 120 \text{ N} \\
\text{200 N} & \quad 150 \text{ mm} & \quad 120 \text{ N} \\
\text{250 mm} & & \\
\end{align*}
\]
Moment Couples

• added just like moments caused by one force
• can replace two couples with a single couple

\[
\begin{align*}
300 \text{ N} & \quad 100 \text{ mm} & \quad 300 \text{ N} \\
200 \text{ N} & \quad 150 \text{ mm} & \quad 240 \text{ N} \quad 240 \text{ N} \\
\end{align*}
\]
Moment Couples

- moment couples in structures

The flanges of a steel beam are welded to the flange of a column. Equal and opposite forces \( T \) and \( C \) in the beam flanges form a couple with moment \( M \) that is transferred into the column.
Equivalent Force Systems

- two forces at a point is equivalent to the resultant at a point
- resultant is equivalent to two components at a point
- resultant of equal & opposite forces at a point is zero
- put equal & opposite forces at a point (sum to 0)
- transmission of a force along action line
**Force-Moment Systems**

- single force causing a moment can be replaced by the same force at a different point by providing the moment that force caused

- moments are shown as arched arrows
Force-Moment Systems

- A force-moment pair can be replaced by a force at another point causing the original moment.

\[ M = F \cdot d \]
Parallel Force Systems

- forces are in the same direction
- can find resultant force
- need to find location for equivalent moments