ARCHITECTURAL STRUCTURES: FORM, BEHAVIOR, AND DESIGN
ARCH 331
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lecture
nine

beam sections - geometric properties
Center of Gravity

- **location of equivalent weight**
- **determined with calculus**

\[
\sum \Delta W = \int dW
\]
Center of Gravity

- “average” x & y from moment

\[
\sum M_y = \sum_{i=1}^{n} x_i \Delta W_i = \bar{x}W 
\Rightarrow \quad \bar{x} = \frac{\sum (x_i \Delta W)}{W}
\]

“bar” means average

\[
\sum M_x = \sum_{i=1}^{n} y_i \Delta W_i = \bar{y}W 
\Rightarrow \quad \bar{y} = \frac{\sum (y_i \Delta W)}{W}
\]
Centroid

• “average” x & y of an area
• for a volume of constant thickness
  \[ \Delta W = \gamma t \Delta A \] where $\gamma$ is weight/volume
  \[ \text{center of gravity} = \text{centroid of area} \]

\[
\bar{x} = \frac{\sum (x \Delta A)}{A}
\]

\[
\bar{y} = \frac{\sum (y \Delta A)}{A}
\]
Centroid

- for a line, sum up length

\[
\bar{x} = \frac{\sum (x \Delta L)}{L} \\
\bar{y} = \frac{\sum (y \Delta L)}{L}
\]
**1st Moment Area**

- **math concept**
- **the moment of an area about an axis**

\[
Q_x = \bar{y}A \\
Q_y = \bar{x}A
\]
Symmetric Areas

• symmetric about an axis

• symmetric about a center point

• mirrored symmetry
Composite Areas

- made up of basic shapes
- areas can be negative
- (centroids can be negative for any area)
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate \( \hat{x} \) and \( \hat{y} \)
# Area Centroids

- **Table 7.1 — pg. 242**

### Centroids of Common Shapes of Areas and Lines

<table>
<thead>
<tr>
<th>Shape</th>
<th>( \bar{x} )</th>
<th>( \bar{y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular area</td>
<td>( \frac{b}{3} )</td>
<td>( \frac{h}{3} )</td>
</tr>
<tr>
<td>Quarter-circular area</td>
<td>( \frac{4r}{3\pi} )</td>
<td>( \frac{4r}{3\pi} )</td>
</tr>
<tr>
<td>Semicircular area</td>
<td>0</td>
<td>( \frac{4r}{3\pi} )</td>
</tr>
<tr>
<td>Semiparabolic area</td>
<td>( \frac{3a}{8} )</td>
<td>( \frac{3h}{5} )</td>
</tr>
<tr>
<td>Parabolic area</td>
<td>0</td>
<td>( \frac{3h}{5} )</td>
</tr>
</tbody>
</table>

- **Note**: Right triangle only.
Moments of Inertia

• 2\textsuperscript{nd} moment area
  – math concept
  – area \times (distance)^2

• need for behavior of
  – beams
  – columns
Moment of Inertia

• about any reference axis
• can be negative

\[ I_y = \int x^2 \, dA \]

\[ I_x = \int y^2 \, dA \]

• resistance to bending and buckling

\[ dA = y \cdot dx \]
Moment of Inertia

- same area moved away a distance
  - larger $I$
Polar Moment of Inertia

- for roundish shapes
- uses polar coordinates (r and $\theta$)
- resistance to twisting

\[ J_o = \int r^2 \, dA \]
Radius of Gyration

- measure of inertia with respect to area

\[ r_x = \sqrt{\frac{I_x}{A}} \]
Parallel Axis Theorem

- can find composite $I$ once composite centroid is known (basic shapes)

\[ I_x = I_{cx} + Ad_y^2 \]

\[ = \bar{I}_x + Ad_y^2 \]

\[ I = \sum \bar{I} + \sum Ad^2 \]

\[ \bar{I} = I - Ad^2 \]
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with $A$, $\bar{x}$, $\bar{x}A$, $\bar{y}$, $\bar{y}A$, $\bar{I}$’s, $d$’s, and $Ad^2$’s
5. Fill in table and get $\hat{x}$ and $\hat{y}$ for composite
6. Sum necessary columns
7. Sum $\bar{I}$’s and $Ad^2$’s

$$\begin{align*}
(d_x &= \hat{x} - \bar{x}) \\
(d_y &= \hat{y} - \bar{y})
\end{align*}$$
Area Moments of Inertia

- Table 7.2 – pg. 252: (bars refer to centroid)
  - $x, y$
  - $x', y'$
  - $C$

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
<th>Centroid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>$I_x = \frac{1}{12}bh^3$</td>
<td>$I_y = \frac{1}{12}b^3h$</td>
</tr>
<tr>
<td>Triangle</td>
<td>$I_x = \frac{1}{36}bh^3$</td>
<td>$I_y = \frac{1}{12}b^3h$</td>
</tr>
<tr>
<td>Circle</td>
<td>$I_x = I_y = \frac{1}{4\pi}r^4$</td>
<td>$J_O = \frac{1}{2}\pi r^4$</td>
</tr>
</tbody>
</table>