Steel construction: columns & tension members

Design Methods (revisited)

- know
  - loads or lengths
- select
  - section or load
  - adequate for strength and no buckling

Allowable Stress Design (ASD)

- AICS 9th ed

\[
F_a = \frac{f_{\text{critical}}}{F.S.} = \frac{12\pi^2 E}{23(Kl/r)^2}
\]

- slenderness ratio \( \frac{Kl}{r} \)
  - for \( kl/r \geq C_c \)
    - \( = 126.1 \) with \( F_y = 36 \text{ ksi} \)
    - \( = 107.0 \) with \( F_y = 50 \text{ ksi} \)

Structural Steel

- standard rolled shapes
  - \( W, C, L, T \)
- tubing
- pipe
- built-up
**C_c and Euler’s Formula**

- **Kl/r < C_c**
  - short and stubby
  - parabolic transition
- **Kl/r > C_c**
  - Euler’s relationship
  - < 200 preferred

\[ C_c = \sqrt{\frac{2\pi^2 E}{F_y}} \]

**Short / Intermediate**

- **L_e/r < C_c**
  
  \[ F_a = 1 - \frac{(Kl/r)^2}{2C_c^2} \]

  - where

  \[ F.S. = \frac{5}{3} + \frac{3(Kl/r)}{8C_c} - \frac{(Kl/r)^3}{8C_c^3} \]

**Unified Design**

- limit states for failure

  \[ P_a \leq \frac{P_n}{\Omega} \]

  \[ \phi_c = 0.90 \quad P_n = F_{cr} A_g \quad P_u \leq \phi_c P_n \]

  1. yielding \[ KL \leq 4.71 \sqrt{\frac{E}{F_y}} \quad \text{or} \quad F_e \geq 0.44F_y \]

  2. buckling \[ KL > 4.71 \sqrt{\frac{E}{F_y}} \quad \text{or} \quad F_e < 0.44F_y \]

\[ F_e \text{ – elastic buckling stress (Euler)} \]
Unified Design

- \( P_n = F_{cr}A_g \)
  - for \( \frac{KL}{r} \leq 4.71 \) \[ F_{cr} = \left( \frac{E}{F_y} \right) \leq 4.71 \left( \frac{F_y}{F_e} \right) \]
  - for \( \frac{KL}{r} > 4.71 \) \[ F_{cr} = 0.877F_e \]
  - where \( F_e = \frac{\pi^2 E}{(KL/r)^2} \)

Procedure for Analysis

1. calculate \( KL/r \)
   - biggest of \( KL/r \) with respect to x axes and y axis
2. find \( F_a \) or \( F_{cr} \) from appropriate equation
   - tables are available
3. compute \( P_{allowable} = F_a \cdot A \) or \( P_n = F_{cr}A_g \)
   - or find \( f_{actual} = P/A \)
4. is \( P \leq P_{allowable} \) (\( P_a \leq P_n / \Omega \))? or is \( P_u \leq \phi P_n \)?
   - yes: ok
   - no: insufficient capacity and no good

Procedure for Design

1. guess a size (pick a section)
2. calculate \( KL/r \)
   - biggest of \( KL/r \) with respect to x axes and y axis
3. find \( F_a \) or \( F_{cr} \) from appropriate equations
   - or find a chart
4. compute \( P_{allowable} = F_a \cdot A \) (\( P_n / \Omega = F_{cr}A_g \))
   - or \( P_n = F_{cr}A_g \)
   - or find \( f_{actual} = P/A \)

Procedure for Design (cont’d)

5. is \( P \leq P_{allowable} \) (\( P_a \leq P_n / \Omega \))? or is \( P_u \leq \phi P_n \)?
   - yes: ok
   - no: pick a bigger section and go back to step 2.
6. check design efficiency
   - percentage of stress = \( \frac{P_r}{P_c} \cdot 100\% \)
   - if between 90-100%: good
   - if < 90%: pick a smaller section and go back to step 2.
Column Charts, $F_a$ (pg. 361-364)

Table 4–1 (continued) Available Strength in Axial Compression, kips W Shapes

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<th>Shape</th>
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<th>W12×</th>
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<td>LRFD</td>
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<td>$\phi P_{u1}$</td>
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</table>

Column Charts, $\phi F_{cr}$

Available Critical Stress, $\phi F_{cr}$, for Compression Members, ksi ($F_y = 60$ ksi and $\phi = 0.90$)

Beam-Column Design

• moment magnification ($P-\Delta$)

$$M_u = B_1M_{max-factored} \quad B_1 = \frac{C_m}{1 - \left(\frac{P_u}{P_{el1}}\right)}$$

$C_m$ – modification factor for end conditions

$$= 0.6 - 0.4(M_1/M_2) \quad \text{or} \quad 0.85 \text{ restrained, } 1.00 \text{ unrestrained}$$

$P_{el1}$ – Euler buckling strength

$$P_{el1} = \frac{\pi^2 EA}{(Kl/r)^2}$$
Beam-Column Design

• LRFD (Unified) Steel
  – for
  \[ \frac{P_r}{P_c} \geq 0.2 : \frac{P_u}{\phi_c P_n} + \frac{8}{9} \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0 \]
  – for
  \[ \frac{P_r}{P_c} < 0.2 : \frac{P_u}{2\phi_c P_n} + \left( \frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}} \right) \leq 1.0 \]

  \( P_r \) is required, \( P_c \) is capacity
  \( \phi_c \) - resistance factor for compression = 0.9
  \( \phi_b \) - resistance factor for bending = 0.9

Design Steps Knowing Loads (revisited)

1. assume limiting stress
   • buckling, axial stress, combined stress
2. solve for \( r, A \) or \( S \)
3. pick trial section
4. analyze stresses
5. section ok?
6. stop when section is ok

Rigid Frame Design (revisited)

• columns in frames
  – ends can be “flexible”
  – stiffness affected by beams and column = \( EI/L \)
  \[ \sum EI \]
  \[ \frac{G = \Psi = \frac{\sum EI}{l_c}}{\sum EI/l_b} \]
  – for the joint
    • \( l_c \) is the column length of each column
    • \( l_b \) is the beam length of each beam
    • measured center to center

Rigid Frame Design (revisited)

• column effective length, \( k \)
Tension Members

- steel members can have holes
- reduced area
  \[ A_n = A_g - A_{of\ all\ holes} + t\sum \frac{s^2}{4g} \]  
  (AISC - Steel Structures of the Everyday)
- increased stress

Effective Net Area

- likely path to “rip” across
- bolts divide transferred force too
- shear lag \( A_e \leq A_n U \)

Tension Members

- limit states for failure
  \[ P_a \leq \frac{P_n}{\Omega} \leq \phi_t P_n \]
  \[ P_n = F_y A_g \]
  \[ P_n = F_u A_e \]
  \( A_g \) - gross area
  \( A_e \) - effective net area
  (holes 3/16” + d)
  \( F_u \) = the tensile strength of the steel (ultimate)