Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for maximum stresses
  - limit state design
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity
  - concrete strength = $f'_c$

Concrete Construction

- cast-in-place
- tilt-up
- prestressing
- post-tensioning

Concrete Beams

- types
  - reinforced
  - precast
  - prestressed
- shapes
  - rectangular, I
  - $T$, double $T$'s, bulb $T$'s
  - box
  - spandrel
Concrete Beams

- shear
  - vertical
  - horizontal
  - combination:
    - tensile stresses at 45°
- bearing
  - crushing

Concrete

- hydration
  - chemical reaction
  - workability
  - water to cement ratio
  - mix design
- fire resistant
- cover for steel
- creep & shrinkage

Concrete

- low strength to weight ratio
- relatively inexpensive
  - Portland cement
    - types I - V
  - aggregate
    - course & fine
  - water
  - admixtures
    - air entraining
    - superplasticizers

Concrete

- placement (not pouring!)
- vibrating
- screeding
- floating
- troweling
- curing
- finishing
Reinforcement

• deformed steel bars (rebar)
  – Grade 40, \( F_y = 40 \text{ ksi} \)
  – Grade 60, \( F_y = 60 \text{ ksi} \) - most common
  – Grade 75, \( F_y = 75 \text{ ksi} \)
  – US customary in # of 1/8” \( \phi \)

• longitudinally placed
  – bottom
  – top for compression reinforcement

Composite Beams

• concrete
  – in compression

• steel
  – in tension

• shear studs

Reinforcement

• prestressing strand
• post-tensioning
• stirrups
• detailing
  – development length
  – anchorage
  – splices

Behavior of Composite Members

• plane sections remain plane
• stress distribution changes

\[
f_1 = \frac{E_1 \varepsilon}{\rho} = - \frac{E_1 y}{\rho} \quad \text{and} \quad f_2 = \frac{E_2 \varepsilon}{\rho} = - \frac{E_2 y}{\rho}
\]
Transformation of Material

- $n$ is the ratio of $E$'s
  \[ n = \frac{E_2}{E_1} \]
- effectively widens a material to get same stress distribution

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Stresses in Composite Section

- with a section transformed to one material, new $I$
  \[ n = \frac{E_2}{E_1} = \frac{E_{\text{steel}}}{E_{\text{concrete}}} \]
  - stresses in that material are determined as usual
  - stresses in the other material need to be adjusted by $n$

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Reinforced Concrete - stress/strain

Reinforced Concrete Analysis

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent stress block
  - concrete takes no tension
  - steel takes tension
  - force ductile failure

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**Location of n.a.**

- ignore concrete below n.a.
- transform steel
- same area moments, solve for $x$

\[ bx \cdot \frac{x}{2} - nA_s (d - x) = 0 \]

**T sections**

- n.a. equation is different if n.a. below flange

\[ b_f h_f \left( x - \frac{h_f}{2} \right) + (x - h_f) b_w \left( x - \frac{h_f}{2} \right) - nA_s (d - x) = 0 \]

**ACI Load Combinations**

- 1.4D
- 1.2D + 1.6L + 0.5(L_r or S or R)
- 1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)
- 1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)
- 1.2D + 1.0E + 1.0L + 0.2S
- 0.9D + 1.0W
- 0.9D + 1.0E

*can also use old ACI factors*

**Reinforced Concrete Design**

- stress distribution in bending

![Stress distribution diagram](image)

**Wang & Salmon, Chapter 3**
### Force Equations
- \( C = 0.85 f'_c b a \)
- \( T = A_s f_y \)
- where
  - \( f'_c \) = concrete compressive strength
  - \( a \) = height of stress block
  - \( \beta_1 \) = factor based on \( f'_c \)
  - \( x \) = location to the n.a.
  - \( b \) = width of stress block
  - \( f_y \) = steel yield strength
  - \( A_s \) = area of steel reinforcement

### Equilibrium
- \( T = C \)
- \( M_n = T(d - a/2) \)
  - \( d \) = depth to the steel n.a.
- with \( A_s \)
  - \( a = \frac{A_s f_y}{0.85 f'_c b} \)
  - \( M_u \leq \phi M_n, \phi = 0.9 \) for flexure*
  - \( \phi M_n = \phi T(d - a/2) = \phi A_s f_y (d - a/2) \)

### Over and Under-reinforcement
- over-reinforced
  - steel won't yield
- under-reinforced
  - steel will yield
- reinforcement ratio
  - \( \rho = \frac{A_s}{bd} \)
  - use as a design estimate to find \( A_s, b, d \)
  - \( \max \rho \) is found with \( \varepsilon_{\text{steel}} \geq 0.004 \) (not \( \rho_{\text{bal}} \))
  - *with \( \varepsilon_{\text{steel}} \geq 0.005, \phi = 0.9 \)

### \( A_s \) for a Given Section
- several methods
  - guess \( a \) and iterate
    1. guess \( a \) (less than n.a.)
    2. \( A_s = \frac{0.85 f'_c b a}{f_y} \)
    3. solve for \( a \) from \( M_u = \phi A_s f_y (d - a/2) \)
       \[ a = 2\left( d - \frac{M_u}{\phi A_s f_y} \right) \]
    4. repeat from 2. until \( a \) from 3. matches \( a \) in 2.

*with \( \varepsilon_{\text{steel}} \geq 0.005, \phi = 0.9 \)
$A_s$ for a Given Section (cont)

- **chart method**
  - Wang & Salmon Fig. 3.8.1 $R_n$ vs. $\rho$
    1. calculate $R_n = \frac{M_R}{bd^2}$
    2. find curve for $f'_c$ and $f_y$ to get $\rho$
    3. calculate $A_s$ and $a$
  - simplify by setting $h = 1.1d$

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**Reinforcement**

- **min for crack control**
- **required**
  $$A_s = \frac{3\sqrt{f'_c}}{f_y} (bd)$$
- **not less than**
  $$A_s = \frac{200}{f_y} (bd)$$
- **$A_{s-max}$**
  $$a = \beta_1 (0.375d)$$
- **typical cover**
  - 1.5 in, 3 in with soil
- **bar spacing**

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**Annunciation Greek Orthodox Church**

- **Wright, 1956**

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**Shells**

- [Image of shells]

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**Concrete Beams  26**

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**Concrete Beams  27**

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**Concrete Beams  28**

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http://nisee.berkeley.edu/godden

http://www.bluffton.edu/~sullivanm/
Annunciation Greek Orthodox Church

• Wright, 1956

Cylindrical Shells

• can resist tension
• shape adds “depth”
• not vaults
• barrel shells

Kimball Museum, Kahn 1972

• outer shell edges
Kimball Museum, Kahn 1972
• skylights at peak

Approximate Depths