Concrete construction: shear & deflection

Shear in Concrete Beams

- flexure combines with shear to form diagonal cracks
- horizontal reinforcement doesn’t help
- stirrups = vertical reinforcement

ACI Shear Values

- $V_u$ is at distance $d$ from face of support
- shear capacity: $V_c = V_u \times b_w \times d$
  - where $b_w$ means thickness of web at n.a.

- shear stress (beams)
  - $\phi V_c = \phi 2 f_c' b_w d$ 
    - $\phi = 0.75$ for shear
    - $f_c'$ is in psi

- shear strength:
  - $V_u \leq \phi V_c + \phi V_s$
  - $V_s$ is strength from stirrup reinforcement
Stirrup Reinforcement

- shear capacity:
  \[ V_s = \frac{A_v f_y d}{s} \]
  - \( A_v \) = area in all legs of stirrups
  - \( s \) = spacing of stirrup

- may need stirrups when concrete has enough strength!

Required Stirrup Reinforcement

- spacing limits

<table>
<thead>
<tr>
<th>( V_s )</th>
<th>( \phi V_s )</th>
<th>( V_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{V_s}{2} )</td>
<td>( \frac{\phi V_s}{2} )</td>
<td>( \frac{V_s}{2} )</td>
</tr>
</tbody>
</table>

- Recommended
  - \( \frac{A_v}{f_y} \)
  - \( \frac{\phi A_v}{f_y} \)

- Minimum
  - \( \frac{d_2}{2} \) or \( 24\text{ in.} \)
  - \( \frac{d_1}{2} \) or \( 12\text{ in.} \)

Torsional Stress & Strain

- can see torsional stresses & twisting of axi-symmetrical cross sections
  - torque
  - remain plane
  - undistorted
  - rotates

- not true for square sections....

Shear Stress Distribution

- depend on the deformation
  - \( \phi \) = angle of twist
  - measure

- can prove planar section doesn’t distort
Shearing Strain

- related to $\phi$
  \[ \gamma = \frac{\rho \phi}{L} \]
- $\rho$ is the radial distance from the centroid to the point under strain
- shear strain varies linearly along the radius: $\gamma_{\text{max}}$ is at outer diameter

Torsional Stress - Strain

- know $f = \tau = G \cdot \gamma$ and $\gamma = \frac{\rho \phi}{L}$
- so
  \[ \tau = G \cdot \frac{\rho \phi}{L} \]
- where $G$ is the Shear Modulus

Shear Stress

- $\tau_{\text{max}}$ happens at outer diameter
- combined shear and axial stresses
  - maximum shear stress at 45° “twisted” plane
Shear Strain

- knowing \( \tau = G \cdot \frac{p \phi}{L} \) and \( \tau = \frac{T \rho}{J} \)
- solve: \( \phi = \frac{TL}{JG} \)
- composite shafts: \( \phi = \sum_i \frac{T_i L_i}{J_i G_i} \)

Noncircular Shapes

- torsion depends on \( J \)
- plane sections don’t remain plane
- \( \tau_{max} \) is still at outer diameter
- \( \tau_{max} = \frac{T}{c_1 ab^2 \phi} = \frac{TL}{c_2 ab^3 G} \)
  - where \( a \) is longer side (> \( b \))

Open Thin-Walled Sections

- with very large \( a/b \) ratios:

\[
\tau_{max} = \frac{T}{\frac{1}{3} ab^2} \quad \phi = \frac{TL}{\frac{1}{3} ab^3 G}
\]

Shear Flow in Closed Sections

- \( q \) is the internal shear force/unit length

\[
\tau = \frac{T}{2 t A} \quad \phi = \frac{TL}{4 t A^2 \sum_i \frac{s_i}{t_i}}
\]
- \( A \) is the area bounded by the centerline
- \( s_i \) is the length segment, \( t_i \) is the thickness
Shear Flow in Open Sections

- each segment has proportion of $T$ with respect to torsional rigidity,

$$\tau_{\text{max}} = \frac{T t_{\text{max}}}{\frac{1}{3} \sum b_i t_i^3}$$

- total angle of twist:

$$\phi = \frac{TL}{\frac{1}{3} G \sum b_i t_i^3}$$

- I beams - web is thicker, so $\tau_{\text{max}}$ is in web

Torsional Shear Stress

- twisting moment

- and beam shear

Torsional Shear Reinforcement

- closed stirrups

- more longitudinal reinforcement

- area enclosed by shear flow

Development Lengths

- required to allow steel to yield ($f_y$)

- standard hooks
  - moment at beam end

- splices
  - lapped
  - mechanical connectors
Development Lengths

- $l_d$, embedment required both sides
- proper cover, spacing:
  - No. 6 or smaller
    \[ l_d = \frac{d_b F_y}{25 \sqrt{f_c'}} \] or 12 in. minimum
  - No. 7 or larger
    \[ l_d = \frac{d_b F_y}{20 \sqrt{f_c'}} \] or 12 in. minimum

Development Lengths

- bars in compression
  \[ l_d = \frac{0.02 d_b F_y}{\sqrt{f_c'}} \leq 0.0003 d_b F_y \]
- splices
  - tension minimum is function of $l_d$ and splice classification
  - compression minimum
  - is function of $d_b$ and $F_y$

Concrete Deflections

- elastic range
  - $l$ transformed
  - $E_c$ (with $f_c'$ in psi)
    - normal weight concrete ($\sim 145$ lb/ft$^3$)
      \[ E_c = 57,000 \sqrt{f_c'} \]
    - concrete between 90 and 160 lb/ft$^3$
      \[ E_c = w^{1.5} 33 \sqrt{f_c'} \]
  - cracked
    - $l$ cracked
    - $E$ adjusted

Development Lengths

- hooks
  - bend and extension

Concrete Shear 21
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Deflection Limits

- relate to whether or not beam supports or is attached to a damageable non-structural element
- need to check service live load and long term deflection against these

<table>
<thead>
<tr>
<th>Deflection Limit</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>L/180</td>
<td>roof systems (typical) – live</td>
</tr>
<tr>
<td>L/240</td>
<td>floor systems (typical) – live + long term</td>
</tr>
<tr>
<td>L/360</td>
<td>supporting plaster – live</td>
</tr>
<tr>
<td>L/480</td>
<td>supporting masonry – live + long term</td>
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</tbody>
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