Beams

• span horizontally
  – floors
  – bridges
  – roofs
• loaded transversely by gravity loads
• may have internal axial force
• will have internal shear force
• will have internal moment (bending)

Beams

• transverse loading
• sees:
  – bending
  – shear
  – deflection
  – torsion
  – bearing
• behavior depends on cross section shape

Beams

• bending
  – bowing of beam with loads
  – one edge surface stretches
  – other edge surface squishes
Beam Stresses

• stress = relative force over an area
  – tensile
  – compressive
  – bending
    • tension and compression + ...

Beam Stresses

• prestress or post-tensioning
  – put stresses in tension area to “pre-compress”
Beam Stresses

- shear – horizontal & vertical

Beam Stresses

- shear – horizontal

Beam Deflections

- depends on
  - load
  - section
  - material

Figure 5.4  Bending (flexural) loads on a beam.
**Beam Deflections**

- “moment of inertia”

**Beam Styles**

- vierendeel

- open web joists

- manufactured

http://nisee.berkeley.edu/godden

**Internal Beam Forces**

**Internal Forces**

- trusses
  - axial only, (compression & tension)

- in general
  - axial force
  - shear force, V
  - bending moment, M

**Beam Loading**

- concentrated force
- concentrated moment
  - spandrel beams
**Beam Loading**

- uniformly distributed load (line load)
- non-uniformly distributed load
  - hydrostatic pressure = \( \gamma h \)
  - wind loads

**Beam Supports**

- statically determinate
- statically indeterminate

**Internal Beam Forces**

- like method of sections / joints
  - no axial forces
- section must be in equilibrium
- want to know where biggest internal forces and moments are for designing
**V & M Diagrams**

- tool to locate $V_{\text{max}}$ and $M_{\text{max}}$ (at $V = 0$)
- necessary for designing
- have a different sign convention than external forces, moments, and reactions

**Sign Convention**

- shear force, $V$:
  - cut section to LEFT
  - if $\sum F_y$ is positive by statics, $V$ acts down and is POSITIVE
  - beam has to resist shearing apart by $V$

**Shear Sign Convention**

- bending moment, $M$:
  - cut section to LEFT
  - if $\sum M_{\text{cut}}$ is clockwise, $M$ acts ccw and is POSITIVE – flexes into a “smiley” beam has to resist bending apart by $M$
Bending Moment Sign Convention

• (+) Moment.
  - compression
  - tension in bottom, compression in top

• (−) Moment.
  - tension
  - tension in top, compression in bottom

Deflected Shape

• positive bending moment
  – tension in bottom, compression in top

• negative bending moment
  – tension in top, compression in bottom

• zero bending moment
  – inflection point

Constructing V & M Diagrams

• along the beam length, plot V, plot M

Mathematical Method

• cut sections with x as width

• write functions of V(x) and M(x)
Method 1: Equilibrium

- cut sections at important places
- plot $V$ & $M$

\[ V \]
\[ L \]
\[ M \]
\[ L/2 \]

Method 2: Semigraphical

- by knowing
  - area under loading curve = change in $V$
  - area under shear curve = change in $M$
  - concentrated forces cause “jump” in $V$
  - concentrated moments cause “jump” in $M$

\[ V_D - V_C = - \int x_P wdx \]
\[ M_D - M_C = \int x_P Vdx \]

Method 1: Equilibrium

- important places
  - supports
  - concentrated loads
  - start and end of distributed loads
  - concentrated moments
- free ends
  - zero forces

Method 2

- relationships

\[ \int Vdx \]
Method 2: Semigraphical

- $M_{\text{max}}$ occurs where $V = 0$ (calculus)

Curve Relationships

- integration of functions
- line with 0 slope, integrates to sloped
- ex: load to shear, shear to moment

Curve Relationships

- line with slope, integrates to parabola
- ex: load to shear, shear to moment

Curve Relationships

- parabola, integrates to 3rd order curve
- ex: load to shear, shear to moment
**Basic Procedure**

1. Find reaction forces & moments
   - Plot axes, underneath beam load diagram

2. Starting at left

3. Shear is 0 at free ends

4. Shear has 2 values at point loads

5. Sum vertical forces at each section

**Basic Procedure**

M:

6. Starting at left

7. Moment is 0 at free ends

8. Moment has 2 values at moments

9. Sum moments at each section

10. Maximum moment is where shear = 0!
    (locate where \( V = 0 \))

**Shear Through Zero**

- slope of \( V \) is \( w \) (-w:1)

\[
\begin{align*}
\text{load} & \quad \text{shear} \\
\text{height} = V_A & \quad \text{width} = x \\
\frac{w \text{ (force/length)}}{V_A} & \quad x = \frac{w}{V_A}
\end{align*}
\]

**Parabolic Shapes**

- cases

\[
\begin{align*}
\text{up fast, then slow} & \quad \text{up slow, then fast} \\
\text{down fast, then slow} & \quad \text{down slow, then fast}
\end{align*}
\]