Center of Gravity

- location of equivalent weight
- determined with calculus

\[ W = \int dW \]

Centroid

- “average” x & y of an area
- for a volume of constant thickness
  - \( \Delta W = \gamma \Delta A \) where \( \gamma \) is weight/volume
  - center of gravity = centroid of area

\[ \bar{x} = \frac{\sum (x\Delta A)}{A} \]
\[ \bar{y} = \frac{\sum (y\Delta A)}{A} \]
Centroid

- for a line, sum up length

\[ \bar{x} = \frac{\sum (x \Delta L)}{L} \]
\[ \bar{y} = \frac{\sum (y \Delta L)}{L} \]

1st Moment Area

- math concept

- the moment of an area about an axis

\[ Q_x = \bar{y}A \]
\[ Q_y = \bar{x}A \]

Symmetric Areas

- symmetric about an axis

- symmetric about a center point

- mirrored symmetry

Composite Areas

- made up of basic shapes

- areas can be negative

- (centroids can be negative for any area)
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate \( \bar{x} \) and \( \bar{y} \)

---

Area Centroids

- **Table 7.1 – pg. 242**

<table>
<thead>
<tr>
<th>Shape</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quarter-circular area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semicircular area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Semicircular parabolic area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parabolic area</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

Moments of Inertia

- 2nd moment area
  - math concept
  - area \( x \) (distance)\(^2\)
- need for behavior of
  - beams
  - columns

---

Moment of Inertia

- about any reference axis
- can be negative

\[
I_y = \int x^2 \, dA
\]
\[
I_x = \int y^2 \, dA
\]

- resistance to bending and buckling
### Moment of Inertia

- **same area moved away a distance**
  - **larger $I$**

### Polar Moment of Inertia

- **for roundish shapes**
- **uses polar coordinates ($r$ and $\theta$)**
- **resistance to twisting**

\[
J_o = \int r^2 \, dA
\]

### Radius of Gyration

- **measure of inertia with respect to area**

\[
r_x = \sqrt{\frac{I_x}{A}}
\]

### Parallel Axis Theorem

- **can find composite $I$ once composite centroid is known (basic shapes)**

\[
I_x = I_{cx} + Ad_y^2
= \bar{I}_x + Ad_y^2
\]

\[
I = \sum \bar{I} + \sum Ad^2
\]

\[
\bar{I} = I - Ad^2
\]
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with $A, \bar{x}, \bar{x}A, \bar{y}, \bar{y}A, \bar{I}'s, d's, \text{ and } Ad^2's$
5. Fill in table and get $\hat{x}$ and $\hat{y}$ for composite
6. Sum necessary columns
7. Sum $I's$ and $Ad^2's$

Area Moments of Inertia

- $x, y$
- $x', y'$
- $C$

Table 7.2 – pg. 252: (bars refer to centroid)

<table>
<thead>
<tr>
<th>Shape</th>
<th>$I_x$</th>
<th>$I_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>$\frac{1}{12}Ah^3$</td>
<td>$\frac{1}{12}Ah^3$</td>
</tr>
<tr>
<td>Triangle</td>
<td>$\frac{1}{3}bh^3$</td>
<td>$\frac{1}{3}bh^3$</td>
</tr>
<tr>
<td>Circle</td>
<td>$\frac{1}{4}\pi r^4$</td>
<td>$\frac{1}{4}\pi r^4$</td>
</tr>
<tr>
<td>Semicircle</td>
<td>$\frac{1}{8}\pi r^4$</td>
<td>$\frac{1}{8}\pi r^4$</td>
</tr>
<tr>
<td>Quarter circle</td>
<td>$\frac{1}{16}\pi r^4$</td>
<td>$\frac{1}{16}\pi r^4$</td>
</tr>
<tr>
<td>Ellipse</td>
<td>$\frac{1}{4}ab$</td>
<td>$\frac{1}{4}ab$</td>
</tr>
</tbody>
</table>

$I = \int_A (x^2 + y^2) dA$