Concrete construction: shear & deflection
Shear in Concrete Beams

• flexure combines with shear to form diagonal cracks

• horizontal reinforcement doesn’t help
• stirrups = vertical reinforcement
ACI Shear Values

- $V_u$ is at distance $d$ from face of support
- shear capacity: $V_c = \nu_c \times b_w d$

where $b_w$ means thickness of web at n.a.
ACI Shear Values

- shear stress (beams)
  \[ \nu_c = 2\sqrt{f'_c} \]
  \[ \phi V_c = \phi 2\sqrt{f'_c} b_w d \]
  \( \phi = 0.75 \) for shear \( f'_c \) is in \( \text{psi} \)

- shear strength:
  \[ V_u \leq \phi V_c + \phi V_s \]
  \( V_s \) is strength from stirrup reinforcement
Stirrup Reinforcement

- shear capacity:

\[ V_s = \frac{A_v f_y d}{s} \]

- \( A_v = \text{area in all legs of stirrups} \)
- \( s = \text{spacing of stirrup} \)

- may need stirrups when concrete has enough strength!
## Required Stirrup Reinforcement

- **spacing limits**

### Table 3-8 ACI Provisions for Shear Design*

<table>
<thead>
<tr>
<th>Stirrup spacing, $s$</th>
<th>$V_u \leq \frac{\phi V_c}{2}$</th>
<th>$\phi V_c \geq V_u &gt; \frac{\phi V_c}{2}$</th>
<th>$V_u &gt; \phi V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required area of stirrups, $A_v^{**}$</td>
<td>$A_v f_y$</td>
<td>$\frac{50 b_w s}{f_y}$</td>
<td>$\frac{(V_u - \phi V_c) s}{\phi f_y d}$</td>
</tr>
<tr>
<td>Required</td>
<td>$\frac{A_v f_y}{50 b_w}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Recommended Minimum$^\dagger$</td>
<td>$\frac{d}{2}$ or 24 in.</td>
<td></td>
<td>4 in.</td>
</tr>
<tr>
<td>Maximum$^{\dagger\dagger}$ (ACI 11.5.4)</td>
<td>$\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \leq 4\sqrt{f'_c b_w d}$</td>
<td></td>
<td>$\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) &gt; 4\sqrt{f'_c b_w d}$</td>
</tr>
</tbody>
</table>

*Members subjected to shear and flexure only; $\phi V_c = \phi 2 \sqrt{f'_c b_w d}$, $\phi = 0.75$ (ACI 11.3.1.1)

**$A_v = 2 \times A_b$ for U stirrups; $f_y \leq 60$ ksi (ACI 11.5.2)

$^\dagger$A practical limit for minimum spacing is $d/4$

$^{\dagger\dagger}$Maximum spacing based on minimum shear reinforcement ($= \frac{A_v f_y}{50 b_w}$) must also be considered (ACI 11.5.5.3).
Torsional Stress & Strain

• can see torsional stresses & twisting of axi-symmetrical cross sections
  – torque
  – remain plane
  – undistorted
  – rotates

• not true for square sections....
Shear Stress Distribution

- depend on the deformation
- $\phi = \text{angle of twist}$
  - measure
- can prove planar section doesn’t distort
Shearing Strain

- related to $\phi$

$$\gamma = \frac{\rho \phi}{L}$$

- $\rho$ is the radial distance from the centroid to the point under strain

- shear strain varies linearly along the radius: $\gamma_{\text{max}}$ is at outer diameter
Torsional Stress - Strain

- know $f_v = \tau = G \cdot \gamma$ and $\gamma = \frac{\rho \phi}{L}$
- so $\tau = G \cdot \frac{\rho \phi}{L}$
- where $G$ is the Shear Modulus
Torsional Stress - Strain

- from
  \[ T = \sum \tau(\rho) \Delta A \]

- can derive
  \[ T = \frac{\tau J}{\rho} \]

  - where \( J \) is the polar moment of inertia
  - elastic range
    \[ \tau = \frac{T \rho}{J} \]
Shear Stress

- $\tau_{\text{max}}$ happens at outer diameter

- combined shear and axial stresses
  - maximum shear stress at 45° “twisted” plane
Shear Strain

• knowing \( \tau = G \cdot \frac{\rho \phi}{L} \) and \( \tau = \frac{T \rho}{J} \)

• solve: \( \phi = \frac{TL}{JG} \)

• composite shafts: \( \phi = \sum_i \frac{T_i L_i}{J_i G_i} \)
Noncircular Shapes

• torsion depends on $J$
• plane sections don’t remain plane
• $\tau_{\text{max}}$ is still at outer diameter

$$\tau_{\text{max}} = \frac{T}{c_1 ab^2} \quad \phi = \frac{TL}{c_2 ab^3 G}$$

– where $a$ is longer side ($> b$)
Open Thin-Walled Sections

• with very large $a/b$ ratios:

\[
\tau_{\text{max}} = \frac{T}{\frac{1}{3} ab^2} \quad \phi = \frac{TL}{\frac{1}{3} ab^3 G}
\]
Shear Flow in Closed Sections

• \( q \) is the internal shear force/unit length

\[
\tau = \frac{T}{2ta}
\]

\[
\phi = \frac{TL}{4ta^2} \sum_i \frac{s_i}{t_i}
\]

• \( a \) is the area bounded by the centerline

• \( s_i \) is the length segment, \( t_i \) is the thickness
Shear Flow in Open Sections

- each segment has proportion of \( T \) with respect to torsional rigidity,

\[
\tau_{\text{max}} = \frac{T t_{\text{max}}}{\frac{1}{3} \sum b_i t_i^3}
\]

- total angle of twist:

\[
\phi = \frac{TL}{\frac{1}{3} G \sum b_i t_i^3}
\]

- \( I \) beams - web is thicker, so \( \tau_{\text{max}} \) is in web
Torsional Shear Stress

- twisting moment
- and beam shear

Fig. R11.6.3.1—Addition of torsional and shear stresses
**Torsional Shear Reinforcement**

- **closed stirrups**
- **more longitudinal reinforcement**
- **area enclosed by shear flow**
Development Lengths

- required to allow steel to yield ($f_y$)
- standard hooks
  - moment at beam end
- splices
  - lapped
  - mechanical connectors
Development Lengths

- $l_d$, embedment required both sides
- proper cover, spacing:
  - No. 6 or smaller
    \[ l_d = \frac{d_b F_y}{25 \sqrt{f'_c}} \quad \text{or 12 in. minimum} \]
  - No. 7 or larger
    \[ l_d = \frac{d_b F_y}{20 \sqrt{f'_c}} \quad \text{or 12 in. minimum} \]
Development Lengths

- **hooks**
  - **bend and extension**

![Diagram of 90° bar hook](image1.png)

*Figure 9-17: Minimum requirements for 90° bar hooks.*

![Diagram of 180° bar hook](image2.png)

*Figure 9-18: Minimum requirements for 180° bar hooks.*

- **minimum**

\[ l_{dh} = \frac{1200d_b}{\sqrt{f'_c}} \]
Development Lengths

- **bars in compression**
  \[
  l_d = \frac{0.02d_b F_y}{\sqrt{f'_c}} \leq 0.0003d_b F_y
  \]

- **splices**
  - tension minimum is function of \( l_d \) and splice classification
  - compression minimum
  - is function of \( d_b \) and \( F_y \)

*Figure 13.24 The lapped splice for steel reinforcing bars.*
Concrete Deflections

- **elastic range**
  - I transformed
  - $E_c$ (with $f'_c$ in psi)
    - normal weight concrete ($\sim 145$ lb/ft$^3$)
      \[ E_c = 57,000 \sqrt{f'_c} \]
    - concrete between 90 and 160 lb/ft$^3$
      \[ E_c = w_c^{1.5} 33 \sqrt{f'_c} \]
- **cracked**
  - I cracked
  - $E$ adjusted
Deflection Limits

• relate to whether or not beam supports or is attached to a damageable non-structural element
• need to check service live load and long term deflection against these

<table>
<thead>
<tr>
<th>Limit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/180</td>
<td>roof systems (typical) – live</td>
</tr>
<tr>
<td>L/240</td>
<td>floor systems (typical) – live + long term</td>
</tr>
<tr>
<td>L/360</td>
<td>supporting plaster – live</td>
</tr>
<tr>
<td>L/480</td>
<td>supporting masonry – live + long term</td>
</tr>
</tbody>
</table>