Steel beams

Lecture 18

Steel construction: materials & beams
Steel Beam Design

- American Institute of Steel Construction
  - Manual of Steel Construction
  - ASD & LRFD
  - combined in 2005
Steel Materials

- smelt iron ore
- add alloying elements
- heat treatments
- iron, carbon
- microstructure

A36 steel, JOM 1998
Steel Materials

- cast into billets
- hot rolled
- cold formed
- residual stress
- corrosion-resistant “weathering” steels
- stainless
Steel Materials

• steel grades
  – ASTM A36 – carbon
    • plates, angles
    • $F_y = 36 \text{ ksi}$ & $F_u = 58 \text{ ksi}$
  – ASTM A572 – high strength low-alloy
    • some beams
    • $F_y = 60 \text{ ksi}$ & $F_u = 75 \text{ ksi}$
  – ASTM A992 – for building framing
    • most beams
    • $F_y = 50 \text{ ksi}$ & $F_u = 65 \text{ ksi}$
Steel Properties

- high strength to weight ratio
- elastic limit – yield ($F_y$)
- inelastic – plastic
- ultimate strength ($F_u$)
- ductile
- strength sensitive to temperature
- can corrode
- fatigue

strain hardening

Winnepeg DOT
Structural Steel

- standard rolled shapes ($W$, $C$, $L$, $T$)
- open web joists
- plate girders
- decking
Steel Construction

- welding
- bolts
Steel Construction

• fire proofing
  – cementicious spray
  – encasement in gypsum
  – intumescent – expands with heat
  – sprinkler system
Unified Steel Design

• ASD

\[ R_a \leq \frac{R_n}{\Omega} \]

- bending (braced) \( \Omega = 1.67 \)
- bending (unbraced*) \( \Omega = 1.67 \)
- shear \( \Omega = 1.5 \) or 1.67
- shear (bolts & welds) \( \Omega = 2.00 \)
- shear (welds) \( \Omega = 2.00 \)

* flanges in compression can buckle
Unified Steel Design

- braced vs. unbraced
LRFD

• loads on structures are
  - not constant
  - can be more influential on failure
  - happen more or less often
  - UNCERTAINTY

\[ R_u = \gamma_D R_D + \gamma_L R_L \leq \phi R_n \]

\( \phi \) - resistance factor
\( \gamma \) - load factor for (D)ead & (L)ive load
LRFD Steel Beam Design

- limit state is yielding all across section
- outside elastic range
- load factors & resistance factors

\[ f_y = 50 \text{ksi} \]

\[ \varepsilon_y = 0.001724 \]
LRFD Load Combinations

- 1.4D
- 1.2D + 1.6L + 0.5(L_r or S or R)
- 1.2D + 1.6(L_r or S or R) + (L or 0.5W)
- 1.2D + 1.0W + L + 0.5(L_r or S or R)
- 1.2D + 1.0E + L + 0.2S
- 0.9D + 1.0W
- 0.9D + 1.0E
  - F has same factor as D in 1-5 and 7
  - H adds with 1.6 and resists with 0.9 (permanent)

ASCE-7 (2010)
Beam Design Criteria (revisited)

• strength design
  – bending stresses predominate
  – shear stresses occur

• serviceability
  – limit deflection
  – stability

• superpositioning
  – use of beam charts
  – elastic range only!
  – “add” moment diagrams
  – “add” deflection CURVES (not maximums)
Steel Beams

- lateral stability - bracing
- local buckling – stiffen, or bigger $I_y$
Local Buckling

- **steel I beams**
- **flange**
  - buckle in direction of smaller radius of gyration
- **web**
  - force
  - “crippling”
Local Buckling

- flange
- web

Figure 2-5. Flange Local Bending Limit State
(Beedle, I.S., Christopher, R., 1964)

Figure 2-7. Web Local Buckling Limit State
(SAC Project)
Shear in Web

- panels in plate girders or webs with large shear
- buckling in compression direction
- add stiffeners
Shear in Web

- plate girders and stiffeners
Steel Beams

- **bearing**
  - provide adequate area
  - prevent local yield of flange and web

*Figure 9.10* Considerations for bearing in beams with thin webs, as related to web crippling (buckling of the thin web in compression).
\[ \sum \gamma_i R_i = M_u \leq \phi_b M_n = 0.9 F_y Z \]

- **\( M_u \)** - maximum moment
- **\( \phi_b \)** - resistance factor for bending = 0.9
- **\( M_n \)** - nominal moment (ultimate capacity)
- **\( F_y \)** - yield strength of the steel
- **\( Z \)** - plastic section modulus*
Internal Moments - at yield

- material hasn’t failed

\[ M_y = \frac{I}{c} f_y = \frac{b h^2}{6} f_y \]

\[ = \frac{b(2c)^2}{6} f_y = \frac{2bc^2}{3} f_y \]
Internal Moments - ALL at yield

- all parts reach yield
- plastic hinge forms
- ultimate moment
- $A_{\text{tension}} = A_{\text{compression}}$

$$M_p = bc^2 f_y = \frac{3}{2} M_y$$
n.a. of Section at Plastic Hinge

- cannot guarantee at centroid
- \( f_y \cdot A_1 = f_y \cdot A_2 \)
- moment found from yield stress times moment area

\[
M_p = f_y A_1 d = f_y \sum_{n.a.} A_i d_i
\]
Plastic Hinge Development

(a) $M < M_Y$

(b) $M = M_Y$

(c) $M > M_Y$

(d) $M = M_p$
Plastic Hinge Examples

- stability can be effected
Plastic Section Modulus

- shape factor, $k$
  
  $= \frac{3}{2}$ for a rectangle
  
  $\approx 1.1$ for an I

- plastic modulus, $Z$
  
  $Z = \frac{M_p}{f_y}$
LRFD – Shear (compact shapes)

\[ \sum \gamma_i R_i = V_u \leq \phi_v V_n = 1.0 \left( 0.6 F_{yw} A_w \right) \]

- \( V_u \) - maximum shear
- \( \phi_v \) - resistance factor for shear = 1.0
- \( V_n \) - nominal shear
- \( F_{yw} \) - yield strength of the steel in the web
- \( A_w \) - area of the web = \( t_w d \)
LRFD – Flexure Design

- limit states for beam failure
  1. yielding
  2. lateral-torsional buckling*
  3. flange local buckling
  4. web local buckling

- minimum $M_n$ governs

\[
\sum \gamma_i R_i = M_u \leq \phi_b M_n
\]
Compact Sections

- plastic moment can form before buckling
- criteria

\[ \frac{b_f}{2t_f} \leq 0.38 \sqrt{\frac{E}{F_y}} \]

- and \[ \frac{h_c}{t_w} \leq 3.76 \sqrt{\frac{E}{F_y}} \]

TABLE A.3 Properties of W Shapes
Lateral Torsional Buckling

\[ M_n = C_b \left[ \text{moment based on lateral buckling} \right] \leq M_p \]

\[ C_b = \frac{12.5 M_{\text{max}}}{2.5 M_{\text{max}} + 3 M_A + 4 M_B + 3 M_C} \]

- \( C_b \) = modification factor
- \( M_{\text{max}} \) = \( |\text{max moment}| \), unbraced segment
- \( M_A \) = \( |\text{moment}| \), 1/4 point
- \( M_B \) = \( |\text{moment}| \), center point
- \( M_C \) = \( |\text{moment}| \), 3/4 point
Beam Design Charts

Table 3-10 (continued)

W Shapes

Available Moment vs. Unbraced Length

<table>
<thead>
<tr>
<th>ASD</th>
<th>LRFD</th>
</tr>
</thead>
<tbody>
<tr>
<td>52</td>
<td>78</td>
</tr>
<tr>
<td>48</td>
<td>72</td>
</tr>
<tr>
<td>44</td>
<td>66</td>
</tr>
<tr>
<td>40</td>
<td>60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Available Moment, $M_{c,0} / M_p$ (1 kip-ft increments)</th>
<th>Unbraced Length (0.5-ft increments)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W8x21</td>
<td>2</td>
</tr>
<tr>
<td>W10x46</td>
<td>4</td>
</tr>
<tr>
<td>W12x22</td>
<td>6</td>
</tr>
<tr>
<td>W14x22</td>
<td>8</td>
</tr>
<tr>
<td>W16x22</td>
<td>10</td>
</tr>
<tr>
<td>W18x22</td>
<td>12</td>
</tr>
<tr>
<td>W20x22</td>
<td>14</td>
</tr>
<tr>
<td>W22x22</td>
<td>16</td>
</tr>
<tr>
<td>W24x22</td>
<td>18</td>
</tr>
</tbody>
</table>
Charts & Deflections

• beam charts
  – solid line is most economical
  – dashed indicates there is another more economical section
  – self weight is NOT included in $M_n$

• deflections
  – no factors are applied to the loads
  – often governs the design
Design Procedure (revisited)

1. **Know unbraced length, material, design method** ($\Omega$, $\phi$)

2. **Draw V & M, finding $M_{\text{max}}$**

3. **Calculate $Z_{\text{req'd}}$**

\[
\begin{align*}
M_a & \leq M_n / \Omega \\
M_u & \leq \phi_b M_n
\end{align*}
\]

4. **Choose (economical) section from section or beam capacity charts**
### Beam Charts by $S_x$ (Appendix A)

Table 11  Listing of W Shapes in Descending Order of $S_x$ for Beam Design.

<table>
<thead>
<tr>
<th>$S_x$—US (in.³)</th>
<th>Section</th>
<th>$S_x$—SI ($10^3 \times$ mm³)</th>
<th>$S_x$—US (in.³)</th>
<th>Section</th>
<th>$S_x$—SI ($10^3 \times$ mm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>448</td>
<td>W33 × 141</td>
<td>7350</td>
<td>188</td>
<td>W18 × 97</td>
<td>3080</td>
</tr>
<tr>
<td>439</td>
<td>W36 × 135</td>
<td>7200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>411</td>
<td>W27 × 146</td>
<td>6740</td>
<td>175</td>
<td>W16 × 100</td>
<td>2870</td>
</tr>
<tr>
<td>406</td>
<td>W33 × 130</td>
<td>6660</td>
<td>173</td>
<td>W14 × 109</td>
<td>2840</td>
</tr>
<tr>
<td>380</td>
<td>W30 × 132</td>
<td>6230</td>
<td>171</td>
<td>W21 × 83</td>
<td>2800</td>
</tr>
<tr>
<td>371</td>
<td>W24 × 146</td>
<td>6080</td>
<td>166</td>
<td>W18 × 86</td>
<td>2720</td>
</tr>
<tr>
<td>359</td>
<td>W33 × 118</td>
<td>5890</td>
<td>157</td>
<td>W14 × 99</td>
<td>2570</td>
</tr>
<tr>
<td>355</td>
<td>W30 × 124</td>
<td>5820</td>
<td>155</td>
<td>W16 × 89</td>
<td>2540</td>
</tr>
<tr>
<td>329</td>
<td>W30 × 116</td>
<td>5400</td>
<td>151</td>
<td>W21 × 73</td>
<td>2480</td>
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<tr>
<td>329</td>
<td>W24 × 131</td>
<td>5400</td>
<td>146</td>
<td>W18 × 76</td>
<td>2390</td>
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<tr>
<td>329</td>
<td>W21 × 147</td>
<td>5400</td>
<td>143</td>
<td>W14 × 90</td>
<td>2350</td>
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</table>
TABLE 9.1 Load Factor Resistance Design Selection for Shapes Used as Beams

<table>
<thead>
<tr>
<th>Designation</th>
<th>$L_p$</th>
<th>$L_r$</th>
<th>$M_p$</th>
<th>$M_r$</th>
<th>$L_p$</th>
<th>$L_r$</th>
<th>$M_p$</th>
<th>$M_r$</th>
<th>$r_y$</th>
<th>$b_y/2t_y$</th>
<th>$h_{ix}$</th>
<th>$X_{ij}$</th>
<th>$X_{ix} \times 10^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W 33 × 141</td>
<td>514</td>
<td>10.1</td>
<td>30.1</td>
<td>1,542</td>
<td>945</td>
<td>8.59</td>
<td>23.1</td>
<td>2,142</td>
<td>1,493</td>
<td>2.43</td>
<td>6.01</td>
<td>49.6</td>
<td>1,800</td>
</tr>
<tr>
<td>W 30 × 148</td>
<td>500</td>
<td>9.50</td>
<td>30.6</td>
<td>1,500</td>
<td>945</td>
<td>8.06</td>
<td>22.8</td>
<td>2,083</td>
<td>1,453</td>
<td>2.28</td>
<td>4.44</td>
<td>41.6</td>
<td>2,310</td>
</tr>
<tr>
<td>W 24 × 162</td>
<td>468</td>
<td>12.7</td>
<td>45.2</td>
<td>1,404</td>
<td>897</td>
<td>10.8</td>
<td>32.4</td>
<td>1,950</td>
<td>1,380</td>
<td>3.05</td>
<td>5.31</td>
<td>30.6</td>
<td>2,870</td>
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<tr>
<td>W 24 × 146</td>
<td>418</td>
<td>12.5</td>
<td>42.0</td>
<td>1,254</td>
<td>804</td>
<td>10.6</td>
<td>30.6</td>
<td>1,742</td>
<td>1,237</td>
<td>3.01</td>
<td>5.92</td>
<td>33.2</td>
<td>2,590</td>
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<tr>
<td>W 33 × 118</td>
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<td>9.67</td>
<td>27.8</td>
<td>1,245</td>
<td>778</td>
<td>8.20</td>
<td>21.7</td>
<td>1,729</td>
<td>1,197</td>
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<td>7.76</td>
<td>54.5</td>
<td>1,510</td>
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<td>408</td>
<td>9.29</td>
<td>28.2</td>
<td>1,224</td>
<td>769</td>
<td>7.88</td>
<td>21.5</td>
<td>1,700</td>
<td>1,183</td>
<td>2.23</td>
<td>5.65</td>
<td>46.2</td>
<td>1,930</td>
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<td>W 21 × 147</td>
<td>373</td>
<td>12.3</td>
<td>46.4</td>
<td>1,119</td>
<td>713</td>
<td>10.4</td>
<td>32.8</td>
<td>1,554</td>
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<td>39.3</td>
<td>1,110</td>
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<td>29.1</td>
<td>1,542</td>
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<td>648</td>
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<td>6.89</td>
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<td>648</td>
<td>7.71</td>
<td>21.3</td>
<td>1,429</td>
<td>997</td>
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<td>5.41</td>
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<tr>
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<td>37.1</td>
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<td>631</td>
<td>10.4</td>
<td>27.9</td>
<td>1,363</td>
<td>970</td>
<td>2.94</td>
<td>7.53</td>
<td>39.2</td>
<td>2,090</td>
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<td>W 21 × 122</td>
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<td>41.0</td>
<td>921</td>
<td>592</td>
<td>10.3</td>
<td>29.8</td>
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<td>910</td>
<td>2.92</td>
<td>6.45</td>
<td>31.3</td>
<td>2,630</td>
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<td>W 18 × 130</td>
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<td>47.7</td>
<td>870</td>
<td>555</td>
<td>9.55</td>
<td>32.8</td>
<td>1,208</td>
<td>853</td>
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<td>4.65</td>
<td>23.9</td>
<td>3,680</td>
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<td>W 30 × 90</td>
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<td>24.8</td>
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<td>531</td>
<td>7.39</td>
<td>19.4</td>
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<td>531</td>
<td>7.04</td>
<td>20.9</td>
<td>1,167</td>
<td>817</td>
<td>1.99</td>
<td>4.59</td>
<td>39.2</td>
<td>2,390</td>
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<td>W 21 × 94</td>
<td>278</td>
<td>8.83</td>
<td>25.9</td>
<td>834</td>
<td>527</td>
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<td>810</td>
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<td>1,740</td>
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<td>260</td>
<td>16.6</td>
<td>81.6</td>
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<td>503</td>
<td>14.1</td>
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<td>1,083</td>
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<td>7.11</td>
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<td>4,400</td>
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<td>W 12 × 94</td>
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<td>8.25</td>
<td>25.9</td>
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<td>740</td>
<td>1.98</td>
<td>5.18</td>
<td>41.9</td>
<td>2,180</td>
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</tbody>
</table>
Beam Design (revisited)

4*. Include self weight for $M_{\text{max}}$
  - it’s dead load
  - and repeat 3 & 4 if necessary

5. Consider lateral stability

Unbraced roof trusses were blown down in 1999 at this project in Moscow, Idaho.

Photo: Ken Carper
Beam Design (revisited)

6. Evaluate shear stresses - horizontal

- \( (V_a \leq V_n / \Omega) \) or \( (V_u \leq \phi_v V_n) \)

- rectangles and W’s
  \[
  f_{v-max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}
  \]

- general
  \[
  V_n = 0.6F_{yw} A_w
  \]

- general
  \[
  f_{v-max} = \frac{VQ}{I_b}
  \]
Beam Design (revisited)

7. Provide adequate bearing area at supports

\[
\begin{align*}
(P_a & \leq P_n / \Omega) \\
(P_u & \leq \phi P_n)
\end{align*}
\]
Beam Design (revisited)

8. Evaluate torsion

\[
(f_v \leq F_v)
\]

- circular cross section
  \[
f_v = \frac{T#}{J}
\]
- rectangular
  \[
f_v = \frac{T}{c_1 ab^2}
\]

**TABLE 3.1. Coefficients for Rectangular Bars in Torsion**

<table>
<thead>
<tr>
<th>a/b</th>
<th>c_1</th>
<th>c_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.208</td>
<td>0.1406</td>
</tr>
<tr>
<td>1.2</td>
<td>0.219</td>
<td>0.1661</td>
</tr>
<tr>
<td>1.5</td>
<td>0.231</td>
<td>0.1958</td>
</tr>
<tr>
<td>2.0</td>
<td>0.246</td>
<td>0.229</td>
</tr>
<tr>
<td>2.5</td>
<td>0.258</td>
<td>0.249</td>
</tr>
<tr>
<td>3.0</td>
<td>0.267</td>
<td>0.263</td>
</tr>
<tr>
<td>4.0</td>
<td>0.282</td>
<td>0.281</td>
</tr>
<tr>
<td>5.0</td>
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</tr>
<tr>
<td>10.0</td>
<td>0.312</td>
<td>0.312</td>
</tr>
<tr>
<td>∞</td>
<td>0.333</td>
<td>0.333</td>
</tr>
</tbody>
</table>
Beam Design (revisited)

9. Evaluate deflections – NO LOAD FACTORS

\[ y_{\text{max}}(x) = \Delta_{\text{actual}} \leq \Delta_{\text{allowable}} \]
Load Tables & Equivalent Load

- uniformly distributed loads
- equivalent “w”

\[ M_{\text{max}} = \frac{w_{\text{equivalent}} L^2}{8} \]

Load for live load deflection limit in RED, total in BLACK
Sloped Beams

- stairs & roofs
- projected live load
- dead load over length

perpendicular load to beam:

$$W_{\perp} = w \cdot \cos \alpha$$

equivalent distributed load:

$$W_{adj.} = \frac{w}{\cos \alpha}$$
Steel Arches and Frames

- solid sections
- or open web

http://nisee.berkeley.edu/godden
Steel Shell and Cable Structures
Approximate Depths

<table>
<thead>
<tr>
<th>Structure</th>
<th>Approximate Depths</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decking</td>
<td>L/30–L/50</td>
</tr>
<tr>
<td>Wide flanges</td>
<td>L/18–L/28</td>
</tr>
<tr>
<td>Plate girders</td>
<td>L/15–L/20</td>
</tr>
<tr>
<td>Open-web joists</td>
<td>L/18–L/22</td>
</tr>
<tr>
<td>Fink truss</td>
<td>L/4–L/5</td>
</tr>
<tr>
<td>Howe truss</td>
<td>L/4–L/5</td>
</tr>
<tr>
<td>Bowstring truss</td>
<td>L/6–L/10</td>
</tr>
<tr>
<td>Special truss</td>
<td>L/4–L/15</td>
</tr>
<tr>
<td>Arches</td>
<td>L/3–L/5</td>
</tr>
<tr>
<td>Ribbed domes</td>
<td>L/3–L/5</td>
</tr>
<tr>
<td>Cables</td>
<td>L/5–L/11</td>
</tr>
<tr>
<td>Space frame (column-supported)</td>
<td>L/12–L/20</td>
</tr>
<tr>
<td>Space frame (wall-supported)</td>
<td>L/12–L/20</td>
</tr>
</tbody>
</table>