concrete construction: shear & deflection
Shear in Concrete Beams

- *flexure combines with shear to form diagonal cracks*  
  ![Diagram showing diagonal cracks in a concrete beam with flexure and shear forces](image)

- *horizontal reinforcement doesn’t help*

- *stirrups = vertical reinforcement*
  
  ![Diagram showing vertical stirrups and a single-loop or U stirrup](image)
ACI Shear Values

- $V_u$ is at distance $d$ from face of support
- shear capacity: $V_c = \nu_c \times b_w d$

- where $b_w$ means thickness of web at n.a.

*Figure 13.16 Layout for shear stress analysis: ACI Code requirements.*
ACI Shear Values

- **shear stress (beams)**
  
  \[ \nu_c = 2\sqrt{f'_c} \]
  \[ \phi V_c = \phi 2\sqrt{f'_c} b_w d \]

- **shear strength:**
  
  \[ V_u \leq \phi V_c + \phi V_s \]

- \( V_s \) is strength from stirrup reinforcement

\[ \phi = 0.75 \text{ for shear } \]

\( f'_c \) is in \textbf{psi}
Stirrup Reinforcement

- **shear capacity:**
  \[ V_s = \frac{A_v f_y d}{s} \]
  - \( A_v \) = area in all legs of stirrups
  - \( s \) = spacing of stirrup

- **may need stirrups when concrete has enough strength!**
**Required Stirrup Reinforcement**

- **Spacing Limits**

### Table 3-8 ACI Provisions for Shear Design*

<table>
<thead>
<tr>
<th>Stirrup spacing, s</th>
<th>$V_u \leq \frac{\phi V_c}{2}$</th>
<th>$\phi V_c \geq V_u &gt; \frac{\phi V_c}{2}$</th>
<th>$V_u &gt; \phi V_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required area of stirrups, $A_v$ **</td>
<td>none</td>
<td>$\frac{50bws}{f_y}$</td>
<td>$\frac{(V_u - \phi V_c)s}{\phi f_yd}$</td>
</tr>
<tr>
<td><strong>Required</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Recommended Minimum†</strong></td>
<td></td>
<td></td>
<td>4 in.</td>
</tr>
<tr>
<td><strong>Maximum‡‡ (ACI 11.5.4)</strong></td>
<td></td>
<td></td>
<td>$\frac{d}{2}$ or 24 in.</td>
</tr>
</tbody>
</table>

| | |  | $\frac{d}{2}$ or 24 in. for $(V_u - \phi V_c) \leq 4\sqrt{f'_c} b_w d$ |
| | |  | $\frac{d}{4}$ or 12 in. for $(V_u - \phi V_c) > 4\sqrt{f'_c} b_w d$ |

*Members subjected to shear and flexure only; $\phi V_c = \phi 2 \sqrt{f'_c} b_w d$, $\phi = 0.75$ (ACI 11.3.1.1)

**$A_v = 2 \times A_b$ for U stirrups; $f_y \leq 60$ ksi (ACI 11.5.2)

† A practical limit for minimum spacing is $d/4$

‡‡ Maximum spacing based on minimum shear reinforcement ($= A_v f_y / 50b_w$) must also be considered (ACI 11.5.5.3).
Torsional Stress & Strain

• can see torsional stresses & twisting of axi-symmetrical cross sections
  – torque
  – remain plane
  – undistorted
  – rotates

• not true for square sections....
Shear Stress Distribution

- depend on the deformation

- $\phi = \text{angle of twist}$
  - measure

- can prove planar section doesn’t distort
Shearing Strain

- related to $\phi$
  \[ \gamma = \frac{\rho \phi}{L} \]
- $\rho$ is the radial distance from the centroid to the point under strain
- shear strain varies linearly along the radius: $\gamma_{\text{max}}$ is at outer diameter
Torsional Stress - Strain

- know \( f_v = \tau = G \cdot \gamma \) and \( \gamma = \frac{\rho \phi}{L} \)
- so \( \tau = G \cdot \frac{\rho \phi}{L} \)
- where \( G \) is the Shear Modulus
Torsional Stress - Strain

- from $T = \Sigma \tau (\rho) \Delta A$

- can derive $T = \frac{\tau J}{\rho}$

  - where $J$ is the polar moment of inertia

  - elastic range $\tau = \frac{T \rho}{J}$
Shear Stress

- $\tau_{\text{max}}$ happens at outer diameter

- combined shear and axial stresses
  - maximum shear stress at 45° “twisted” plane
Shear Strain

- knowing \( \tau = G \cdot \frac{\rho \phi}{L} \) and \( \tau = \frac{T\rho}{J} \)

- solve: \( \phi = \frac{TL}{JG} \)

- composite shafts: \( \phi = \sum \frac{T_i L_i}{J_i G_i} \)
Noncircular Shapes

- Torsion depends on $J$
- Plane sections don’t remain plane
- $\tau_{\text{max}}$ is still at outer diameter

$$\tau_{\text{max}} = \frac{T}{c_1 a b^2} \quad \phi = \frac{TL}{c_2 a b^3 G}$$

- Where $a$ is longer side (> $b$)

<table>
<thead>
<tr>
<th>$a/b$</th>
<th>$c_1$</th>
<th>$c_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.208</td>
<td>0.1406</td>
</tr>
<tr>
<td>1.2</td>
<td>0.219</td>
<td>0.1661</td>
</tr>
<tr>
<td>1.5</td>
<td>0.231</td>
<td>0.1958</td>
</tr>
<tr>
<td>2.0</td>
<td>0.246</td>
<td>0.229</td>
</tr>
<tr>
<td>2.5</td>
<td>0.258</td>
<td>0.249</td>
</tr>
<tr>
<td>3.0</td>
<td>0.267</td>
<td>0.263</td>
</tr>
<tr>
<td>4.0</td>
<td>0.282</td>
<td>0.281</td>
</tr>
<tr>
<td>5.0</td>
<td>0.291</td>
<td>0.291</td>
</tr>
<tr>
<td>10.0</td>
<td>0.312</td>
<td>0.312</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.333</td>
<td>0.333</td>
</tr>
</tbody>
</table>
Open Thin-Walled Sections

- with very large $a/b$ ratios:

$$\tau_{\text{max}} = \frac{T}{\frac{1}{3}ab^2}$$

$$\phi = \frac{TL}{\frac{1}{3}ab^3G}$$
Shear Flow in Closed Sections

- \( q \) is the internal shear force/unit length

\[
\tau = \frac{T}{2ta}
\]

\[
\phi = \frac{TL}{4ta^2} \sum_i \frac{s_i}{t_i}
\]

- \( a \) is the area bounded by the centerline
- \( s_i \) is the length segment, \( t_i \) is the thickness
Shear Flow in Open Sections

- each segment has proportion of $T$ with respect to torsional rigidity,

\[ \tau_{\text{max}} = \frac{T t_{\text{max}}}{\frac{1}{3} \sum b_i t_i^3} \]

- total angle of twist:

\[ \phi = \frac{TL}{\frac{1}{3} G \sum b_i t_i^3} \]

- I beams - web is thicker, so $\tau_{\text{max}}$ is in web
Torsional Shear Stress

- twisting moment
- and beam shear

*Design torque may not be reduced because moment redistribution is not possible*

Fig. R11.6.3.1—Addition of torsional and shear stresses
Torsional Shear Reinforcement

- closed stirrups
- more longitudinal reinforcement
- area enclosed by shear flow
Development Lengths

- **required to allow steel to yield** $(f_y)$
- **standard hooks**
  - moment at beam end
- **splices**
  - lapped
  - mechanical connectors
Development Lengths

- $l_d$, embedment required both sides
- proper cover, spacing:
  - No. 6 or smaller
    \[ l_d = \frac{d_b F_y}{25 \sqrt{f'_c}} \] or 12 in. minimum
  - No. 7 or larger
    \[ l_d = \frac{d_b F_y}{20 \sqrt{f'_c}} \] or 12 in. minimum
Development Lengths

- hooks
  - bend and extension

\[ l_{dh} = \frac{1200d_b}{\sqrt{f'_c}} \]

Figure 9-17: Minimum requirements for 90° bar hooks.

Figure 9-18: Minimum requirements for 180° bar hooks.
Development Lengths

- **bars in compression**
  
  \[ l_d = \frac{0.02 d_b F_y}{\sqrt{f'_{c}}} \leq 0.0003 d_b F_y \]

- **splices**
  
  - tension minimum is function of \( l_d \) and splice classification
  - compression minimum
  - is function of \( d_b \) and \( F_y \)
Concrete Deflections

• elastic range
  – I transformed
  – \( E_c \) (with \( f'_c \) in psi)
    • normal weight concrete (~ 145 lb/ft\(^3\))
      \[
      E_c = 57,000 \sqrt{f'_c}
      \]
    • concrete between 90 and 160 lb/ft\(^3\)
      \[
      E_c = w_c^{1.5} 33 \sqrt{f'_c}
      \]
• cracked
  – I cracked
  – \( E \) adjusted
Deflection Limits

- relate to whether or not beam supports or is attached to a damageable non-structural element
- need to check service live load and long term deflection against these

<table>
<thead>
<tr>
<th>Deflection</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/180</td>
<td>roof systems (typical) – live</td>
</tr>
<tr>
<td>L/240</td>
<td>floor systems (typical) – live + long term</td>
</tr>
<tr>
<td>L/360</td>
<td>supporting plaster – live</td>
</tr>
<tr>
<td>L/480</td>
<td>supporting masonry – live + long term</td>
</tr>
</tbody>
</table>