beam sections - geometric properties
Center of Gravity

- location of equivalent weight
- determined with calculus

\[ \sum \Delta W = \int dW \]
Center of Gravity

• “average” x & y from moment

\[
\sum M_y = \sum_{i=1}^{n} x_i \Delta W_i = \bar{x}W \quad \Rightarrow \quad \bar{x} = \frac{\sum (x \Delta W)}{W}
\]

“bar” means average

\[
\sum M_x = \sum_{i=1}^{n} y_i \Delta W_i = \bar{y}W \quad \Rightarrow \quad \bar{y} = \frac{\sum (y \Delta W)}{W}
\]
Centroid

• “average” x & y of an area
• for a volume of constant thickness
  – $\Delta W = \gamma t \Delta A$ where $\gamma$ is weight/volume
  – center of gravity = centroid of area

$$\bar{x} = \frac{\sum(x\Delta A)}{A}$$

$$\bar{y} = \frac{\sum(y\Delta A)}{A}$$
Centroid

• for a line, sum up length

\[
\bar{x} = \frac{\sum(x \Delta L)}{L}
\]

\[
\bar{y} = \frac{\sum(y \Delta L)}{L}
\]
1\textsuperscript{st} Moment Area

- math concept
- the moment of an area about an axis

\[ Q_x = \bar{y}A \]
\[ Q_y = \bar{x}A \]
Symmetric Areas

- symmetric about an axis
- symmetric about a center point
- mirrored symmetry
Composite Areas

- *made up of basic shapes*
- *areas can be negative*
- *(centroids can be negative for any area)*
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate \( \hat{x} \) and \( \hat{y} \)
# Area Centroids

- **Table 7.1 – pg. 242**

## Centroids of Common Shapes of Areas and Lines

<table>
<thead>
<tr>
<th>Shape</th>
<th>$\bar{x}$</th>
<th>$\bar{y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular area</td>
<td>$\frac{b}{3}$</td>
<td>$\frac{h}{3}$</td>
</tr>
<tr>
<td>Quarter-circular area</td>
<td>$\frac{4r}{3\pi}$</td>
<td>$\frac{4r}{3\pi}$</td>
</tr>
<tr>
<td>Semicircular area</td>
<td>0</td>
<td>$\frac{4r}{3\pi}$</td>
</tr>
<tr>
<td>Semiparabolic area</td>
<td>$\frac{3u}{8}$</td>
<td>$\frac{3h}{5}$</td>
</tr>
<tr>
<td>Parabolic area</td>
<td>0</td>
<td>$\frac{3h}{5}$</td>
</tr>
</tbody>
</table>

- Right triangle only

---

**Sections 10**  
**Lecture 9**  
**Architectural Structures**  
**ARCH 331**  
**S2014abn**
Moments of Inertia

• 2\textsuperscript{nd} moment area
  – math concept
  – area \times (distance)^2

• need for behavior of
  – beams
  – columns
Moment of Inertia

- about any reference axis
- can be negative

\[ I_y = \int x^2 \, dA \]

\[ I_x = \int y^2 \, dA \]

- resistance to bending and buckling
Moment of Inertia

- same area moved away a distance
  - larger $I$
Polar Moment of Inertia

- for roundish shapes
- uses polar coordinates \((r \text{ and } \theta)\)
- resistance to twisting

\[ J_o = \int r^2 \, dA \]
Radius of Gyration

- measure of inertia with respect to area

$$r_x = \sqrt{\frac{I_x}{A}}$$
Parallel Axis Theorem

- can find composite $I$ once composite centroid is known (basic shapes)

\[ I_x = I_{cx} + Ad y^2 \]
\[ = \bar{I}_x + Ad y^2 \]

\[ I = \sum \bar{I} + \sum Ad^2 \]

\[ \bar{I} = I - Ad^2 \]
Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with $A$, $\bar{x}$, $\bar{x}A$, $\bar{y}$, $\bar{y}A$, $\bar{I}$'s, $d$'s, and $Ad^2$'s
5. Fill in table and get $\hat{x}$ and $\hat{y}$ for composite
6. Sum necessary columns
7. Sum $\bar{I}$'s and $Ad^2$'s

\[
\begin{align*}
    d_x & = \hat{x} - \bar{x} \\
    d_y & = \hat{y} - \bar{y}
\end{align*}
\]
Area Moments of Inertia

- Table 7.2 – pg. 252: (bars refer to centroid)
  - $x$, $y$
  - $x'$, $y'$
  - $C$

\[ I_x = \frac{1}{12}bh^3 \]
\[ I_y = \frac{1}{12}b'h'^3 \]
\[ I_x = \frac{1}{36}bh^3 \]
\[ I_y = \frac{1}{12}b^3h \]
\[ J_C = \frac{1}{12}bh(b^2 + h^2) \]
\[ J_x = \frac{1}{3} \pi r^4 \]
\[ J_y = \frac{1}{4} \pi r^4 \]