Beams: bending and shear stress

Beam Bending
- Galileo
  - relationship between stress and depth^2
- can see
  - top squishing
  - bottom stretching
- what are the stress across the section?

Pure Bending
- bending only
- no shear
- axial normal stresses from bending can be found in
  - homogeneous materials
  - plane of symmetry
  - follow Hooke’s law

Bending Moments
- sign convention:
- size of maximum internal moment will govern our design of the section
Normal Stresses

- geometric fit
  - plane sections remain plane
  - stress varies linearly

Neutral Axis

- stresses vary linearly
- zero stress occurs at the centroid
- neutral axis is line of centroids (n.a.)

Derivation of Stress from Strain

- pure bending = arc shape

\[ L = R \theta \]

\[ L_{\text{outside}} = (R + y)\theta \]

\[ \varepsilon = \frac{\delta}{L} = \frac{L_{\text{outside}} - L}{L} = \frac{(R + y)\theta - R\theta}{R\theta} = \frac{y}{R} \]

Derivation of Stress

- zero stress at n.a.

\[ f = E\varepsilon = \frac{Ey}{R} \]

\[ f_{\text{max}} = \frac{Ec}{R} \]

\[ f = \frac{y}{c} f_{\text{max}} \]
**Bending Moment**

- resultant moment from stresses = bending moment!

\[ M = \Sigma f_y \Delta A \]

\[ = \Sigma \frac{y f_{\text{max}}}{c} y \Delta A = \frac{f_{\text{max}}}{c} \Sigma y^2 \Delta A = \frac{f_{\text{max}}}{c} I = f_{\text{max}} S \]

**Bending Stress Relations**

\[ \frac{1}{R} = \frac{M}{EI} \]

\[ f_b = \frac{My}{I} \]

\[ S = \frac{I}{c} \]

- curvature
- general bending stress
- section modulus

- maximum bending stress
- required section modulus for design

**Transverse Loading and Shear**

- perpendicular loading
- internal shear
- along with bending moment

**Bending vs. Shear in Design**

- bending stresses dominate
- shear stresses exist horizontally with shear
- no shear stresses with pure bending
Shear Stresses

- horizontal & vertical

Beam Stresses

- horizontal with bending

Equilibrium

- horizontal force $V$ needed

$$V_{\text{longitudinal}} = \frac{V_T Q}{I} \Delta x$$

- $Q$ is a moment area
Moment of Area

- $Q$ is a moment area with respect to the n.a. of area above or below the horizontal

- $Q_{\text{max}}$ at $y=0$ (neutral axis)

- $q$ is shear flow: $$q = \frac{V_{\text{longitudinal}}}{\Delta x} = \frac{V}{I}$$

Shearing Stresses

- $f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x}$

- $f_{v-\text{ave}} = \frac{VQ}{Ib}$

- $f_{v-\text{ave}} = 0$ on the top/bottom
- $b_{\text{min}}$ may not be with $Q_{\text{max}}$
- with $h/4 \geq b$, $f_{v-\text{max}} \leq 1.008 f_{v-\text{ave}}$

Rectangular Sections

- $I = \frac{bh^3}{12}$
- $Q = A\bar{y} = bh^2/8$

- $f_v = \frac{VQ}{Ib} = \frac{3V}{2A}$

- $f_{v-\text{max}}$ occurs at n.a.

Steel Beam Webs

- $W$ and $S$ sections
  - $b$ varies
  - stress in flange negligible
  - presume constant stress in web

- $f_{v-\text{max}} = \frac{3V}{2A} \approx \frac{V}{A_{\text{web}}}$
Shear Flow

- loads applied in plane of symmetry
- cut made perpendicular

\[ q = \frac{VQ}{I} \]

Connectors Resisting Shear

- plates with
  - nails
  - rivets
  - bolts
- splices

\[ \frac{V_{\text{longitudinal}}}{p} = \frac{VQ}{I} \]

Vertical Connectors

- isolate an area with vertical interfaces

\[ n F_{\text{connector}} \geq \frac{VQ_{\text{connected area}}}{I} \cdot p \]
Unsymmetrical Shear or Section

• member can bend and twist
  – not symmetric
  – shear not in that plane

• shear center
  – moments balance