Rigid Frames

- rigid frames have no pins
- frame is all one body
- joints transfer moments and shear
- typically statically indeterminate
- types
  - portal
  - gable

Rigid Frames

- moments get redistributed
- deflections are smaller
- effective column lengths are shorter
- very sensitive to settling
Moment Redistribution

- continuous slabs & beams with uniform loading
  - joints similar to fixed ends, but can rotate
- change in moment to center = \( \frac{wL^2}{8} \)
  - \( M_{\text{max}} \) for simply supported beam

Rigid Frames

- resists lateral loadings
- shape depends on stiffness of beams and columns
- 90° maintained

Rigid Frames

- staggered truss
  - rigidity
  - clear stories

Rigid Frames

- connections
  - steel
  - concrete
Braced Frames

- pin connections
- bracing to prevent lateral movements

Braced Frames

- types of bracing
  - knee-bracing
  - diagonal
  - X
  - K or chevron
  - shear walls

Shear Walls

- resist lateral load in plane with wall

Compression Members

- designed for strength & stresses
- designed for serviceability & deflection
- need to design for stability
  - ability to support a specified load without sudden or unacceptable deformations
Column Buckling

- axially loaded columns
- long & slender
  - unstable equilibrium = buckling
  - sudden and not good

Modeling

- can be modeled with a spring at mid-height
- when moment from deflection exceeds the spring capacity ... “boing”
- critical load $P$

Effect of Length

- long & slender
- short & stubby

Buckling Load

- related to deflected shape ($P\Delta$)
- shape of sine wave
- Euler’s Formula
- smallest $I$ governs

$$P_{\text{critical}} = \frac{\pi^2 EI}{(L)^2}$$
Critical Stress

- short columns
  \[ f_{\text{critical}} = \frac{P_{\text{actual}}}{A} < F_a \]
- slenderness ratio = \( L_e/r \) (L/d)
- radius of gyration = \( r = \sqrt{\frac{I}{A}} \)

\[ f_{\text{critical}} = \frac{P_{\text{critical}}}{A} = \frac{\pi^2 EA r^2}{A (L_e/r)^2} = \frac{\pi^2 E}{(L_e/r)^2} \]
\[ P_{\text{critical}} = \frac{\pi^2 EA}{(L_e/r)^2} \]

Effective Length

- end conditions affect shape
- effective length factor, \( K \)
  \[ L_e = K \cdot L \]

Critical Stresses

- when a column gets stubby, \( F_y \) will limit the load
- real world has loads with eccentricity
- \( C_c \) for steel and allowable stress

\[ L_e/r > C_c = \sqrt{\frac{2\pi^2 E}{F_y}} \]

Bracing

- bracing affects shape of buckle in one direction
- both should be checked!
**Centric & Eccentric Loading**

- **centric**
  - allowable stress from strength or buckling

- **eccentric**
  - combined stresses

**Combined Stresses**

- axial + bending
  \[ f_{\text{max}} = \frac{P}{A} + \frac{Mc}{I} \]
  \[ M = P \cdot e \]

- design
  \[ f_{\text{max}} \leq F_{\text{cr}} = \frac{f_{\text{cr}}}{F.S.} \]

**Stress Limit Conditions**

- ASD interaction formula
  \[ \frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \]

- with biaxial bending
  \[ \frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \]

**Stress Limit Conditions**

- in reality, as the column flexes, the moment increases

  - \( P-\Delta \) effect
    \[ \frac{f_a}{F_a} + \frac{f_b \times (\text{Magnification factor})}{F_{bx}} \leq 1.0 \]
Rigid Frame Analysis

- members see
  - shear
  - axial force
  - bending
- V & M diagrams
  - plot on “outside”

Rigid Frame Analysis

- need support reactions
- free body diagram each member
- end reactions are equal and opposite on next member
- “turn” member like beam
- draw V & M

Rigid Frame Design

- loads and combinations
  - usually uniformly distributed gravity loads
  - worst case for largest moments...
  - wind direction can increase moments
Rigid Frame Design

• frames & floors
  – rigid frame can have slab floors or slab with connecting beams

• other
  – slabs or plates on columns

Rigid Frame Design

• floors – plates & slabs
  – one-way behavior
    • side ratio > 1.5
    • “strip” beam
  – two-way behavior
    • more complex

Rigid Frame Design

• columns in frames
  – ends can be “flexible”
  – stiffness affected by beams and column = $EI/L$

\[ G = \Psi = \frac{\sum EI}{l_c} \]

– for the joint
  • $l_c$ is the column length of each column
  • $l_b$ is the beam length of each beam
  • measured center to center
Tools – Multiframe

• in computer lab

- frame window
  - define frame members
    - or pre-defined frame
  - select points, assign supports
  - select members, assign section
  - load window
  - select point or member, add point or distributed loads

Tools – Multiframe

• to run analysis choose
  - Analyze menu
    - Linear
  - plot
    - choose options
  - results
    - choose options