Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for maximum stresses
  - limit state design
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity
  - concrete strength = $f'_c$

Concrete Construction

- cast-in-place
- tilt-up
- prestressing
- post-tensioning

Concrete Beams

- types
  - reinforced
  - precast
  - prestressed
- shapes
  - rectangular, I
  - T, double T's, bulb T's
  - box
  - spandrel
Concrete Beams

- shear
  - vertical
  - horizontal
  - combination:
    - tensile stresses at 45°
- bearing
  - crushing

Concrete

- hydration
  - chemical reaction
  - workability
  - water to cement ratio
  - mix design
- fire resistant
- cover for steel
- creep & shrinkage

Concrete

- low strength to weight ratio
- relatively inexpensive
  - Portland cement
    - types I - V
  - aggregate
    - course & fine
  - water
  - admixtures
    - air entraining
    - superplasticizers

Concrete

- placement (not pouring!)
- vibrating
- screeding
- floating
- troweling
- curing
- finishing
Reinforcement

• deformed steel bars (rebar)
  – Grade 40, $F_y = 40$ ksi
  – Grade 60, $F_y = 60$ ksi - most common
  – Grade 75, $F_y = 75$ ksi
  – US customary in # of 1/8” $\phi$ (nominal)
• longitudinally placed
  – bottom
  – top for compression reinforcement

Composite Beams

• concrete
  – in compression
• steel
  – in tension
• shear studs

Behavior of Composite Members

• plane sections remain plane
• stress distribution changes

$$f_1 = E_1 \varepsilon = - \frac{E_1 y}{\rho}$$
$$f_2 = E_2 \varepsilon = - \frac{E_2 y}{\rho}$$
Transformation of Material

- $n$ is the ratio of $E$'s \[ n = \frac{E_2}{E_1} \]
- effectively widens a material to get same stress distribution

Stresses in Composite Section

- with a section transformed to one material, new $I$
  \[ n = \frac{E_2}{E_1} = \frac{E_{\text{steel}}}{E_{\text{concrete}}} \]
  \[ f_c = -\frac{M_y}{I_{\text{transformed}}} \]
  \[ f_s = -\frac{M_y}{I_{\text{transformed}}} \]

Reinforced Concrete - stress/strain

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent stress block
  - concrete takes no tension
  - steel takes tension
  - force ductile failure
Location of n.a.

• ignore concrete below n.a.
• transform steel
• same area moments, solve for $x$

$$bx \cdot \frac{x}{2} - nA_s (d - x) = 0$$

T sections

• n.a. equation is different if n.a. below flange

$$b_f h_f \left( x - \frac{h_f}{2} \right) + (x - h_f) b_w \left( \frac{x - h_f}{2} \right) - nA_s (d - x) = 0$$

ACI Load Combinations*

• 1.4D
• 1.2D + 1.6L + 0.5(L_r or S or R)
• 1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)
• 1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)
• 1.2D + 1.0E + 1.0L + 0.2S
• 0.9D + 1.0W
• 0.9D + 1.0E

*can also use old ACI factors

Reinforced Concrete Design

• stress distribution in bending

Wang & Salmon, Chapter 3
Force Equations

- $C = 0.85 \ f'_c b a$
- $T = A_s f_y$
- where
  - $f'_c$ = concrete compressive strength
  - $a$ = height of stress block
  - $\beta_1$ = factor based on $f'_c$
  - $x$ = location to the n.a.
  - $b$ = width of stress block
  - $f_y$ = steel yield strength
  - $A_s$ = area of steel reinforcement

Equilibrium

- $T = C$
- $M_n = T(d-a/2)$
  - $d$ = depth to the steel n.a.
- with $A_s$
  - $a = \frac{A_s f_y}{0.85 f'_c b}$
  - $M_u \leq \phi M_n$, $\phi = 0.9$ for flexure*
  - $\phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2)$

Over and Under-reinforcement

- over-reinforced
  - steel won’t yield
- under-reinforced
  - steel will yield
- reinforcement ratio
  - $\rho = \frac{A_s}{bd}$
  - use as a design estimate to find $A_s, b, d$
  - max $\rho$ is found with $\varepsilon_{steel} \geq 0.004$ (not $\rho_{bal}$)
  - *with $\varepsilon_{steel} \geq 0.005$, $\phi = 0.9$

$A_s$ for a Given Section

- several methods
  - guess a and iterate
    1. guess a (less than n.a.)
    2. $A_s = \frac{0.85 f'_c b a}{f_y}$
    3. solve for a from $M_u = \phi A_s f_y (d-a/2)$
    4. repeat from 2. until a from 3. matches a in 2.
\( A_s \) for a Given Section (cont)

- chart method
  - Wang & Salmon Fig. 3.8.1  \( R_n \) vs. \( \rho \)
    1. calculate \( R_n = \frac{M_n}{bd^2} \)
    2. find curve for \( f'_c \) and \( f_y \) to get \( \rho \)
    3. calculate \( A_s \) and \( a \)
- simplify by setting \( h = 1.1d \)

Reinforcement

- min for crack control
- required
  \[
  A_s = \frac{3\sqrt{f'_c}}{f_y} (bd) 
  \]
- not less than
  \[
  A_i = \frac{200}{f_y} (bd) 
  \]
- \( A_{s_{-max}} : a = \beta_1 (0.375d) \)
- typical cover
  - 1.5 in, 3 in with soil
- bar spacing

Shells

- Annunciation Greek Orthodox Church
  - Wright, 1956

http://nisee.berkeley.edu/godden

http://www.bluffton.edu/~sullivanm/
Annunciation Greek Orthodox Church
• Wright, 1956

Cylindrical Shells
• can resist tension
• shape adds “depth”
• not vaults
• barrel shells

Kimball Museum, Kahn 1972
• outer shell edges
Kimball Museum, Kahn 1972

- skylights at peak

Approximate Depths

[Diagram of concrete beams with labeled depths and specifications]