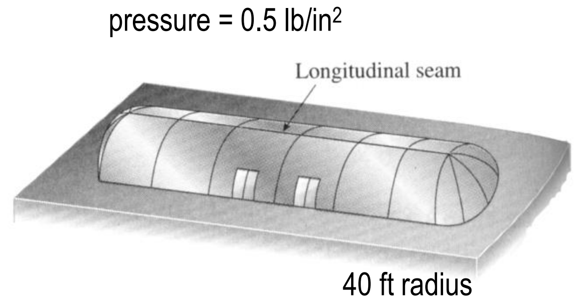
Examples:
Membranes, Nets & Shells

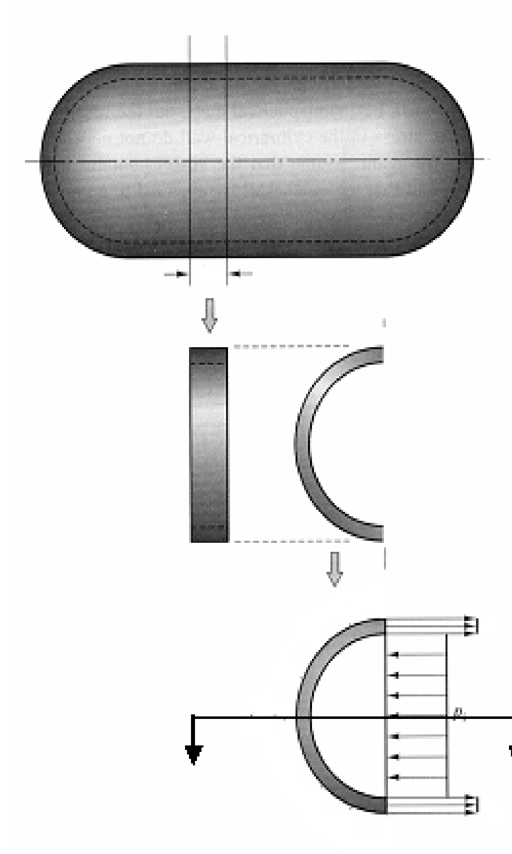
Example 1

An inflatable structure used by a traveling circus has the shape of a half-circular cylinder with closed ends. The fabric and plastic structure is inflated by a small blower and has a radius of 40 ft when fully inflated. A longitudinal seam runs the entire length of the "ridge" of the structure.



If the seam tears open when it is subjected to a tensile load of 540 pounds per inch of seam, what is the factor of safety against tearing when the internal pressure is 0.5 psi and the structure is fully inflated?

What is the force on the seam at the intersection with the quarter spheres? If the thickness of the membrane is 0.025 in, what is the stress?



SOLUTION:
Find the tensile load for a one inch section of the membrane structure. A free body diagram is helpful to show the pressure:

T for a circular membrane for a unit width carrying an internal pressure p_r is:
T = p_rR

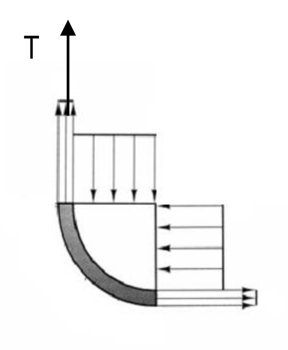
It doesn't matter where we cut a section, the force will still be T.

T = $0.5\, {}^{lb}\!/_{in^2}(40\,ft)\cdot(\frac{12in}{1ft})$ = 240 lb/in

The factor of safety is the ratio of limit load to actual load:

F.S. = $\frac{540\, {}^{lb}\!/_{in}}{240\, {}^{lb}\!/_{in}}$ = 2.25

The ends are spherical, so the equation for force is T= p_rR/2.
The force will be ½ (240 lb/in) = 120 lb/in



The stress is equal to the force per length divided by the thickness, f = T/t

f = T/t = (120 lb/in)/(0.025 in) = 4,800 lb/in^2

Example 2

Investigate with computer modeling the stresses and behavior of a hyperbolic paraboloid under uniform roof loading with column supports away from the edges, as actually built for a residence with glazing between columns. (Ref. Architectural Structures, Wayne Place, 2007, Wiley, NJ.)

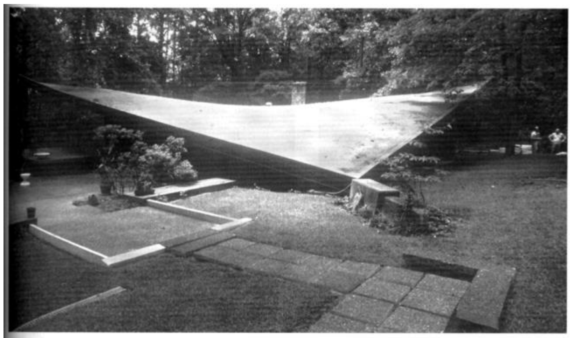


Figure 8.260 Residence with hyperbolic paraboloid roof, designed by architect Eduardo Catalano, in Raleigh, North Carolina.

Figure 8.261 Residence with hyperbolic paraboloid roof, showing the ample overhang of the roof, the boundary members, and the mullions in the glass walls.

SOLUTION:

The axial force diagram (b) shows that the axial forces appear to be uniform, as the discussion in the text indicates, but that the edge members have higher axial forces.

The deflection diagram (b) indicates negative bending over the columns, which indicates there are probably significant bending moments (which should be minimal in a shell), verified by the bending moment diagram (d).
This house had significant problems.

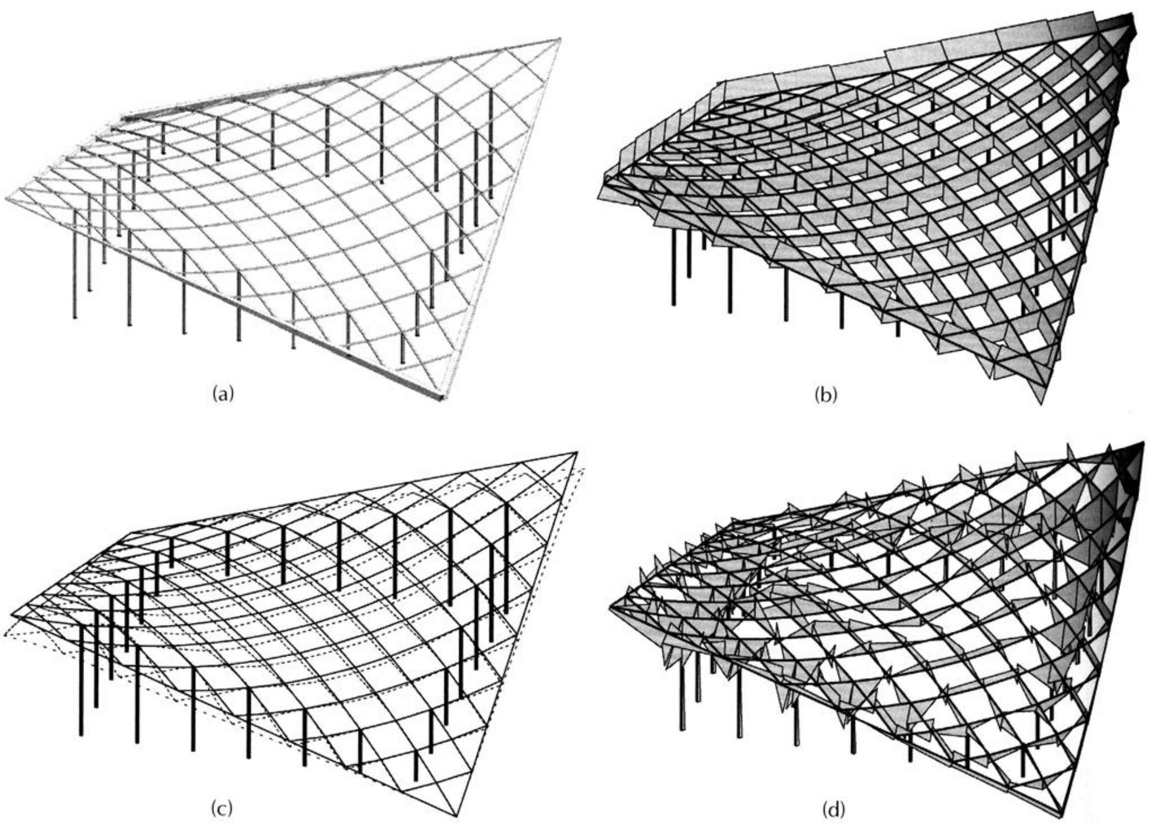


Figure 8.262 Hyperbolic paraboloid under uniform load, showing stabilizing columns (a), axial forces (b), deflection (c), and bending stress (d).

Example 3 (page 407)

EXAMPLE

Consider a dome having a spherical radius of 100 ft (30.48 m), a thickness of 4 in. (100 mm), and an aspect angle of 45°. Determine the meridional and hoop forces at the base of the shell for a loading of 100 lb/ft^2 (4788 N/m^2). The load includes all applicable loads.

Solution:

Meridional forces:

$$N_\phi = \frac{Rw}{1 + \cos\phi} = \frac{(100\text{ ft})(100\text{ lb/ft}^2)}{1 + 0.707} = 5858\text{ lb/ft in compression}$$
$$= \frac{(30.48\text{ m})(4788\text{ N/m}^2)}{1.707} = 85{,}494\text{ N/m}$$

Meridional stresses:

$$f_\phi = \frac{N_\phi}{t} = \frac{(5858\text{ lb/ft})/12\text{ in./ft}}{4\text{ in.}} = \frac{488\text{ lb/in.}}{4\text{ in.}} = 122\text{ lb/in.}^2\text{ in compression}$$
$$= \frac{85{,}494\text{ N/m}}{(1000\text{ mm/m})(100\text{ mm})} = 0.85\text{ N/mm}^2$$

Hoop forces:

$$N_\theta = Rw\left(-\frac{1}{1 + \cos\phi} + \cos\phi\right) = (100\text{ ft})\left(100\text{ lb/ft}^2\right)\left[-\left(\frac{1}{1.707}\right) + 0.707\right]$$
$$= 1212\text{ lb/ft in compression}$$
$$= (30.48\text{ m})(4788\text{ N/m}^2)\left[-\left(\frac{1}{1.707}\right) + 0.707\right] = 17{,}684\text{ N/m}$$

Hoop stresses:

$$f_\theta = \frac{N_\theta}{t} = \frac{1212\text{ lb/ft}}{(12\text{ in./ft})(4\text{ in.})} = 25.3\text{ lb/in.}^2\text{ in compression}$$
$$= \frac{17{,}684\text{ N/m}}{(1000\text{ mm/m})(100\text{ mm})} = 0.177\text{ N/mm}^2$$

The stresses in the sphere at the point in question are thus extremely low, a characteristic of most shell structures.