Examples:  
Timber

Example 1
Design a Flat Roof joist, 16 in. on center (o.c.), 18 ft span with Douglas fir-larch No. 2. Snow load is 30 psf. Dead load (including ballast, roofing, sheathing, joists & ceiling) = 18.9 psf. C_r = 1.15 for bending only.

F_b = 875 psi; F_v = 95 psi; E = 1.6 x 10^6 psi

Also design the glulam girder supporting the joists if it spans 35 ft (simply supported) and F_b = 2400 psi.

Assume the density of the glulam timber is 32 lb/ft^3.

SOLUTION:
The load case that is most likely to govern the design is Dead + Live. Because the live load is from snow, C_D = 1.15:

\[
\frac{18.9 \text{ psf}}{0.9} = 21 \text{ psf} < \frac{(18.9 \text{ psf} + 30 \text{ psf})}{1.15} = 42.5 \text{ psf}
\]

Joist
The distributed load for each joist needs to be found by multiplying the area load by the tributary width:

\[
w = (30 \text{ lb/ft}^2 + 18.9 \text{ lb/ft}^2)(16 \text{ in})(1 \text{ ft/12 in}) = 65.2 \text{ lb/ft}
\]

\[
M_{\text{max}} = \frac{wL^2}{8} = \frac{(65.2 \text{ lb/ft})(18 \text{ ft})^2}{8} = 2641 \text{ lb-ft}
\]

Allowable stress is the tabulated stress multiplied by all applicable adjustment factors, which would be C_D and C_r:

\[
F_b' = F_b C_D C_r = 875 \text{ lb/ft}^2 (1.15)(1.15) = 1157 \text{ lb/ft}^2
\]

\[
S_{\text{req'd}} \geq \frac{M}{F_b'} = \frac{2641 \text{ lb-ft}}{1157 \text{ lb/ft}^2} \left( \frac{12 \text{ in}}{\text{ft}} \right) = 27.4 \text{ in}^3
\]

Shear can quite often govern the design of timber beams:

\[
V_{\text{max}} = \frac{wL}{2} = \frac{(65.2 \text{ lb/ft})(18 \text{ ft})}{2} = 587 \text{ lb}
\]

Allowable stress is the tabulated stress multiplied by all applicable adjustment factors, which would be C_D only:

\[
F_v' = F_v C_D = 95 \text{ lb/ft}^2 (1.15) = 109 \text{ lb/ft}^2
\]

Shear stress in a rectangular beam is found from 3V/2A:

\[
A_{\text{req'd}} \geq \frac{3V}{2F_v'} = \frac{3 \times 587 \text{ lb}}{2 \times (109 \text{ lb/ft}^2)} = 8.1 \text{ in}^2
\]
Allowable deflection is not known, but $I_{req}$ could be determined from $\Delta = \frac{5wl^4}{384EI} \leq \Delta_{limit}$ then $I_{req} \geq \frac{5wl^4}{384E\Delta_{limit}}$

From the section property table, a 2 x 12 satisfies $A_{req}$ and $S_{req}$ (bending governs)

**Girder**

The distributed load on the girder is the reaction of each joist over the 16 inch spacing plus the self weight of the girder.

Guessing a self weight of 40 lb/ft ($\approx 32$ lb/ft$^2 \times 1$ ft$^3$):

$$w = \frac{V}{\text{spacing}} + \text{s.w.} = \frac{5871b}{16in} \cdot \frac{12in}{ft} + 40 \frac{lb}{ft} = 480 \frac{lb}{ft}$$

$$M_{max} = \frac{wI^2}{8} = \left(\frac{480 \frac{lb}{ft} \times (35 \text{ ft})^2}{8}\right) = 73,500 \frac{lb\cdot \text{in}}{ft}$$

Allowable stress is the tabulated stress multiplied by all applicable adjustment factors, which would be $C_F$. The charts provided say that $C_F$ has been included in the section modulus. If we didn't have a chart that included $C_F$ and we don't know the depth, we could guess - say 18 inches:

$$C_F = \left(\frac{12}{d}\right)^{\frac{1}{9}} = \left(\frac{12}{18}\right)^{\frac{1}{9}} = 0.956 \lt 1$$

$$S_{req} \geq \frac{M}{F_b} \geq \frac{73,500 \frac{lb\cdot \text{in}}{ft} \times (12 \text{ in}/ft )}{2400 \frac{lb}{in^2}} = 367.5 \frac{in^3}{ft}$$

No information is available to evaluate shear or deflection. Based on that, try a 5 1/8 x 22.5. It has a smaller area than the 8 3/4 section with a big enough adjusted S. (Real S = 5.125x22.5/6 = 432.42 in$^3$, $C_F = 0.932$, $S_{adjusted} = 403.2$ in$^3$)

Check self weight: $\gamma \cdot A = 32 \frac{lb}{ft} \cdot \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)^2 = \frac{26 \frac{lb}{ft}}{\text{in}}$ which is less than what was used.

We could try a smaller section, which would mean calculating a new self weight, then moment, then $S_{req}$ and comparing $S_{actual}$ to $S_{req}$.

The lower self weight means a lower design moment, but the smaller $C_F$ means a smaller allowed stress, so we might end up with the same section.

$$w_{revised} = 480 \text{ lb/ft} + (26 - 40 \text{ lb/ft}), \quad M_{revised} = 71,356 \text{ lb-ft}$$

$w_{req} = 356.8 \text{ in}^3$ and the 5 1/8 x 22.5 is the choice for bending.

Of course, we need to satisfy shear and deflection criteria as well.
Example 2

Example Problem 10.20:
Design of Wood Columns (Figure 10.66)

A 22'-tall glu-lam column is required to support a roof load (including snow) of 40 kips. Assuming $8\frac{3}{4}''$ in one dimension (to match the beam width above), determine the minimum column size if the top and bottom are pin supported.

Select from the following sizes:

- $8\frac{3}{4}'' 	imes 9''$ \( (A = 78.75 \text{ in.}^2) \)
- $8\frac{3}{4}'' 	imes 10\frac{1}{2}''$ \( (A = 91.88 \text{ in.}^2) \)
- $8\frac{3}{4}'' 	imes 12''$ \( (A = 105.00 \text{ in.}^2) \)

Solution:

Glu-lam column: \( (F_c = 1650 \text{ psi}, E = 1.8 \times 10^6 \text{ psi}) \)

Try $8\frac{3}{4}'' 	imes 10\frac{1}{2}''$ \( (A = 105.00 \text{ in.}^2) \)

\[
\frac{L_c}{d} = \frac{(22' \times 12 \text{ in./ft.})}{8.75 \text{ in.}}
\]

\[= 30.2 < 50 \text{ (max. slenderness ratio)}\]

\[
F_{cE} = \frac{0.418E}{(L_c/d)^2} = \frac{0.418(1.8 \times 10^6 \text{ lb./in.}^2)}{(30.2)^2} = 825 \text{ psi}
\]

\[
F_c' = F_cC_D = (1650 \text{ psi}) \times (1.15) = 1900 \text{ psi (snow)}
\]

\[
\frac{F_{cE}}{F_c'} = \frac{825}{1900} = 0.43
\]

From Appendix Table 14: \( C_p = 0.403 \)

\[
F_c' = F_{cE}C_p = (1900 \text{ lb./in.}^2) \times (0.403) = 765 \text{ psi}
\]

\[
P_a = F_c' \times A = (765 \text{ lb./in.}^2) \times (91.9 \text{ in.}^2) = 70,300 \text{ lb.} > 40,000 \text{ lb.}
\]

Cycle again, trying a smaller, more economical section. Try $8\frac{3}{4}'' 	imes 9''$ \( (A = 78.8 \text{ in.}^2) \)

Since the critical dimension is still $8\frac{3}{4}''$, the values for $F_{cE}, F_c'$, and $F_c''$ all remain the same as in trial 1. The only change that affects the capability of the column is the available cross-sectional area.

\[
\therefore P_a = F_c' \times A = (765 \text{ lb./in.}^2) \times (78.8 \text{ in.}^2) = 60,300 \text{ lb.}
\]

\[
P_a = 60.3 \text{ k} > 40 \text{ k}
\]

Use $8\frac{3}{4}'' 	imes 9''$ glu-lam section.
Example 3

**EXAMPLE 7.16 Combined Bending and Compression in a Stud Wall**

Check the 2 × 6 stud in the first-floor bearing wall in the building shown in Fig. 7.20a. Consider the given vertical loads and lateral forces. Lumber is No. 2 DF-L. MC ≤ 19 percent and normal temperatures apply. Allowable stresses are to be in accordance with the NDS. $F_b = 2152$ psi $F_c = 1350$ psi

**COLUMN CAPACITY:**

Sheathing provides lateral support about the weak axis of the stud. Therefore, check column buckling about the x axis only ($L = 10.5$ ft and $d_x = 5.5$ in.):

$$\frac{l_x}{d} = 0 \quad \text{because of sheathing}$$

$$\left(\frac{l_y}{d}\right)_{\text{max}} = \frac{10.5 \text{ ft} \times 12 \text{ in./ft}}{5.5 \text{ in.}} = 22.9$$

$$E = 1,600,000 \text{ psi}$$

For visually graded sawn lumber:

$$K_{ex} = 0.3$$

$$c = 0.8$$

$$F_{eb} = \frac{K_{ex}E'}{(l/d)^2} = \frac{0.3 \times 1600000}{(22.9)^2} = 915 \text{ psi}$$

$$F_c' = F_c(C_D) \quad C_D = 1.6 \text{ from wind loading}$$

$$= 1350(1.6) = 2376 \text{ psi}$$

$$\frac{F_{eb}}{F_c'} = \frac{915}{2376} = 0.385 \quad C_r = 0.35$$

$$F_e' = F_c(C_D)(C_P) = 2376(0.35) = 832 \text{ psi}$$

**Load Case 2: Gravity Loads + Lateral Forces**

**BENDING:**

Wind governs over seismic. Force to one stud:

Wind = 27.8 psf

$$w = 27.8 \text{ psf} \times \frac{16\text{ in}}{12\text{ in./ft}} = 37.0 \text{ lb/ft}$$

$$M = \frac{wL^2}{8} = \frac{37.0(10.5)^2}{8} = 510 \text{ ft·lb} = 6115 \text{ in·lb}$$

**AXIAL:**

$$f_b = \frac{M}{S} = \frac{6115}{7.56} = 809 \text{ psi} \quad F_e' = 2152 \text{ psi}$$

D + W: $f_e = \frac{P}{A} = \frac{378}{8.25} = 46 \text{ psi}$

**COMBINED STRESS:**

The simplified interaction formula from Example 7.13 (Sec. 7.12) applies:

$$\left(\frac{f_e}{F_e'}\right)^2 + \frac{f_{ex}}{F_{ex}(1 - f_e/F_{ex})} \leq 1.0$$

$$F_{ex} = F_{eb} = 915 \text{ psi}$$

**D + W:**

In this load combination, D produces the axial stress $f_e$ and W results in the bending stress $f_{ex}$.

$$\left(\frac{f_e}{F_e'}\right)^2 + \left(\frac{1}{1 - f_e/F_{ex}}\right)\frac{f_{ex}}{F_{ex}} = \left(\frac{46}{832}\right)^2 + \left(\frac{1}{1 - 46/915}\right)\frac{809}{2152} = 0.399 < 1.0$$

2 × 6 No. 2 DF-L exterior bearing wall OK