Masonry Design

Notation:

- \( A \) = name for area
- \( A_n \) = net area, equal to the gross area subtracting any reinforcement
- \( A_{nv} \) = net shear area of masonry
- \( A_s \) = area of steel reinforcement in masonry design
- \( A_{st} \) = area of steel reinforcement in masonry column design
- \( A_v \) = area of concrete shear stirrup reinforcement
- ACI = American Concrete Institute
- ASCE = American Society of Civil Engineers
- \( b \) = width, often cross-sectional = total width of material at a horizontal section
- \( C_m \) = compression force in the masonry for masonry design
- CMU = shorthand for concrete masonry unit
- \( d \) = effective depth from the top of a reinforced masonry beam to the centroid of the tensile steel
- \( D \) = shorthand for dead load
- \( e \) = eccentric distance of application of a force (\( P \)) from the centroid of a cross section
- \( E \) = shorthand for earthquake load
- \( E_m \) = modulus of elasticity of masonry
- \( E_s \) = modulus of elasticity of steel
- \( f_a \) = axial stress
- \( f_b \) = bending stress
- \( f_m \) = calculated compressive stress in masonry
- \( f_m' \) = masonry design compressive stress
- \( f_s \) = stress in the steel reinforcement for masonry design
- \( f_v \) = shear stress
- \( F_a \) = allowable axial stress
- \( F_b \) = allowable bending stress
- \( F_s \) = allowable tensile stress in reinforcement for masonry design
- \( F_t \) = allowable tensile stress
- \( F_v \) = allowable shear stress
- \( F_{vm} \) = allowable shear stress of the masonry
- \( F_{vs} \) = allowable shear stress of the shear reinforcement
- \( h \) = name for height = effective height of a wall or column
- \( I_n \) = moment of inertia of the net section
- \( j \) = multiplier by effective depth of masonry section for moment arm, \( j_d \)
- \( k \) = multiplier by effective depth of masonry section for neutral axis, \( k_d \)
- \( K \) = type of masonry mortar
- \( L \) = shorthand for live load
- \( M \) = internal bending moment = type of masonry mortar
- \( M_m \) = moment capacity of a reinforced masonry beam governed by steel stress
- \( M_s \) = moment capacity of a reinforced masonry beam governed by masonry stress
- MSJC = Masonry Structural Joint Council
- \( n \) = modulus of elasticity transformation coefficient for steel to masonry
- n.a. = shorthand for neutral axis (N.A.)
- \( N \) = type of masonry mortar
- NCMA = National Concrete Masonry Association
- \( P \) = name for axial force vector
- \( P_a \) = allowable axial load in columns
- \( P_e \) = critical (Euler) buckling load
- \( Q \) = first area moment about a neutral axis
- \( r \) = radius of gyration
- \( s \) = spacing of stirrups in reinforced masonry
- \( S \) = type of masonry mortar = section modulus
- \( t \) = name for thickness
- \( T_s \) = tension force in the steel reinforcement for masonry design
- TMS = The Masonry Society
- \( V \) = internal shear force
- \( W \) = shorthand for wind load
\[ \beta_i = \text{coefficient for determining stress block height, } c, \text{ in masonry LRFD design} \]
\[ \rho = \text{reinforcement ratio in masonry design} \]
\[ \rho_b = \text{balanced reinforcement ratio in masonry design} \]
\[ \epsilon_m = \text{strain in the masonry} \]
\[ \epsilon_s = \text{strain in the steel} \]
\[ \Sigma = \text{summation symbol} \]

**Masonry Design**

Structural design standards for reinforced masonry are established by the *Masonry Standards Joint Committee* consisting of ACI, ASCE and The Masonry Society (TMS), and presents allowable stress design as well as limit state (strength) design.

**Materials**

Masonry mortars are mixtures of water, masonry cement, lime, and sand. The strengths are categorized by letter designations (from MaSoNwOrK).

<table>
<thead>
<tr>
<th>Designation</th>
<th>strength range</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>2500 psi</td>
</tr>
<tr>
<td>S</td>
<td>1800 psi</td>
</tr>
<tr>
<td>N</td>
<td>750 psi</td>
</tr>
<tr>
<td>O</td>
<td>350 psi</td>
</tr>
<tr>
<td>K</td>
<td>75 psi</td>
</tr>
</tbody>
</table>

\[ f_m = \text{masonry prism test compressive strength} \]

Deformed reinforcing bars come in grades 40, 50 & 60 (for 40 ksi, 50 ksi and 60 ksi yield strengths). Sizes are given nominally as \# of 1/8”.

Clay and concrete masonry units are porous, and their durability with respect to weathering is an important consideration. The amount of water in the mortar is important as well as the absorption capacity of the units for good bond; both for strength and for weatherproofing. Because of the moisture and tendency for shrinkage and swelling, it is critical to provide control joints for expansion and contraction.

**Masonry Walls**

Masonry walls can be reinforced or unreinforced, grouted or ungrouted, single wythe or cavity, prestressed or not. Cavity walls will require ties to force the two walls separated by the cavity to act as one.

From centuries of practice, the height to thickness ratio is limited because of slenderness (h/t < 25 or 35 depending on code). Most walls will see bending from wind or eccentricity along with bearing (combined stresses).
Allowable Stresses

- If tension stresses result, the allowable tensile strength for unreinforced walls must not be exceeded. These are relatively low (40 – 70 psi) and are shown in Table 2.2.3.2.
- If compression stresses result, the allowable strength (in bending) for unreinforced masonry \( F_b = \frac{1}{3} f_m' \)
- If compression stresses result, the allowable strength (in bending) for reinforced masonry \( F_b = 0.45 f_m' \)
- Shear stress in unreinforced masonry cannot exceed \( F_v = 1.5\sqrt{f_m'} \leq 120 \text{ psi} \).
- Shear stress in reinforced masonry for \( M/(Vd) \leq 0.25 \) cannot exceed \( F_v = 3.0 \sqrt{f_m'} \).
- Shear stress in reinforced masonry for \( M/(Vd) \geq 1.0 \) cannot exceed \( F_v = 2.0 \sqrt{f_m'} \).
- Allowable tensile stress, \( F_s \), in grades 40 & 50 steel is 20 ksi, grade 60 is 32 ksi, and wire joint reinforcement is 30 ksi.

where \( f_m' \) = specified compressive strength of masonry

<table>
<thead>
<tr>
<th>Direction of flexural tensile stress and masonry type</th>
<th>Mortar types</th>
<th>Mortar cement or air entrained portland cement/lime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M or S</td>
<td>N</td>
</tr>
<tr>
<td>Normal to bed joints</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid units</td>
<td>53 (366)</td>
<td>40 (276)</td>
</tr>
<tr>
<td>Hollow units(^1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ungrooved</td>
<td>33 (228)</td>
<td>25 (172)</td>
</tr>
<tr>
<td>Fully grouted</td>
<td>86 (593)</td>
<td>84 (579)</td>
</tr>
<tr>
<td>Parallel to bed joints in running bond</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid units</td>
<td>106 (731)</td>
<td>80 (552)</td>
</tr>
<tr>
<td>Hollow units</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ungrooved and partially grouted</td>
<td>66 (455)</td>
<td>50 (345)</td>
</tr>
<tr>
<td>Fully grouted</td>
<td>106 (731)</td>
<td>80 (552)</td>
</tr>
<tr>
<td>Parallel to bed joints in masonry not laid in running bond</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continuous grout section parallel to bed joints</td>
<td>133 (917)</td>
<td>133 (917)</td>
</tr>
<tr>
<td>Other</td>
<td>0 (0)</td>
<td>0 (0)</td>
</tr>
</tbody>
</table>

\(^1\) For partially grouted masonry, allowable stresses shall be determined on the basis of linear interpolation between fully grouted hollow units and ungrouted hollow units based on amount (percentage) of grouting.

Loads on Lintels in Masonry Walls

*Arching action* is present in masonry walls when there is an opening and sufficient wall width on either side of the opening to resist the arch thrust. A lintel is required to support the weight of the wall material above the opening. When arching action is present, the weight that must be supported can be determined from a 45 degree angle. This area may be a triangle, or trapezoid if the wall height above the lintel is less than half the opening width. The distributed load is calculated as height x wall thickness x specific weight of the masonry.
When there are concentrated loads on the wall, the load can be distributed to a width at the lintel height based on a 60 degree angle.

**Reinforced Masonry Members**

For stress analysis in masonry flexural members
- the strain is linear
- the compressive stress in the masonry is linear
- the tensile stress in the steel is not at yield
- any masonry in tension is assumed to have no strength
- the steel can be in tension, and is placed in the bottom of a beam that has positive bending moment

**Load Combinations**

\[
\begin{align*}
D \\
D+L \\
D + 0.75(L_r \text{ or } S \text{ or } R) \\
D + 0.75L + 0.75(L_r \text{ or } S \text{ or } R) \\
D + (0.6W \text{ or } 0.7E) \\
D + 0.75L + 0.75(0.6W) + 0.75(L_r \text{ or } S \text{ or } R) \\
D + 0.75L + 0.75(0.6W) + 0.75(L_r \text{ or } S \text{ or } R) \\
0.6D + 0.6W \\
0.6D + 0.7E
\end{align*}
\]

**Internal Equilibrium**

\[
C_m = \text{compression in masonry} = \text{stress x area} = f_m \frac{b(kd)}{2}
\]

\[
T_s = \text{tension in steel} = \text{stress x area} = A_s f_s
\]

\[
C_m = T_s \quad \text{and} \quad M_m = T_s(d-kd/3) = T_s(jd) \quad \text{and} \quad M_s = C_m(jd)
\]

\[
\rho = \frac{A_s f_s}{bd} \quad \Sigma F=0: \quad A_s f_s = f_m b \frac{kd}{2}
\]
where

\( f_m \) = stress in mortar at extreme fiber
\( k_d \) = height to neutral axis
\( b \) = width of section
\( f_s \) = stress in steel at d
\( A_s \) = area of steel reinforcement
\( d \) = depth to n.a. of reinforcement
\( j = (1 - k/3) \)

For flexure design:

\[ M \leq M_m \text{ or } M_s \]

so,

\[ M_m = T(jd) = 0.5f_mb^2jd \text{ and } M_s = \rho bd^2jfs \]

The design is adequate when \( f_b \leq F_b \) in the masonry and \( f_s \leq F_s \) in the steel.

**Shear Strength**

Shear stress is determined by \( f_v = V/A_{nv} \) where \( A_{nv} \) is net shear area. Shear strength is determined from the shear capacity of the masonry and the stirrups: \( F_v = F_{vm} + F_{vs} \). Stirrup spacings are limited to \( d/2 \) but not to exceed 48 in.

\[
F_{vm} = \frac{1}{2} \left[ 4.0 - 1.75 \left( \frac{M}{Vd} \right) \right] \sqrt{f_m} + 0.25 \frac{P}{A_n} \text{ where } M/(Vd) \text{ is positive and cannot exceed 1.0}
\]

\[
F_{vs} = 0.5 \left( \frac{A_s A_{nv}}{A_{nv}} \right) \frac{d}{f_s} \text{ (when } M/(Vd) \geq 0.25 )
\]

\[
F_{v} = 2.0 \sqrt{f_m} \text{ (when } M/(Vd) \geq 1.0) \) Values can be linearly interpolated.

**Table B.2 BALANCED SECTION PROPERTIES FOR RECTANGULAR MASONRY SECTIONS WITH TENSION REINFORCEMENT**

<table>
<thead>
<tr>
<th>Reinforcement</th>
<th>f_m (psi)</th>
<th>n = E_s/E_m</th>
<th>F_s = f_s/3 (psi)</th>
<th>Balanced Section Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>k,j,K,p = A_s/bd</td>
</tr>
<tr>
<td>Grades 40</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1350</td>
<td>22</td>
<td>450</td>
<td>0.333</td>
</tr>
<tr>
<td></td>
<td>1500</td>
<td>20</td>
<td>500</td>
<td>0.333</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>15</td>
<td>667</td>
<td>0.333</td>
</tr>
<tr>
<td></td>
<td>4000</td>
<td>7.5</td>
<td>1333</td>
<td>0.333</td>
</tr>
<tr>
<td>Grades 60</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1350</td>
<td>22</td>
<td>450</td>
<td>0.273</td>
</tr>
<tr>
<td></td>
<td>1500</td>
<td>20</td>
<td>500</td>
<td>0.273</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>15</td>
<td>667</td>
<td>0.273</td>
</tr>
<tr>
<td></td>
<td>4000</td>
<td>7.5</td>
<td>1333</td>
<td>0.273</td>
</tr>
</tbody>
</table>

**Reinforcement Ratio**

The amount of steel reinforcement is limited. Too much reinforcement, or over-reinforced will not allow the steel to yield before the concrete crushes and there is a sudden failure. A beam with the proper amount of steel to allow it to yield at failure is said to be under-reinforced.

The reinforcement ratio is a fraction: \( \rho = \frac{A_s}{bd} \) and must be less than \( \rho_b \) where the balanced reinforcement ratio is a function of steel strength and masonry strength.
**Flexure Design of Reinforcement**

One method is to choose a reinforcement ratio, find steel area, check stresses and moment:

1. find $\rho_b$ and assume a value of $\rho < \rho_b$
2. find $k, j$ and calculate $b\sigma^2 = \frac{M}{\rho j F_s}$ where $F_s$ is allowed stress in steel.
   
   Choose nice $b$ & $d$ values.
3. find $A_s = \frac{M}{F_s j d}$
4. check design for $M < M_s = A_s F_s (jd)$
5. check masonry flexural stress against allowable: $f_m = \frac{M}{0.5 b \sigma^2 jk} < F_b$

**Load and Resistance Factor Design**

The design methodology is similar to reinforced concrete ultimate strength design. It is useful with high shear values and for seismic design. The limiting masonry strength is $0.80f_m$.

**Force-Moment Interaction**

Combined stresses and the reduction of axial load with moment is similar to that for reinforced concrete column design as shown in the interaction diagram:

Reinforcement is typically placed in the center of walls. Grouting is placed in hollows with reinforcing, while other hollows may be empty. Stirrups are avoided.

Biaxial bending can occur in columns and stresses must satisfy:

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1$$

When maximum moment occurs somewhere other than at the end of the column or wall, a “virtual” eccentricity can be determined from $e = M/P$. 

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**Masonry Columns**

Columns are classified as having b/t < 3 and h/t > 4. Slender columns have a minimum side dimension of 8” and must have h/t ≤ 25. They must be designed with an eccentricity of 10% of the side dimension, and satisfy the interaction relationship of \( \frac{f_a}{F_a} + \frac{f_b}{F_b} ≤ 1 \), the tensile stress cannot exceed the allowable: \( f_a - f_a ≤ F_i \) and the compressive stress exceed allowable for reinforced masonry: \( f_a + f_b ≤ F_b \) provided \( f_a ≤ F_a \).

For purely axial loading, the capacity \( P_a \) depends on the slenderness ratio of h/r:

- **unreinforced**
  \[
P_a = \left[ 0.25 f'_m A_n \left( 1 - \left( \frac{h}{140r} \right)^2 \right) \right] \quad \text{for h/r} ≤ 99
  \]
  \[
P_a = \left[ 0.25 f'_m A_n \left( \frac{70r}{h} \right)^2 \right] \quad \text{for h/r} > 99
  \]

- **reinforced**
  \[
P_a = \left[ 0.25 f'_m A_n + 0.65 A_{st} F_s \left( 1 - \left( \frac{h}{140r} \right)^2 \right) \right] \quad \text{for h/r} ≤ 99
  \]
  \[
P_a = \left[ 0.25 f'_m A_n + 0.65 A_{st} F_s \left( \frac{70r}{h} \right)^2 \right] \quad \text{for h/r} > 99
  \]

where

- \( h \) = effective length
- \( r \) = least radius of gyration
- \( A_n \) = net area of masonry
- \( A_{st} \) = area of steel reinforcement
- \( f'_m \) = specified masonry compressive strength
- \( F_s \) = allowed compressive strength of reinforcement

The least radius of gyration can be found with \( \sqrt[3]{\frac{I}{A}} \) for a rectangle with side dimensions of b & d as:

\[
r = \sqrt[3]{\frac{db^3}{12bd}} = \sqrt[3]{\frac{b^4}{12}} = \frac{b}{\sqrt[3]{12}}
\]

where \( b \) is the smaller of the two side dimensions.