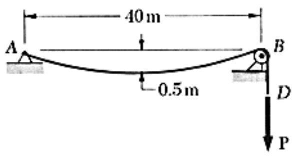
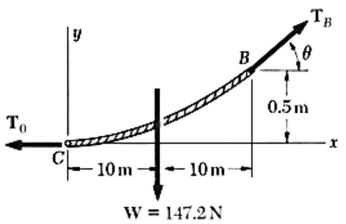
Examples:
Cables and Arches

Example 1



SAMPLE PROBLEM 7.9

A light cable is attached to a support at *A*, passes over a small pulley at *B*, and supports a load **P**. Knowing that the sag of the cable is 0.5 m and that the mass per unit length of the cable is 0.75 kg/m, determine (*a*) the magnitude of the load **P**, (*b*) the slope of the cable at *B*, and (*c*) the total length of the cable from *A* to *B*. Since the ratio of the sag to the span is small, assume the cable to be parabolic. Also, neglect the weight of the portion of cable from *B* to *D*.



***a.* Load P.** We denote by *C* the lowest point of the cable and draw the free-body diagram of the portion *CB* of cable. Assuming the load to be uniformly distributed along the horizontal, we write

$$w = (0.75\ \text{kg/m})(9.81\ \text{m/s}^2) = 7.36\ \text{N/m}$$

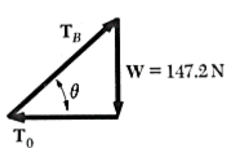
The total load for the portion *CB* of the cable is

$$W = wx_B = (7.36\ \text{N/m})(20\ \text{m}) = 147.2\ \text{N}$$

and is applied halfway between *C* and *B*.

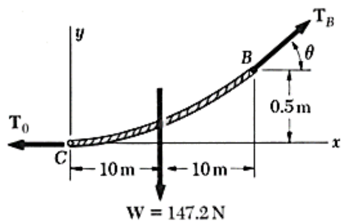
Summing moments about *B*, we write

$$+\circlearrowleft \Sigma M_B = 0: \quad (147.2\ \text{N})(10\ \text{m}) - T_0(0.5\ \text{m}) = 0 \qquad T_0 = 2944\ \text{N}$$



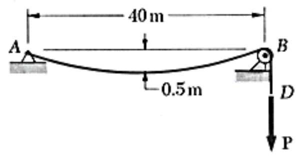
From the force triangle we obtain

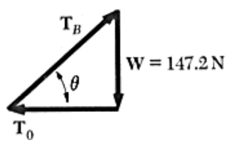
$$T_B = \sqrt{T_0^2 + W^2}$$
$$= \sqrt{(2944\ \text{N})^2 + (147.2\ \text{N})^2} = 2948\ \text{N}$$



Since the tension on each side of the pulley is the same, we find

$$P = T_B = 2948 \text{ N}$$

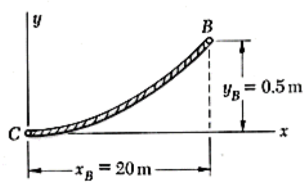




b. **Slope of Cable at *B*.** We also obtain from the force triangle

$$\tan\theta = \frac{W}{T_0} = \frac{147.2 \text{ N}}{2944 \text{ N}} = 0.05$$

$$\theta = 2.9°$$



c. **Length of Cable.** Applying Eq. (7.10) between *C* and *B*, we write

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 + \cdots\right]$$

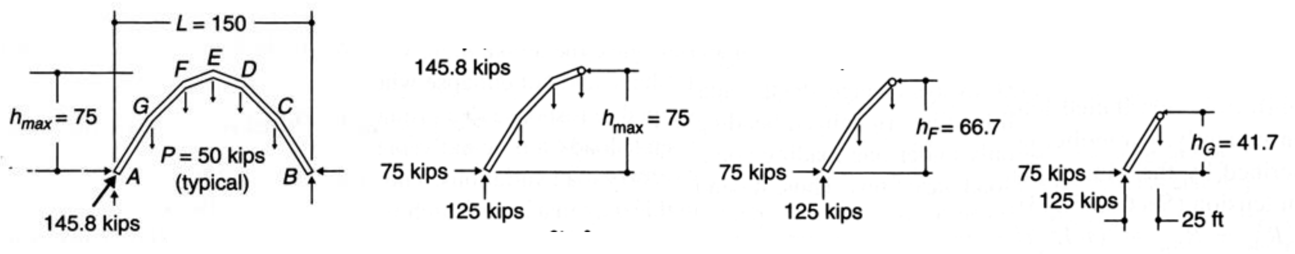
$$= (20 \text{ m})\left[1 + \frac{2}{3}\left(\frac{0.5 \text{ m}}{20 \text{ m}}\right)^2 + \cdots\right] = 20.00833 \text{ m}$$

The total length of the cable between *A* and *B* is twice this value,

$$\text{Length} = 2s_B = 40.0167 \text{ m}$$

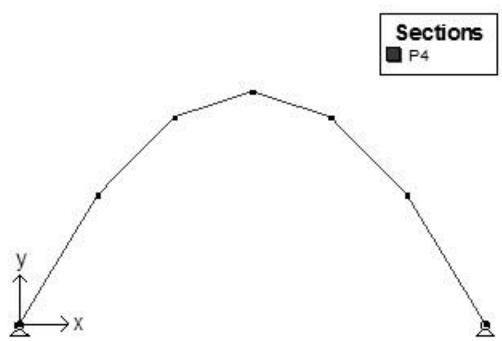
Example 2 (pg. 193, figure 5.23(b))

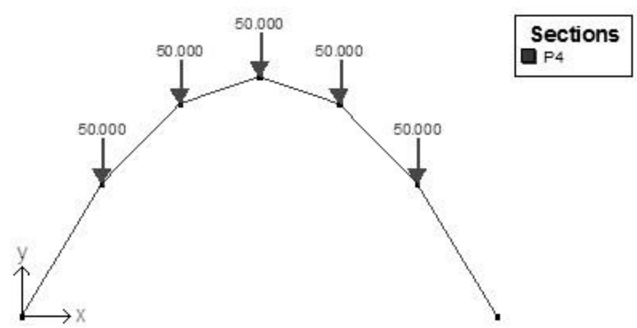
Using Multiframe, verify the axial force, shear and bending moment for the funicular shape with P = 50,000 lb, L = 150 ft, and h_{max} = 75 ft.



Joint Coordinates

Joint	Label	x ft	y ft	z ft	Type
1		0.000	0.000	0.000	Rigid
2		25.000	41.700	0.000	Rigid
3		50.000	66.700	0.000	Rigid
4		75.000	75.000	0.000	Rigid
5		100.000	66.700	0.000	Rigid
6		125.000	41.700	0.000	Rigid
7		150.000	0.000	0.000	Rigid



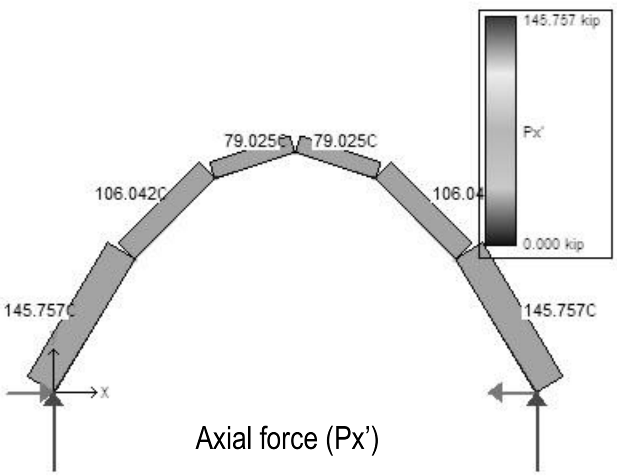


Joint Reactions

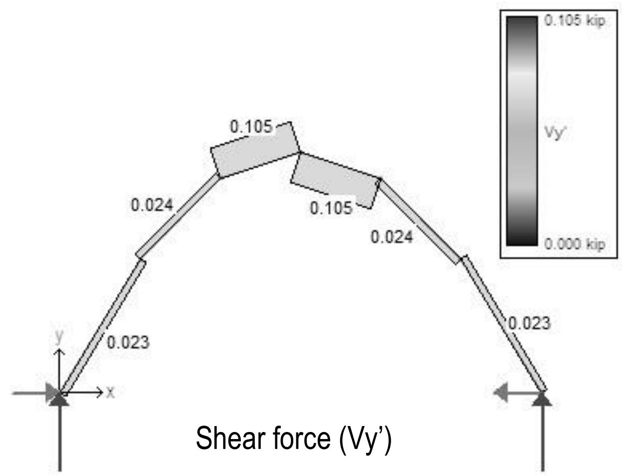
Joint	Label	Load Case	Rx' kip	Ry' kip	Rz' kip	Mx' kip-ft	My' kip-ft	Mz' kip-ft
1		Load Case 1	74.967	125.000	0.000	0.000	0.000	-0.000
2		Load Case 1	-0.000	-0.000	0.000	0.000	0.000	-0.000
3		Load Case 1	0.000	0.000	0.000	0.000	0.000	-0.000
4		Load Case 1	-0.000	0.000	0.000	0.000	0.000	0.000
5		Load Case 1	0.000	-0.000	0.000	0.000	0.000	0.000
6		Load Case 1	-0.000	0.000	0.000	0.000	0.000	-0.000
7		Load Case 1	-74.967	125.000	0.000	0.000	0.000	-0.000

Member Actions

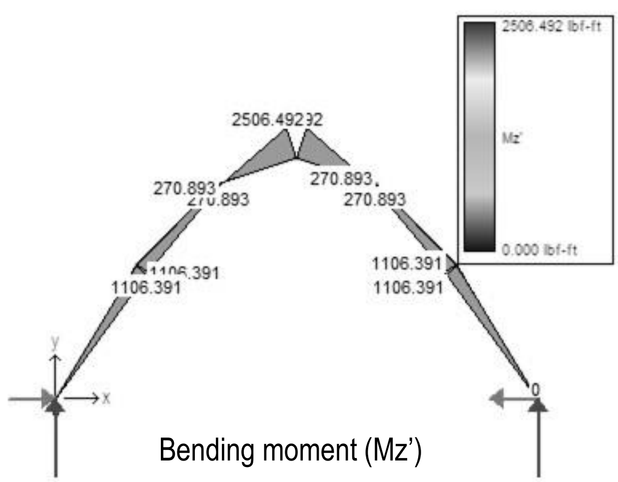
Member	Load Case	Px' kip	Vy' kip	Vz' kip	Tx' kip-ft	My' kip-ft	Mz' kip-ft
1	Load Case 1	145.757	-0.023	0.000	0.000	0.000	-0.000
		-145.757	0.023	0.000	0.000	0.000	-1.106
2	Load Case 1	106.042	0.024	0.000	0.000	0.000	1.106
		-106.042	-0.024	0.000	0.000	0.000	-0.271
3	Load Case 1	79.025	0.105	0.000	0.000	0.000	0.271
		-79.025	-0.105	0.000	0.000	0.000	2.506
4	Load Case 1	79.025	-0.105	0.000	0.000	0.000	-2.506
		-79.025	0.105	0.000	0.000	0.000	-0.271
5	Load Case 1	106.042	-0.024	0.000	0.000	0.000	0.271
		-106.042	0.024	0.000	0.000	0.000	-1.106
6	Load Case 1	145.757	0.023	0.000	0.000	0.000	1.106
		-145.757	-0.023	0.000	0.000	0.000	-0.000



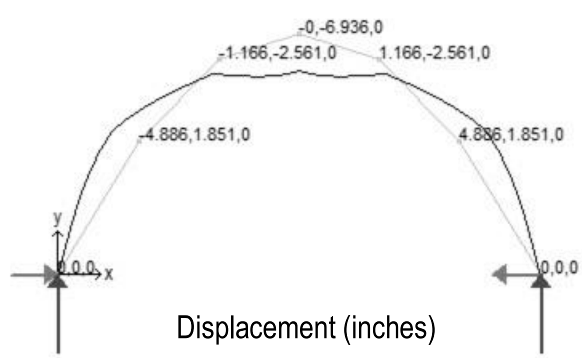
Axial force (Px')



Shear force (Vy')



Bending moment (Mz')



Displacement (inches)