

**Examples:
Rigid Frames**

Example 1 From eStructures v1.1, Schodek and Pollalis, 2000 Harvard College

Lateral Loading STEP 1

**RIGID FRAME STRUCTURES: LATERAL LOADING
PINNED BASE CONNECTIONS**

Determine axial forces, shear forces, and bending moments in each member of the rigid frame shown.

Lateral Loading STEP 2

RIGID FRAME STRUCTURES

DETERMINE REACTIONS

*Assumed directions of reactions:
Horizontal components balance applied force
Vertical components act as shown to prevent overturning*

$$\Sigma M_A = 0$$

$$+ 2000(12) - R_{C_y}(30) = 0$$

$$R_{C_y} = 800 \downarrow$$

$$\Sigma F_y = 0$$

$$+ R_{A_y} - 800 = 0 \quad \text{or} \quad R_{A_y} = 800 \uparrow$$

$$\Sigma F_x = 0$$

$$R_{A_x} + R_{C_x} = 2000$$

This last equation cannot be solved by statics alone. The structure is actually statically indeterminate. As shown on the following slides, an approximate method of analysis can be used to find the unknown reactions.

Example 1 (continued)

Lateral Loading STEP 3

RIGID FRAME STRUCTURES

DRAW DEFLECTED SHAPE OF STRUCTURE

Each of the top rigid joints translates and rotates as a unit.

A "point of inflection" naturally develops at the midspan of the horizontal member. This is a point of reverse curvature in the member, and hence is a point of zero moment.

If use is made of the point of inflection as a point of known zero moment, the structure is now statically determinate and can be analyzed much like a three-hinged arch.

Lateral Loading STEP 4

RIGID FRAME STRUCTURES

ANALYZE RIGHT PART

The forces shown at B are internal to the structure

Example 1 (continued)

Lateral Loading STEP 5

RIGID FRAME STRUCTURES

Point of Inflection
P = 2000 lb
12 ft
30 ft
A B C
 R_{Ax} R_{Ay} R_{Cx} R_{Cy}

ANALYZE RIGHT PART

15ft P = 2000 lb
12 ft
 R_{Bx} R_{By}
 R_{Cx} $R_{Cy} = 800$

FOR RIGHT PART:

$$\Sigma M_B = 0$$

$$-R_{Cy}(15) + R_{Cx}(12) = 0$$

or $R_{Cx} = 1000$ →

Lateral Loading STEP 5

RIGID FRAME STRUCTURES

Point of Inflection
P = 2000 lb
12 ft
30 ft
A B C
 R_{Ax} R_{Ay} R_{Cx} R_{Cy}

ANALYZE RIGHT PART

15ft P = 2000 lb
12 ft
 R_{Bx} R_{By}
 R_{Cx} $R_{Cy} = 800$

FOR RIGHT PART:

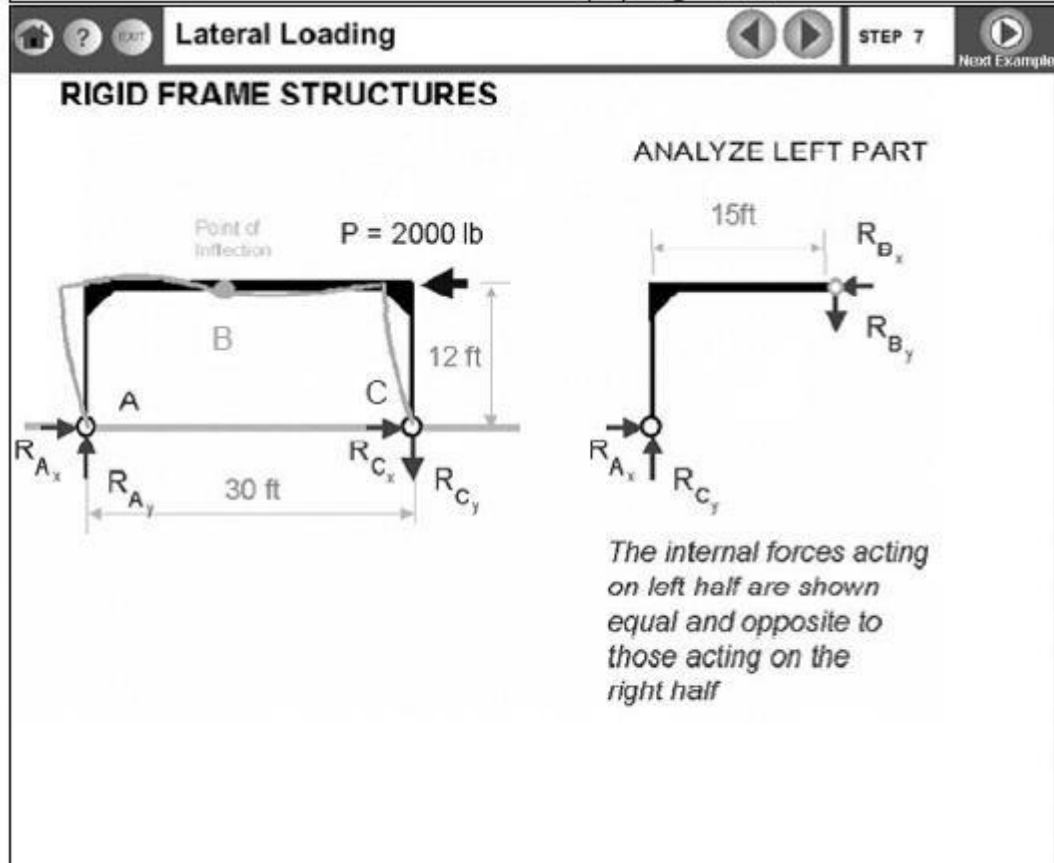
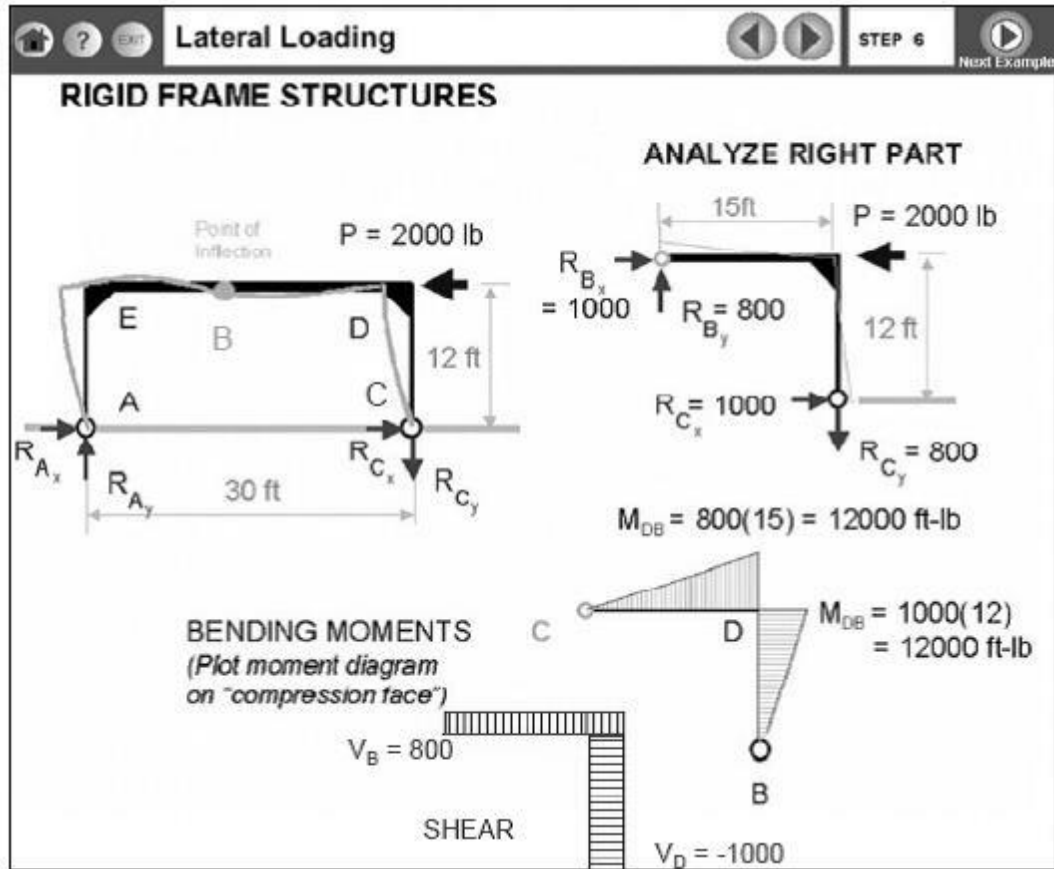
$$\Sigma F_y = 0$$

$$R_{By} - R_{Cy} = 0 \text{ or } R_{By} = 800 \uparrow$$

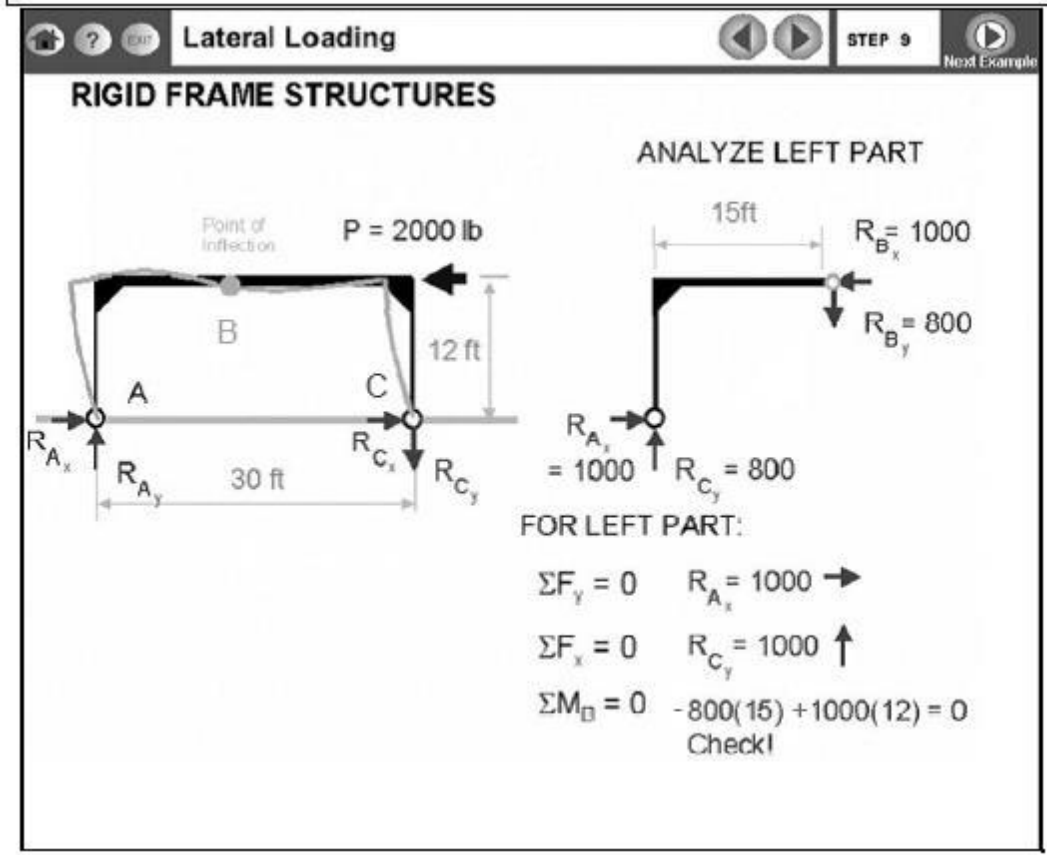
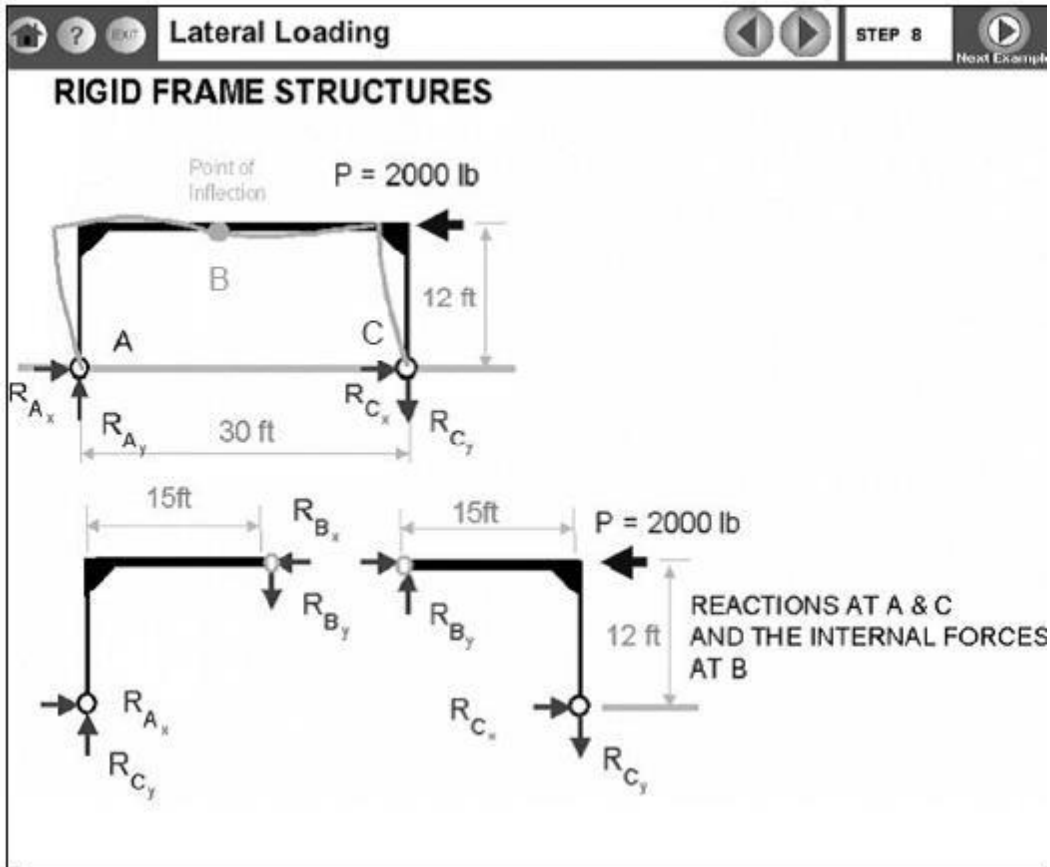
$$\Sigma F_x = 0$$

$$R_{Bx} + R_{Cx} = 2000 \text{ or } R_{Bx} = 1000 \rightarrow$$

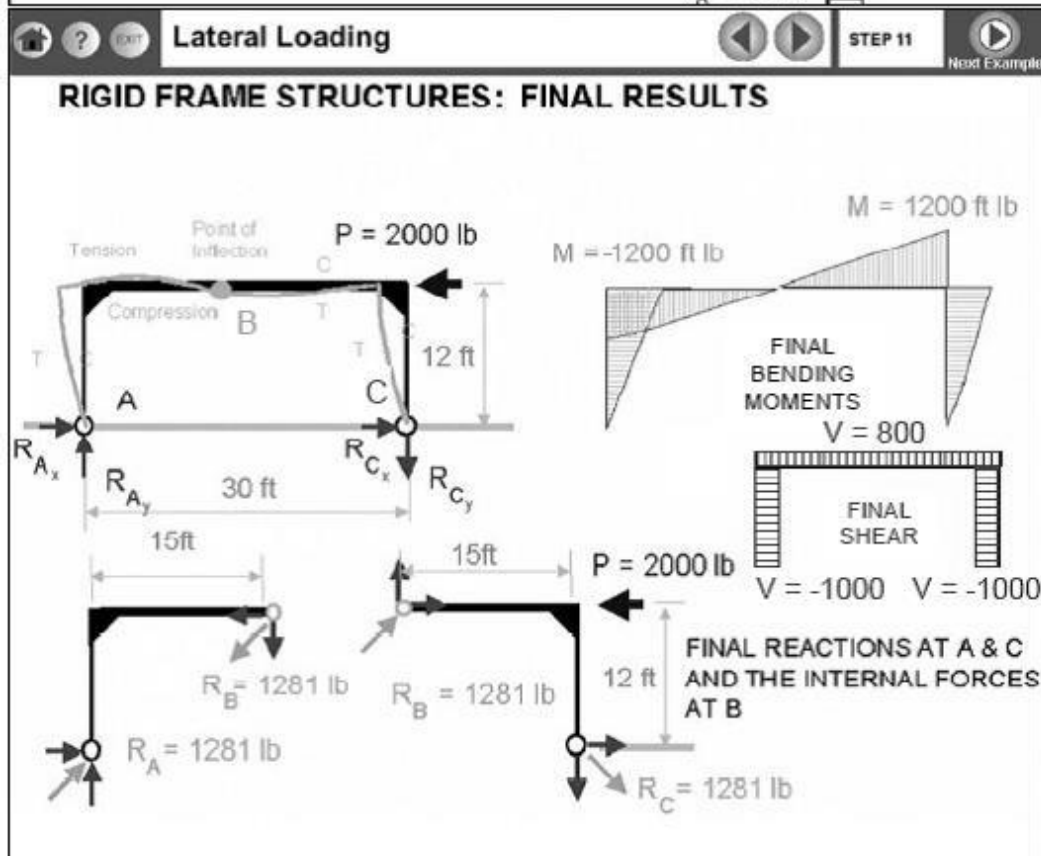
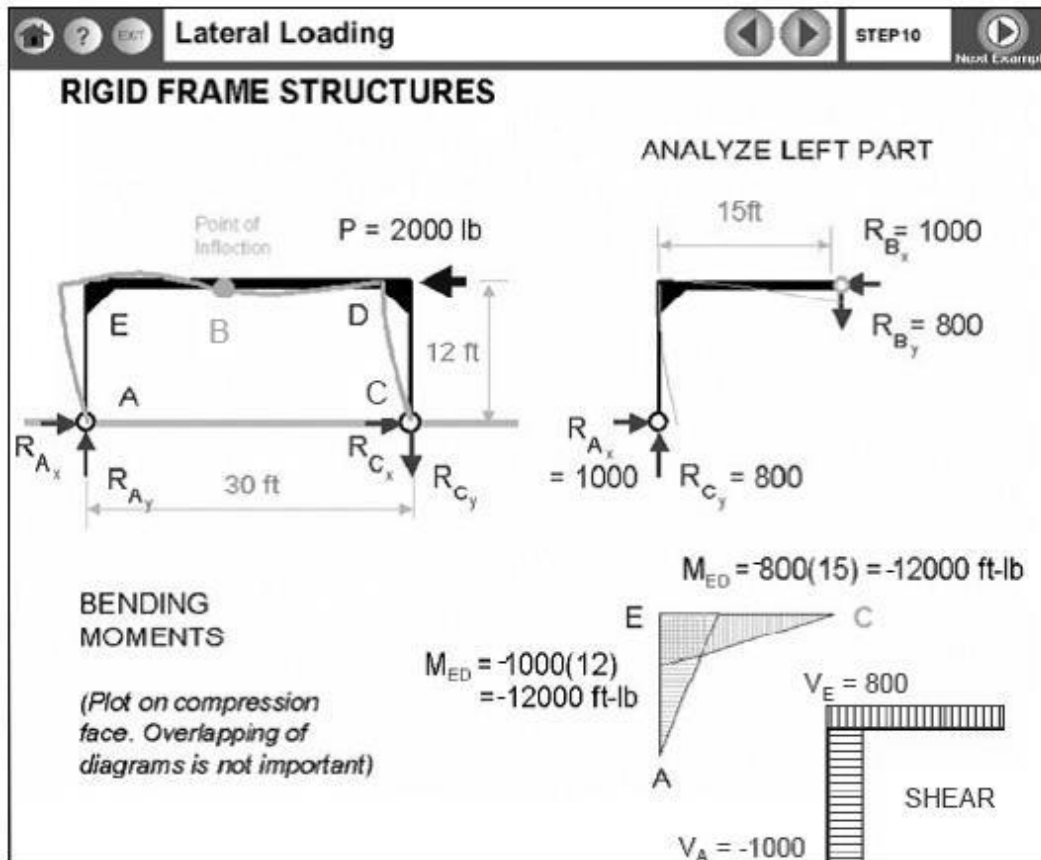
Example 1 (continued)



Example 1 (continued)

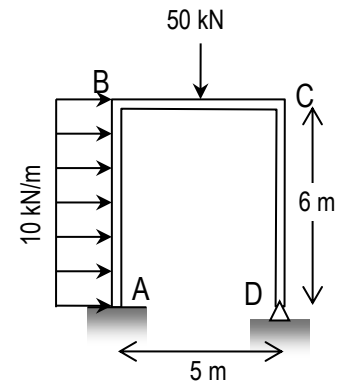


Example 1 (continued)



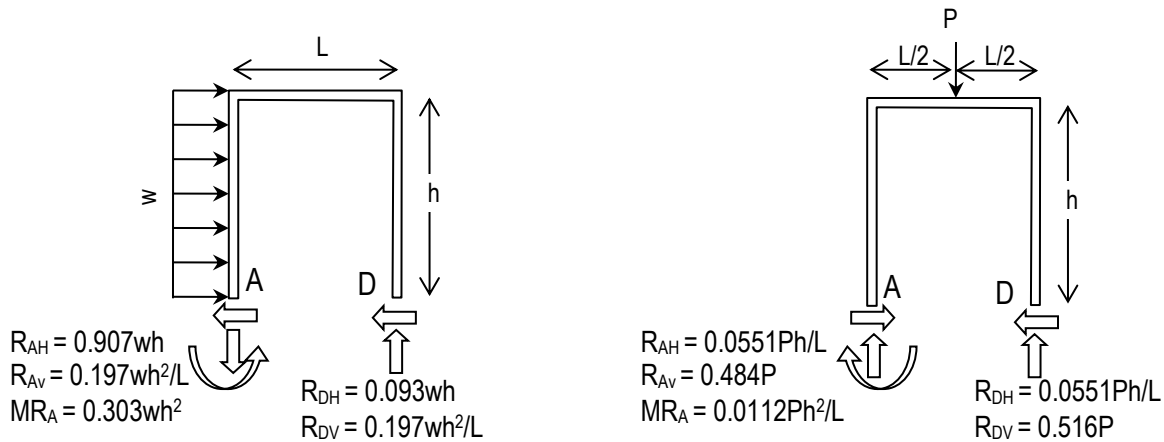
Example 2

The rigid frame shown at the right has the loading and supports as show. Using superpositioning from approximate analysis methods, draw the shear and bending moment diagrams.



Solution:

Reactions The two loading situations for which approximate reaction values are available are shown below. These values must be calculated *and added together* (allowed by superpositioning).



$$R_{AH} = -0.907wh + 0.0551Ph/L = -0.907(10^{kN/m})(6m) + \frac{0.0551(50kN)(6m)}{5m} = -51.11 \text{ kN}$$

$$R_{AV} = -0.197wh^2/L + 0.484P = \frac{-0.197(10^{kN/m})(6m)^2}{5m} + 0.484(50kN) = 10.02 \text{ kN}$$

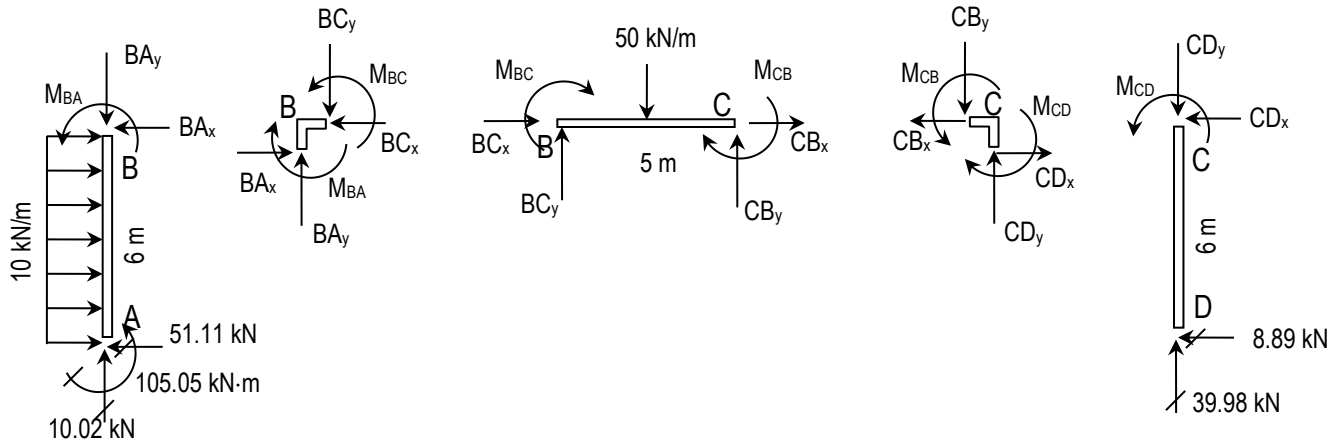
$$MR_A = 0.303wh^2 - 0.0112Ph^2/L = 0.303(10^{kN/m})(6m)^2 - \frac{0.0112(50kN)(6m)^2}{5m} = 105.05 \text{ kN-m}$$

$$R_{DH} = -0.093wh - 0.0551Ph/L = -0.093(10^{kN/m})(6m) - \frac{0.0551(50kN)(6m)}{5m} = -8.89 \text{ kN}$$

$$R_{DV} = 0.197wh^2/L + 0.516P = \frac{0.197(10^{kN/m})(6m)^2}{5m} + 0.516(50kN) = 39.98 \text{ kN}$$

Example 2 (continued)

Member End Forces The free-body diagrams of all the members and joints of the frame are shown below. The unknowns on the members are drawn as anticipated, and the opposite directions are drawn on the joint. We can begin the computation of internal forces with either member AB or CD, both of which have only three unknowns.



Member AB With the magnitudes of reaction forces at A known, the unknowns are at end B of BA_x , BA_y , and M_{BA} , which can be determined by applying $\sum F_x = 0$, $\sum F_y = 0$, and $\sum M_B = 0$. (NOTE: The reaction directions are assumed. When the results are positive, the assumption is verified.) Thus,

$$\begin{aligned}\sum F_x &= -51.11\text{kN} + 10\text{kN}(6\text{m}) - BA_x = 0 & BA_x &= 8.89\text{ kN}, & \sum F_y &= 10.02\text{kN} - BA_y = 0 & BA_y &= 10.02\text{ kN} \\ \sum M_A &= 105.05\text{ kN}\cdot\text{m} - 10\text{ kN}/\text{m}(6\text{m})(3\text{m}) + 8.89\text{kN}(6\text{m}) + M_{BA} = 0 & M_{BA} &= 21.16\text{kN}\cdot\text{m}\end{aligned}$$

Joint B Because the forces and moments must be equal and opposite, $BC_x = 8.89\text{ kN}$, $BC_y = 10.02\text{ kN}$ and $M_{BC} = 21.16\text{ kN}\cdot\text{m}$. (NOTE: The directions are opposite on the other side of the section cut.)

Member CD With the magnitudes of reaction forces at D known, the unknowns are at end C of CD_x , CD_y , and M_{CD} , which can be determined by applying $\sum F_x = 0$, $\sum F_y = 0$, and $\sum M_C = 0$. Thus,

$$\begin{aligned}\sum F_x &= -8.89\text{kN} - CD_x = 0 & CD_x &= -8.89\text{ kN (opposite arrow direction!)} \\ \sum F_y &= 39.98\text{kN} - CD_y = 0 & CD_y &= 39.98\text{ kN} \\ \sum M_D &= -8.89\text{kN}(6\text{m}) + M_{CD} = 0 & M_{CD} &= 53.34\text{ kN}\cdot\text{m}\end{aligned}$$

Joint C Because the forces and moments must be equal and opposite, $CB_x = -8.89\text{ kN}$ (again, the actual direction is opposite what is shown on the FBD), $CB_y = 39.98\text{ kN}$ and $M_{CB} = 53.34\text{ kN}\cdot\text{m}$

Example 2 (continued)

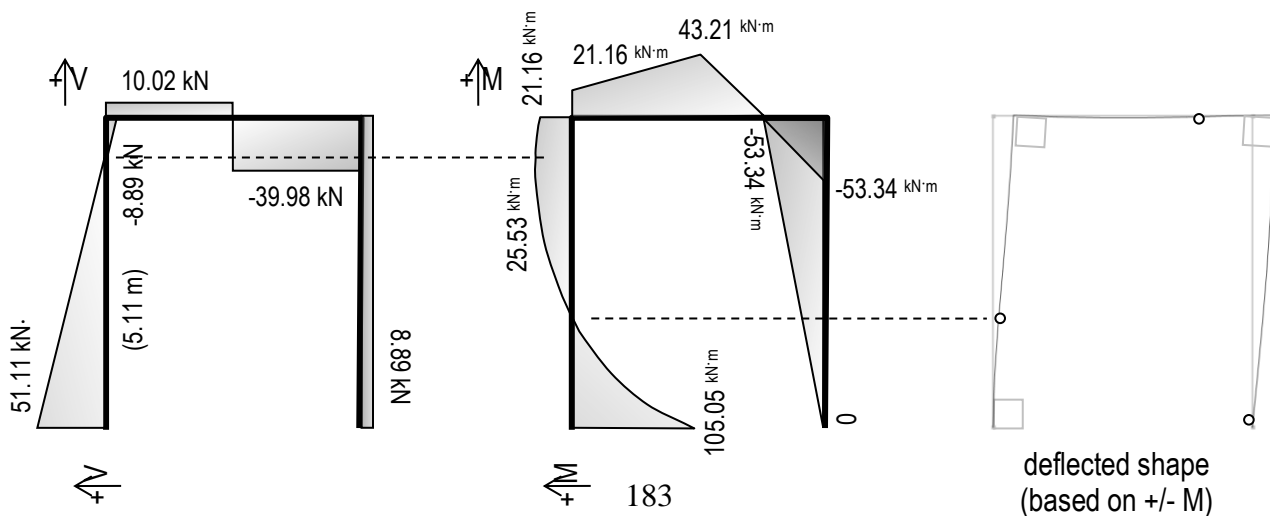
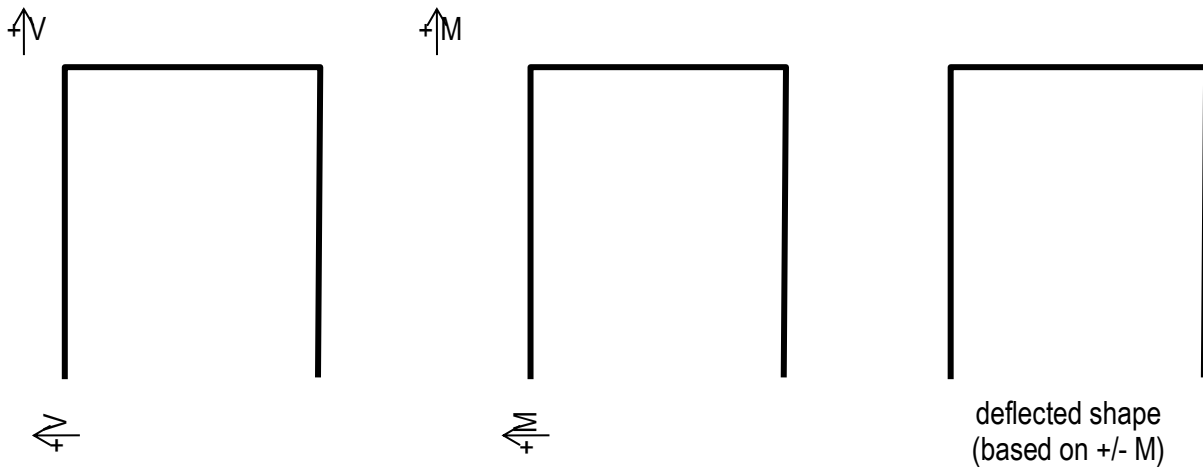
Member BC All forces are known, so equilibrium can be checked.

$$\sum F_x = 8.89kN + (-8.89kN) = 0 \quad (CB_x \text{ is shown to the right in the FBD but has a negative value.})$$

$$\sum F_y = 10.02kN - 50kN + 39.98kN = 0$$

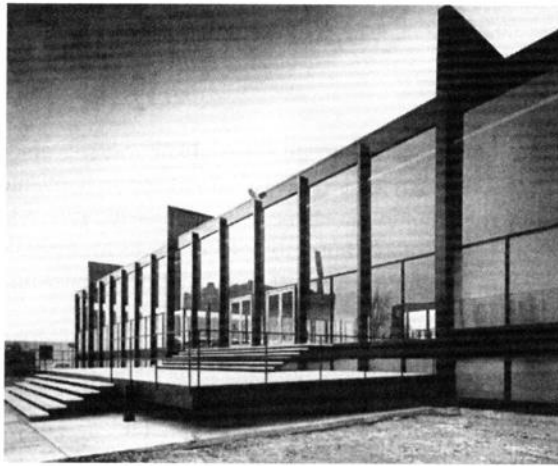
$$\sum M_B = -21.16^{kN\cdot m} - 50kN(2.5m) - 53.34^{kN\cdot m} + 39.98kN(5m) = 0.4 \approx 0$$

(Remember: To find the point of zero shear with a distributed load, divide the peak {triangle} shear by the distributed load; ex. $51.11kN / (10^{kN/m}) = 5.11 \text{ m}$)

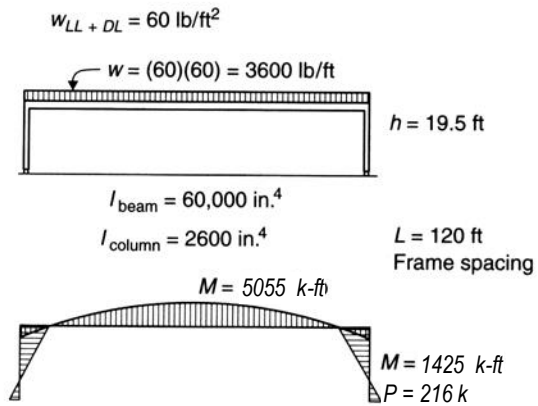


Example 3

Using Multiframe, verify the bending moment diagram for the example on page 335 in Figure 9.9:



(a) Crown Hall



(b) Results of structural analysis

Figure 9.9 The moment distribution illustrates the importance of relative stiffness considerations. The values obtained are quite different from those obtained by estimating points of inflection and using hand calculations.

Joint Coordinates (ft)

Joint	Label	x	y	z
1		0.000	0.000	0.000
2		0.000	19.500	0.000
3		120.000	19.500	0.000
4		120.000	0.000	0.000

Assuming steel ($E = 29,000$ ksi)

Section Properties

Section	I_x in ⁴
mies-slender	2380.000
mies-stiff	58700.001

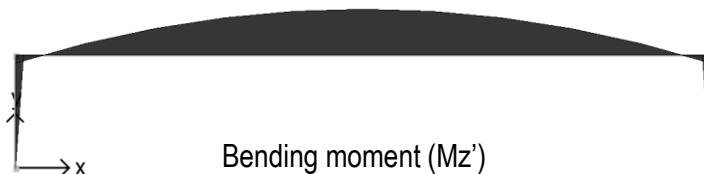
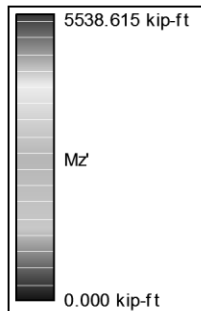
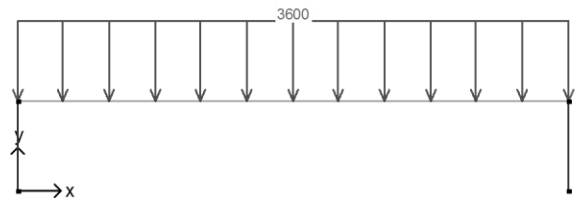
Sections

- mies-slender
- mies-stiff



Sections

- mies-slender
- mies-stiff



Example 3 (continued)

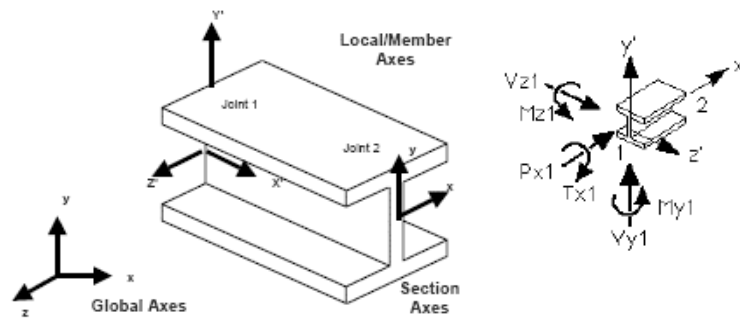
Displacement:



Maximum Actions for all members (column-1, beam-2, column-3):

	Memb	Label	Section	Sign	Px' kip	Vy' kip	Vz' kip	Tx' kip-ft	My' kip-ft	Mz' kip-ft	dy' in	dz' in
1	1		mies-slender	+ve	216.000	0.000	0.000	0.000	0.000	1424.716	0.486	0.000
2	1		mies-slender	-ve	0.000	-109.079	0.000	0.000	0.000	-702.318	-0.032	0.000
3	1		mies-slender	abs	216.000	109.079	0.000	0.000	0.000	1424.716	0.486	0.000
4	2		mies-stiff	+ve	109.079	216.000	0.000	0.000	0.000	1424.716	0.000	0.000
5	2		mies-stiff	-ve	0.000	-216.000	0.000	0.000	0.000	-5055.282	-7.326	0.000
6	2		mies-stiff	abs	109.079	216.000	0.000	0.000	0.000	5055.282	7.326	0.000
7	3		mies-slender	+ve	216.000	109.079	0.000	0.000	0.000	702.318	0.032	0.000
8	3		mies-slender	-ve	0.000	0.000	0.000	0.000	0.000	-1424.716	-0.486	0.000
9	3		mies-slender	abs	216.000	109.079	0.000	0.000	0.000	1424.716	0.486	0.000

(axes orientation reference)



Maximum Stresses for all members (column-1, beam-2, column-3):

	Memb	Label	Section	Sign	Sbz' top ksi	Sbz' bot ksi	Sx' ksi	Sx'+Sbz' top ksi	Sx'+Sbz' bot ksi	dy' in	dz' in
1	1		mies-sl	+ve	42.494	86.203	7.714	50.208	93.917	0.486	0.000
2	1		mies-slen	-ve	-86.203	-42.494	0.000	-78.489	-34.780	-0.032	0.000
3	1		mies-slen	abs	86.203	86.203	7.714	78.489	93.917	0.486	0.000
4	2		mies-sti	+ve	38.237	10.776	1.283	39.521	12.060	0.000	0.000
5	2		mies-stiff	-ve	-10.776	-38.237	0.000	-9.493	-36.954	-7.326	0.000
6	2		mies-stiff	abs	38.237	38.237	1.283	39.521	36.954	7.326	0.000
7	3		mies-sl	+ve	86.203	42.494	7.714	93.917	50.208	0.032	0.000
8	3		mies-slen	-ve	-42.494	-86.203	0.000	-34.780	-78.489	-0.486	0.000
9	3		mies-slen	abs	86.203	86.203	7.714	93.917	78.489	0.486	0.000

Beam-Column stress verification (combined stresses) when $d = 24$ in, $A = 28$ in². $I_x = 2380$ in⁴:

$$f_{\max} = \frac{P}{A} + \frac{M}{S} = \frac{P}{A} + \frac{Mc}{I} = \frac{216k}{28in^2} + \frac{1425^{k-ft} \cdot (24in/2)}{2380in^4} \cdot \frac{12in}{ft} = 7.71ksi + 86.22ksi = 93.93ksi$$