

Examples: Plate and Grids

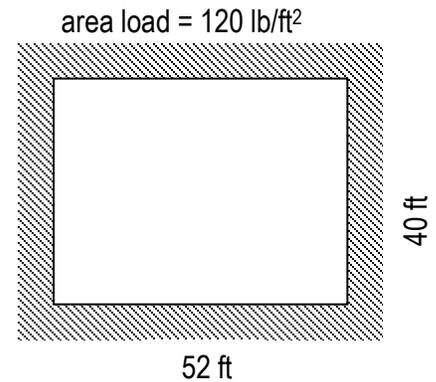
Example 1

What is the maximum positive and negative bending moments developed in a 52 x 40 ft fully fixed plate that carries a load of 120 lb/ft²?

SOLUTION:

The aspect ratio of the side lengths, a/b , must be determined and an appropriate coefficient chart must be found:

$a/b = 52/40 = 1.3$ (no units, and a is always the *bigger* number).



BENDING MOMENTS IN RECTANGULAR PLATES

Aspect ratio $\frac{a}{b}$	Simply supported on all four sides	Fixed on all four sides		Corner slabs fixed on two adjacent sides and free on two sides
		C_a	C_b	
1.0	$C_a = +0.0479$ $C_b = +0.0479$	$C_a = +0.0231$	$C_b = +0.0231$	$C_a = -0.29$ $C_b = -0.29$
1.3	$C_a = +0.0298$ $C_b = +0.0694$	$C_a = +0.0131$	$C_b = +0.0327$	$C_a = -0.35$ $C_b = -0.35$
1.5	$C_a = +0.0221$ $C_b = +0.0812$	$C_a = +0.0090$	$C_b = +0.0368$	$C_a = -0.37$ $C_b = -0.37$
2.0	$C_a = +0.0116$ $C_b = +0.1017$	$C_a = +0.0039$	$C_b = +0.0412$	$C_a = -0.43$ $C_b = -0.43$

Note: In all cases,
 $M_a = C_a wa^2$
 $M_b = C_b wb^2$

The coefficients for moment for the a side length and b side length for fixed support all sides and $a/b = 1.3$ are:

$$C_a = +0.0131 \text{ and } C_a = -0.0333 \quad C_b = +0.0327 \text{ and } C_b = -0.0687$$

The maximum moments are calculated with the formula in the table:

$$M_a(\text{positive}) = C_a wa^2 = 0.0131(120 \frac{\text{lb}}{\text{ft}^2})(52 \text{ ft})^2 = 4251 \frac{\text{lb-ft}}{\text{ft}}$$

$$M_a(\text{negative}) = C_a wa^2 = -0.0333(120 \frac{\text{lb}}{\text{ft}^2})(52 \text{ ft})^2 = -10,805 \frac{\text{lb-ft}}{\text{ft}}$$

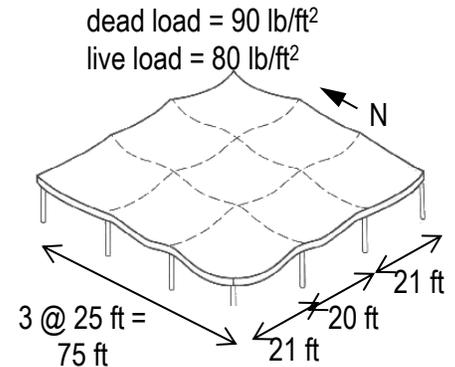
$$M_b(\text{positive}) = C_b wb^2 = 0.0327(120 \frac{\text{lb}}{\text{ft}^2})(40 \text{ ft})^2 = 6278 \frac{\text{lb-ft}}{\text{ft}}$$

$$M_b(\text{negative}) = C_b wb^2 = -0.0687(120 \frac{\text{lb}}{\text{ft}^2})(40 \text{ ft})^2 = -13,190 \frac{\text{lb-ft}}{\text{ft}}$$

Example 2

A two-way interior-bay flat plate (concrete) with the dimensions shown supports a live loading of 80 lb/ft² and has a dead load of 90 lb/ft². The columns can be assumed to be 18 inches square. Determine the design moments based on ACI-318, (ASCE-7) and the Direct Design method.

Also compare design moments for an exterior-interior bay



SOLUTION:

Determine the distributed load combinations:

$$w_u = 1.2D + 1.6L = 1.2(90 \text{ lb/ft}^2) + 1.6(80 \text{ lb/ft}^2) = 236 \text{ lb/ft}^2$$

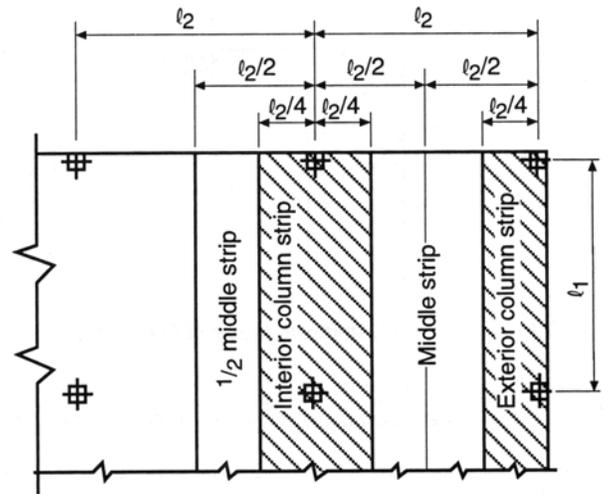
Determine the clear span length for the N-S direction:

$$l_n = l_1 - \frac{1}{2} \text{ column width} - \frac{1}{2} \text{ column width} = 25 \text{ ft} - \frac{1}{2} (18 \text{ in}/12 \text{ in/ft}) - \frac{1}{2} (18 \text{ in}/12 \text{ in/ft}) = 23.5 \text{ ft}$$

Because l_2 is not the same width on either side of an interior panel, it is taken as the average = $(21 \text{ ft} + 20 \text{ ft})/2 = 20.5 \text{ ft}$.

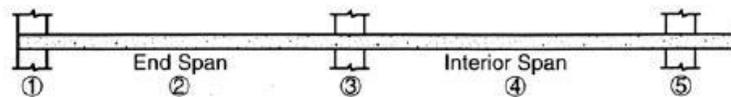
Total moment (to distribute to middle and interior column strip):

$$M_o = \frac{w_u l_2 l_n^2}{8} = \frac{(236 \text{ lb/ft}^2)(20.5 \text{ ft})(23.5 \text{ ft})^2}{8} = 333,973 \text{ lb-ft}$$



(a) Column strip for $l_2 \leq l_1$

Table 4-2 Flat Plate or Flat Slab Supported Directly on Columns



Slab Moments	End Span			Interior Span	
	1 Exterior Negative	2 Positive	3 First Interior Negative	4 Positive	5 Interior Negative
Total Moment	0.26 M_o	0.52 M_o	0.70 M_o	0.35 M_o	0.65 M_o
Column Strip	0.26 M_o	0.31 M_o	0.53 M_o	0.21 M_o	0.49 M_o
Middle Strip	0	0.21 M_o	0.17 M_o	0.14 M_o	0.16 M_o

Note: All negative moments are at face of support.

Interior Column Strip ($l_2 \leq l_1$):

The column strip width is $\frac{1}{4}$ the smaller of l_2 either side of the column:

$$\text{strip width} = \frac{1}{4} (21 \text{ ft}) + \frac{1}{4} (20 \text{ ft}) = 10.25 \text{ ft}$$

Example 2 (continued)

From Table 4.2, the maximum positive moment occurs in an end span:

$$M(\text{positive}) = 0.31M_o = (0.31)(333,973^{\text{lb-ft}}) = 103,532^{\text{lb-ft}}, \text{ distributed over } 10.25 \text{ ft} = 103,532 \text{ lb-ft}/(10.25 \text{ ft}) = 10,101 \text{ lb-ft/ft}$$

The positive design moment for an interior span is:

$$M(\text{positive}) = 0.21M_o = (0.21)(333,973^{\text{lb-ft}}) = 70,134^{\text{lb-ft}}, \text{ distributed over } 10.25 \text{ ft} = 70,134 \text{ lb-ft}/(10.25 \text{ ft}) = 6842 \text{ lb-ft/ft}$$

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

$$M(\text{negative}) = 0.53M_o = (0.53)(333,973^{\text{lb-ft}}) = 177,006^{\text{lb-ft}}, \text{ distributed over } 10.25 \text{ ft} = 177,006 \text{ lb-ft}/(10.25 \text{ ft}) = 17,269 \text{ lb-ft/ft}$$

The negative design moment at the exterior of an end span is:

$$M(\text{negative}) = 0.26M_o = (0.26)(333,973^{\text{lb-ft}}) = 86,833^{\text{lb-ft}}, \text{ distributed over } 10.25 \text{ ft} = 86,833 \text{ lb-ft}/(10.25 \text{ ft}) = 8472 \text{ lb-ft/ft}$$

The negative design moment for an interior span is:

$$M(\text{negative}) = 0.49M_o = (0.49)(333,973^{\text{lb-ft}}) = 163,647^{\text{lb-ft}}, \text{ distributed over } 10.25 \text{ ft} = 163,647 \text{ lb-ft}/(10.25 \text{ ft}) = 15,966 \text{ lb-ft/ft}$$

Middle Strip:

The width is the remaining width of I_2 between column strips:

$$\text{strip width} = 21 \text{ ft} - \frac{1}{4}(20 \text{ ft}) - \frac{1}{4}(21 \text{ ft}) = 10.75 \text{ ft}$$

From Table 4.2, the maximum positive moment occurs in an end span:

$$M(\text{positive}) = 0.21M_o = (0.21)(333,973^{\text{lb-ft}}) = 70,134^{\text{lb-ft}}, \text{ distributed over } 10.75 \text{ ft} = 70,134 \text{ lb-ft}/(10.75 \text{ ft}) = 6524 \text{ lb-ft/ft}$$

The positive design moment for an interior span is:

$$M(\text{positive}) = 0.14M_o = (0.14)(333,973^{\text{lb-ft}}) = 46,756^{\text{lb-ft}}, \text{ distributed over } 10.75 \text{ ft} = 46,756 \text{ lb-ft}/(10.75 \text{ ft}) = 4349 \text{ lb-ft/ft}$$

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

$$M(\text{negative}) = 0.17M_o = (0.17)(333,973^{\text{lb-ft}}) = 56,775^{\text{lb-ft}}, \text{ distributed over } 10.75 \text{ ft} = 56,775 \text{ lb-ft}/(10.75 \text{ ft}) = 5281 \text{ lb-ft/ft}$$

Example 2 (continued)

There is no negative design moment at the exterior of an end span.

The negative design moment for an interior span is:

$$M(\text{negative}) = 0.16M_o = (0.16)(333,973^{\text{lb-ft}}) = 53,436^{\text{lb-ft}}, \text{ distributed over } 10.75 \text{ ft} = 53,436 \text{ lb-ft}/(10.75 \text{ ft}) = 4971 \text{ lb-ft/ft}$$

Exterior Column Strip:

The value to use for l_2 for an edge strip includes the distance to the outside of the columns = $21 \text{ ft} + \frac{1}{2}(18 \text{ in}/12 \text{ in/ft}) = 21.75 \text{ ft}$

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} = \frac{(236 \text{ lb/ft}^2)(21.75 \text{ ft})(23.5 \text{ ft})^2}{8} = 354,337^{\text{lb-ft}}$$

The width is $\frac{1}{4} l_2$ one side of the column plus the distance to the slab edge:

$$\text{strip width} = \frac{1}{4}(21 \text{ ft}) + \frac{1}{2}(18 \text{ in}/12 \text{ in/ft}) = 6 \text{ ft}$$

So a comparison to the interior column strip maximum positive moment occurring in an end span is:

$$M(\text{positive}) = 0.31M_o = (0.31)(354,337^{\text{lb-ft}}) = 109,844^{\text{lb-ft}}, \text{ distributed over } 6 \text{ ft} = 109,844 \text{ lb-ft}/(6 \text{ ft}) = 18,307 \text{ lb-ft/ft} \text{ (as opposed to } 10,101 \text{ lb-ft/ft)}$$

For the E-W direction:

Because the adjacent spans are not the same length, the longer span, which is the END span will be larger:

$$l_n = l_1 - \frac{1}{2} \text{ column width} - \frac{1}{2} \text{ column width} = 21 \text{ ft} - \frac{1}{2}(18 \text{ in}/12 \text{ in/ft}) - \frac{1}{2}(18 \text{ in}/12 \text{ in/ft}) = 19.5 \text{ ft}$$

Because l_2 is 25 ft.

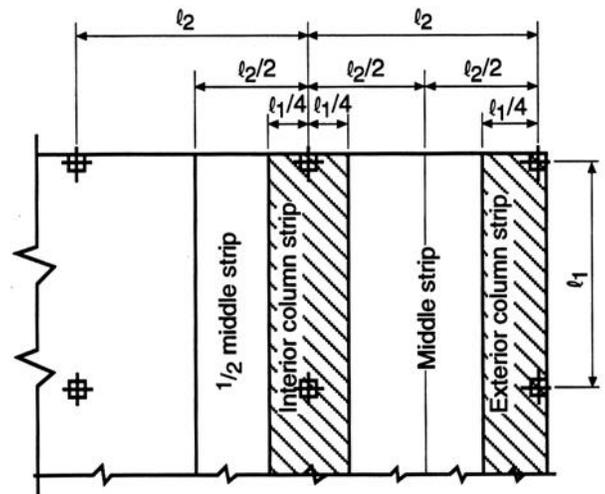
Total moment (to distribute to middle and interior column strip):

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} = \frac{(236 \text{ lb/ft}^2)(25 \text{ ft})(19.5 \text{ ft})^2}{8} = 280,434^{\text{lb-ft}}$$

Interior Column Strip END Spans ($l_2 > l_1$):

The column strip width is $\frac{1}{4}$ the **smaller** of l_1 and l_2 either side of the column:

$$\text{strip width} = \frac{1}{4}(21 \text{ ft}) + \frac{1}{4}(21 \text{ ft}) = 10.5 \text{ ft}$$



(b) Column strip for $l_2 > l_1$

From Table 4.2, the maximum positive moment occurs in an end span:

$$M(\text{positive}) = 0.31M_o = (0.31)(280,434^{\text{lb-ft}}) = 86,935^{\text{lb-ft}}, \text{ distributed over } 10.5 \text{ ft} = 86,935 \text{ lb-ft}/(10.5 \text{ ft}) = 8279 \text{ lb-ft/ft}$$

Example 2 (continued)

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

$$M(\text{negative}) = 0.53M_o = (0.53)(280,434^{\text{lb-ft}}) = 148,630^{\text{lb-ft}}, \text{ distributed over } 10.5 \text{ ft} = 148,630 \text{ lb-ft}/(10.5 \text{ ft}) = 14,155 \text{ lb-ft/ft}$$

The negative design moment at the exterior of an end span is:

$$M(\text{negative}) = 0.26M_o = (0.26)(280,434^{\text{lb-ft}}) = 72,913^{\text{lb-ft}}, \text{ distributed over } 10.5 \text{ ft} = 72,913 \text{ lb-ft}/(10.5 \text{ ft}) = 6944 \text{ lb-ft/ft}$$

Middle Strip END Spans:

The width is the remaining width of l_2 between column strips:

$$\text{strip width} = 25 \text{ ft} - \frac{1}{4}(21 \text{ ft}) - \frac{1}{4}(21 \text{ ft}) = 14.5 \text{ ft}$$

From Table 4.2, the maximum positive moment occurs in an end span:

$$M(\text{positive}) = 0.21M_o = (0.21)(280,434^{\text{lb-ft}}) = 58,891^{\text{lb-ft}}, \text{ distributed over } 14.5 \text{ ft} = 58,891 \text{ lb-ft}/(14.5 \text{ ft}) = 4061 \text{ lb-ft/ft}$$

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

$$M(\text{negative}) = 0.17M_o = (0.17)(280,434^{\text{lb-ft}}) = 47,674^{\text{lb-ft}}, \text{ distributed over } 14.5 \text{ ft} = 47,674 \text{ lb-ft}/(14.5 \text{ ft}) = 3288 \text{ lb-ft/ft}$$

There is no negative design moment at the exterior of an end span.

Exterior Column Strip END Spans:

The value to use for l_2 for an edge strip includes the distance to the outside of the columns = $25 \text{ ft} + \frac{1}{2}(18 \text{ in}/12 \text{ in/ft}) = 25.75 \text{ ft}$

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} = \frac{(236 \text{ lb/ft}^2)(25.75 \text{ ft})(19.5 \text{ ft})^2}{8} = 288,847^{\text{lb-ft}}$$

The width is $\frac{1}{4}l_1$ (because it is smaller than l_2) one side of the column plus the distance to the slab edge:

$$\text{strip width} = \frac{1}{4}(21 \text{ ft}) + \frac{1}{2}(18 \text{ in}/12 \text{ in/ft}) = 6 \text{ ft}$$

So a comparison to the interior column END strip maximum positive moment occurring in an end span is:

$$M(\text{positive}) = 0.31M_o = (0.31)(288,847^{\text{lb-ft}}) = 89,543^{\text{lb-ft}}, \text{ distributed over } 6 \text{ ft} = 89,543 \text{ lb-ft}/(6 \text{ ft}) = 14,923 \text{ lb-ft/ft} \\ \text{(as opposed to } 8279 \text{ lb-ft/ft)}$$

Example 2 (continued)

TABLE OF DESIGN MOMENTS

slab moments / ft	End Span			Interior Span	
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
NS column strip - interior	8472 lb-ft/ft	10,101 lb-ft/ft	17,269 lb-ft/ft	6842 lb-ft/ft	15,966 lb-ft/ft
NS middle strip	0	6524 lb-ft/ft	5281 lb-ft/ft	4349 lb-ft/ft	4971 lb-ft/ft
NS column strip - edge	15,355 lb-ft/ft	18,307 lb-ft/ft	31,300 lb-ft/ft	12,402 lb-ft/ft	28,937 lb-ft/ft
EW column strip - interior	6944 lb-ft/ft	8279 lb-ft/ft	14,155 lb-ft/ft	5048 lb-ft/ft	11,779 lb-ft/ft
EW middle strip	0	4061 lb-ft/ft	3288 lb-ft/ft	2437 lb-ft/ft	5686 lb-ft/ft
EW column strip - edge	12,517 lb-ft/ft	14,923 lb-ft/ft	25,515 lb-ft/ft	6066 lb-ft/ft	6933 lb-ft/ft