Examples:
Plate and Grids

Example 1

What is the maximum positive and negative bending moments developed in a 52 x 40 ft fully fixed plate that carries a load of 120 lb/ft$^2$?

SOLUTION:
The aspect ratio of the side lengths, $a/b$, must be determined and an appropriate coefficient chart must be found:

$a/b = 52/40 = 1.3$ (no units, and $a$ is always the bigger number).

<table>
<thead>
<tr>
<th>Aspect ratio $a/b$</th>
<th>Simply supported on all four sides</th>
<th>Fixed on all four sides</th>
<th>Corner slabs fixed on two adjacent sides and free on two sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>$C_a = +0.0479$, $C_a = +0.0231$, $C_a = -0.0513$</td>
<td>$C_a = -0.29$</td>
<td>$C_b = +0.0479$, $C_b = +0.0231$, $C_b = -0.0513$</td>
</tr>
<tr>
<td>1.3</td>
<td>$C_a = +0.0298$, $C_a = +0.0131$, $C_a = -0.0333$</td>
<td>$C_a = -0.35$</td>
<td>$C_b = +0.0694$, $C_b = +0.0327$, $C_b = -0.0687$</td>
</tr>
<tr>
<td>1.5</td>
<td>$C_a = +0.0221$, $C_a = +0.0090$, $C_a = -0.0253$</td>
<td>$C_a = -0.37$</td>
<td>$C_b = +0.0812$, $C_b = +0.0368$, $C_b = -0.0757$</td>
</tr>
<tr>
<td>2.0</td>
<td>$C_a = +0.0116$, $C_a = +0.0039$, $C_a = -0.0143$</td>
<td>$C_a = -0.43$</td>
<td>$C_b = +0.1017$, $C_b = +0.0412$, $C_b = -0.0829$</td>
</tr>
</tbody>
</table>

Note: In all cases,

$M_a = C_a wa^2$

$M_b = C_b wb^2$

The coefficients for moment for the $a$ side length and $b$ side length for fixed support all sides and $a/b = 1.3$ are:

$C_a = +0.0131$ and $C_a = -0.0333$  
$C_b = +0.0327$ and $C_b = -0.0687$

The maximum moments are calculated with the formula in the table:

$M_a(positive) = C_a wa^2 = 0.0131(120 \frac{lb}{ft})(52 ft)^2 = 4251 \frac{lb-ft}{ft}$

$M_a(negative) = C_a wa^2 = -0.0333(120 \frac{lb}{ft})(52 ft)^2 = -10,805 \frac{lb-ft}{ft}$

$M_b(positive) = C_b wb^2 = 0.0327(120 \frac{lb}{ft})(40 ft)^2 = 6278 \frac{lb-ft}{ft}$

$M_b(negative) = C_b wb^2 = -0.0687(120 \frac{lb}{ft})(40 ft)^2 = -13,190 \frac{lb-ft}{ft}$
Example 2

A two-way interior-bay flat plate (concrete) with the dimensions shown supports a live loading of 80 lb/ft² and has a dead load of 90 lb/ft². The columns can be assumed to be 18 inches square. Determine the design moments based on ACI-318, (ASCE-7) and the Direct Design method.

Also compare design moments for an exterior-interior bay

SOLUTION:

Determine the distributed load combinations:

\[ w_u = 1.2D + 1.6L = 1.2(90 \text{ lb/ft}^2) + 1.6(80 \text{ lb/ft}^2) = 236 \text{ lb/ft}^2 \]

Determine the clear span length for the N-S direction:

\[ l_n = l_1 - \frac{1}{2} \text{ column width} - \frac{1}{2} \text{ column width} \]
\[ = 25 \text{ ft} - \frac{1}{2} (18 \text{ in/12 in/ft}) - \frac{1}{2} (18 \text{ in/12 in/ft}) = 23.5 \text{ ft} \]

Because \( l_2 \) is not the same width on either side of an interior panel, it is taken as the average = \( (21 \text{ ft} + 20 \text{ ft})/2 = 20.5 \text{ ft} \).

Total moment (to distribute to middle and interior column strip):

\[ M_o = \frac{w_u l_2 l_n^2}{8} = \frac{(236 \text{ lb/ft}^2)(20.5 \text{ ft})(23.5 \text{ ft})^2}{8} = 333,973 \text{ lb-ft} \]

Table 4-2  Flat Plate or Flat Slab Supported Directly on Columns

<table>
<thead>
<tr>
<th>Slab Moments</th>
<th>End Span</th>
<th>Interior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 Exterior Negative</td>
<td>2 Positive</td>
</tr>
<tr>
<td>Total Moment</td>
<td>0.26 ( M_0 )</td>
<td>0.52 ( M_0 )</td>
</tr>
<tr>
<td>Column Strip</td>
<td>0.26 ( M_0 )</td>
<td>0.31 ( M_0 )</td>
</tr>
<tr>
<td>Middle Strip</td>
<td>0</td>
<td>0.21 ( M_0 )</td>
</tr>
</tbody>
</table>

Note: All negative moments are at face of support.

Interior Column Strip \( l_2 \leq l_1 \):

The column strip width is \( \frac{1}{4} \) the smaller of \( l_2 \) either side of the column:

strip width = \( \frac{1}{4} (21 \text{ ft}) + \frac{1}{4} (20 \text{ ft}) = 10.25 \text{ ft} \)
Example 2 (continued)
From Table 4.2, the maximum positive moment occurs in an end span:
\[ M_{(positive)} = 0.31M_o = (0.31)(333,973^{lb-ft}) = 103,532^{lb-ft} \text{, distributed over 10.25 ft} = 103,532 \text{ lb-ft/(10.25 ft)} = 10,101 \text{ lb-ft/ft} \]

The positive design moment for an interior span is:
\[ M_{(positive)} = 0.21M_o = (0.21)(333,973^{lb-ft}) = 70,134^{lb-ft} \text{, distributed over 10.25 ft} = 70,134 \text{ lb-ft/(10.25 ft)} = 6842 \text{ lb-ft/ft} \]

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:
\[ M_{(negative)} = 0.53M_o = (0.53)(333,973^{lb-ft}) = 177,006^{lb-ft} \text{, distributed over 10.25 ft} = 177,006 \text{ lb-ft/(10.25 ft)} = 17,269 \text{ lb-ft/ft} \]

The negative design moment at the exterior of an end span is:
\[ M_{(negative)} = 0.26M_o = (0.26)(333,973^{lb-ft}) = 86,833^{lb-ft} \text{, distributed over 10.25 ft} = 86,833 \text{ lb-ft/(10.25 ft)} = 8472 \text{ lb-ft/ft} \]

The negative design moment for an interior span is:
\[ M_{(negative)} = 0.49M_o = (0.49)(333,973^{lb-ft}) = 163,647^{lb-ft} \text{, distributed over 10.25 ft} = 163,647 \text{ lb-ft/(10.25 ft)} = 15,966 \text{ lb-ft/ft} \]

Middle Strip:

The width is the remaining width of \( l_2 \) between column strips:

\[ \text{strip width} = 21 \text{ ft} - \frac{1}{4} (20 \text{ ft}) - \frac{1}{4} (21 \text{ ft}) = 10.75 \text{ ft} \]

From Table 4.2, the maximum positive moment occurs in an end span:
\[ M_{(positive)} = 0.21M_o = (0.21)(333,973^{lb-ft}) = 70,134^{lb-ft} \text{, distributed over 10.75 ft} = 70,134 \text{ lb-ft/(10.75 ft)} = 6524 \text{ lb-ft/ft} \]

The positive design moment for an interior span is:
\[ M_{(positive)} = 0.14M_o = (0.14)(333,973^{lb-ft}) = 46,756^{lb-ft} \text{, distributed over 10.75 ft} = 46,756 \text{ lb-ft/(10.75 ft)} = 4349 \text{ lb-ft/ft} \]

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:
\[ M_{(negative)} = 0.17M_o = (0.17)(333,973^{lb-ft}) = 56,775^{lb-ft} \text{, distributed over 10.75 ft} = 56,775 \text{ lb-ft/(10.75 ft)} = 5281 \text{ lb-ft/ft} \]
Example 2 (continued)
There is no negative design moment at the exterior of an end span.

The negative design moment for an interior span is:

\[ M(\text{negative}) = 0.16M_o = (0.16)(333,973^{lb-ft}) = 53,436^{lb-ft} \text{, distributed over 10.75 ft} = 4971 \text{ lb-ft/ft} \]

Exterior Column Strip:

The value to use for \( l_2 \) for an edge strip includes the distance to the outside of the columns \( = 21 \text{ ft} + \frac{1}{2}(18 \text{ in/12 in/ft}) = 21.75 \text{ ft} \)

\[ M_o = \frac{w_o \ell_2^2 \ell_n^2}{8} = \frac{(236^{lb/ft})(21.75 \text{ ft})(23.5 \text{ ft})^2}{8} = 354,337^{lb-ft} \]

The width is \( \frac{1}{4}l_2 \) one side of the column plus the distance to the slab edge:

\[ \text{strip width} = \frac{1}{4}(21 \text{ ft}) + \frac{1}{2}(18 \text{ in/12 in/ft}) = 6 \text{ ft} \]

So a comparison to the interior column strip maximum positive moment occurring in an end span is:

\[ M(\text{positive}) = 0.31M_o = (0.31)(354,337^{lb-ft}) = 109,844^{lb-ft} \text{, distributed over 6 ft} = 109,844 \text{ lb-ft/(6 ft)} = 18,307 \text{ lb-ft/ft} \]

(as opposed to 10,101 lb-ft/ft)

For the E-W direction:

Because the adjacent spans are not the same length, the longer span, which is the END span will be larger:

\[ l_n = l_1 - \frac{1}{2} \text{ column width} - \frac{1}{2} \text{ column width} = 21 \text{ ft} - \frac{1}{2}(18 \text{ in/12 in/ft}) - \frac{1}{2}(18 \text{ in/12 in/ft}) = 19.5 \text{ ft} \]

Because \( l_2 \) is 25 ft.

Total moment (to distribute to middle and interior column strip):

\[ M_o = \frac{w_o \ell_2^2 \ell_n^2}{8} = \frac{(236^{lb/ft})(25 \text{ ft})(19.5 \text{ ft})^2}{8} = 280,434^{lb-ft} \]

Interior Column Strip END Spans \( l_2 > l_1 \):

The column strip width is \( \frac{1}{4} \) the smaller of \( l_1 \) and \( l_2 \) either side of the column:

\[ \text{strip width} = \frac{1}{4}(21 \text{ ft}) + \frac{1}{4}(21 \text{ ft}) = 10.5 \text{ ft} \]

From Table 4.2, the maximum positive moment occurs in an end span:

\[ M(\text{positive}) = 0.31M_o = (0.31)(280,434^{lb-ft}) = 86,935^{lb-ft} \text{, distributed over 10.5 ft} = 86,935 \text{ lb-ft/(10.5 ft)} = 8279 \text{ lb-ft/ft} \]

208
Example 2 (continued)
From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:
\[ M(\text{negative}) = 0.53 M_o = (0.53)(280,434 \text{lb-ft}) = 148,630 \text{lb-ft}, \text{ distributed over } 10.5 \text{ ft} = 148,630 \text{ lb-ft/(10.5 ft)} = 14,155 \text{ lb-ft/ft} \]

The negative design moment at the exterior of an end span is:
\[ M(\text{negative}) = 0.26 M_o = (0.26)(280,434 \text{lb-ft}) = 72,913 \text{lb-ft}, \text{ distributed over } 10.5 \text{ ft} = 72,913 \text{ lb-ft/(10.5 ft)} = 6944 \text{ lb-ft/ft} \]

Middle Strip END Spans:
The width is the remaining width of \( l_2 \) between column strips:
\[ \text{strip width} = 25 \text{ ft} - \frac{1}{4} (21 \text{ ft}) - \frac{1}{4} (21 \text{ ft}) = 14.5 \text{ ft} \]

From Table 4.2, the maximum positive moment occurs in an end span:
\[ M(\text{positive}) = 0.21 M_o = (0.21)(280,434 \text{lb-ft}) = 58,891 \text{lb-ft}, \text{ distributed over } 14.5 \text{ ft} = 58,891 \text{ lb-ft/(14.5 ft)} = 4061 \text{ lb-ft/ft} \]

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:
\[ M(\text{negative}) = 0.17 M_o = (0.17)(280,434 \text{lb-ft}) = 47,674 \text{lb-ft}, \text{ distributed over } 14.5 \text{ ft} = 47,674 \text{ lb-ft/(14.5 ft)} = 3288 \text{ lb-ft/ft} \]

There is no negative design moment at the exterior of an end span.

Exterior Column Strip END Spans:
The value to use for \( l_2 \) for an edge strip includes the distance to the outside of the columns = 25 ft + \( \frac{1}{4} \) (12 in/12 in/ft) = 25.75 ft
\[ M_o = \frac{w_n l_1^2 l_2^2}{8} = \frac{(236 \text{ lb/ft}) (25.75 \text{ ft})(19.5 \text{ ft})^2}{8} = 288,847 \text{lb-ft} \]

The width is \( \frac{1}{4} \) \( l_1 \) (because it is smaller than \( l_2 \)) one side of the column plus the distance to the slab edge:
\[ \text{strip width} = \frac{1}{4} (21 \text{ ft}) + \frac{1}{2} (18 \text{ in/12 in/ft}) = 6 \text{ ft} \]

So a comparison to the interior column END strip maximum positive moment occurring in an end span is:
\[ M(\text{positive}) = 0.31 M_o = (0.31)(288,847 \text{lb-ft}) = 89,543 \text{lb-ft}, \text{ distributed over 6 ft} = 89,543 \text{ lb-ft/(6 ft)} = 14,923 \text{ lb-ft/ft} \]
(as opposed to 8279 lb-ft/ft)
### Example 2 (continued)

#### TABLE OF DESIGN MOMENTS

<table>
<thead>
<tr>
<th>slab moments / ft</th>
<th>End Span</th>
<th>Interior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exterior Negative</td>
<td>Positive</td>
</tr>
<tr>
<td>NS column strip - interior</td>
<td>8472 lb-ft/ft</td>
<td>10,101 lb-ft/ft</td>
</tr>
<tr>
<td>NS middle strip</td>
<td>0</td>
<td>6524 lb-ft/ft</td>
</tr>
<tr>
<td>NS column strip - edge</td>
<td>15,355 lb-ft/ft</td>
<td>18,307 lb-ft/ft</td>
</tr>
<tr>
<td>EW column strip - interior</td>
<td>6944 lb-ft/ft</td>
<td>8279 lb-ft/ft</td>
</tr>
<tr>
<td>EW middle strip</td>
<td>0</td>
<td>4061 lb-ft/ft</td>
</tr>
<tr>
<td>EW column strip - edge</td>
<td>12,517 lb-ft/ft</td>
<td>14,923 lb-ft/ft</td>
</tr>
</tbody>
</table>