Columns & Trusses

APPLIED ARCHITECTURAL STRUCTURES:
STRUCTURAL ANALYSIS AND SYSTEMS
ARCH 631
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Lecture five

trusses & columns

www.nyc-architecture.com
Calibrated Peer Review / Turnitin
Trusses

- ancient (?) wood
- 1800’s analysis
- efficient
- long spans
Trusses

• comprised of straight members
• geometry with triangles is stable
• loads applied only at joints
Trusses

- 2 force members
  - compression
  - tension

- 3 members connected by 3 joints

- 2 more members need 1 more joint

\[
b = 2n - 3
\]
\[
(n = 2j - 3)
\]
Truss Analysis

• visualize compression and tension from deformed shape
Truss Analysis

• **Method of Joints**
  - $\sum F = 0$

• **Method of Sections**
  - $\sum F = 0$
  - $\sum M = 0$

• **Graphical Methods**

• **all rely on equilibrium**
  – of bodies
  – internal equilibrium

http://nisee.berkeley.edu/godden
Truss Connections

- “pins” – can’t resist rotation

Figure 4.8: Truss joints.

http://nisee.berkeley.edu/godden
Trusses

• require lateral bracing
• consider buckling
• indeterminate trusses
  – extra members
  – solvable with statics
    • cables can’t hold compression
  – displacement methods
    • elastic elongation
  – too few members, unstable
Space Trusses

- pyramid
- tetrahedron

http://nisee.berkeley.edu/godden
Space Trusses

- connections

- supports

(a) UNISTRUT (System U)  
(b) TRIODETIC  
(c) MERC (KK-ball)

(a) CORNER SUPPORTS  
(b) PERIMETER SUPPORTS  
(c) CROSSHEAD BEAMS

(a) COLUMN (POINT) SUPPORT  
(b) INVERTED PYRAMID  
PLAN (crosshead beam support)
Crystal Cathedral, Johnson 1980

http://nisee.berkeley.edu/godden
Expo 70 Festival Plaza

- Tange & Kamiya 1970
Jacob K. Javits Convention Center

- I.M. Pei 1980
Jacob K. Javits Convention Center

Figure 5.10: Javits Center, column details: (a) elevation, and (b through d) plan sections.
Louvre Museum Addition, I. M. Pei 1989
Truss Design

- variables
  - spans
  - depths
  - length of members
  - spacing
  - transverse beam spacing
  - pattern
  - materials; size & strength efficiency
Trusses

- common designs

- King post
- Pitched Pratt truss
- Pitched Howe truss
- Queen post
- Pitched Fink truss
- Parallel chord Pratt truss
- Parallel chord Howe truss
- Parallel chord crossed-diagonal truss

Constant forces in upper chords and no forces in diagonals (normally built with rigid joints if diagonals are omitted).
Trusses

- common designs
Truss Configurations

- **external factors**
  - roof form
  - openings

- **basic forms**
  - symmetrical loading
  - maximum bending
  - cables in tension only
Truss Configurations

• **parallel chords**
  – **verticals common**
  – **longer members in tension**
  – **often cross members - indeterminate**

• **funicular shapes**
  – **efficient**
  – **similar sized forces**
  – **some zero force**
Truss Configurations

• special shapes
  – Vierendeel
  – “frame”

• depth
  – depends on loads, span
  – rules of thumb, charts

• structurally
  – tension members can have holes!
  – compression members can buckle
  – 3D trusses stable

http://nisee.berkeley.edu/godden
Tools – Multiframe

• in classrooms and open access labs
Tools – Multiframe

- frame window
  - define truss members
    - or pre-defined truss
  - select points, assign supports
  - select members, assign section & assign pin ends

- load window
  - select points, add point load
Tools – Multiframe

- to run analysis choose
  - Analyze menu
    - Linear

- plot
  - choose options

- results
  - choose options
Columns

• compression members

• column behavior is length dependent
  – short: crush
  – long: buckle (sudden) in slender direction
  – end restraints

• bearing walls
  – continuous in one direction
  – can resist lateral forces
Columns

- column buckling
- crushing
Columns

- **stability**
  - **stable equilibrium**
    - \( P < P_{cr} \)
  - **neutral equilibrium**
    - \( P = P_{cr} \)
  - **unstable equilibrium**
    - \( P > P_{cr} \)
Buckling Load

- related to deflected shape \((P\Delta)\)
- shape of sine wave
- Euler’s Formula
- \(I\) minimum

\[
P_{\text{critical}} = \frac{\pi^2 EI_{\text{min}}}{(L)^2}
\]

- stiffness related to \(E, I, & L\)
Critical Stress

- short columns

\[ f_{\text{critical}} = \frac{P_{\text{actual}}}{A} < F_a \]

- slenderness ratio = \( \frac{L_e}{r} \) (L/d)

- radius of gyration = \( r = \sqrt{\frac{I}{A}} \)

\[ f_{\text{critical}} = \frac{P_{\text{critical}}}{A} = \frac{\pi^2 E A r^2}{A (L_e)^2} = \frac{\pi^2 E}{\left( \frac{L_e}{r} \right)^2} \]

\[ P_{\text{critical}} = \frac{\pi^2 E A}{\left( \frac{L_e}{r} \right)^2} \]
Critical Stresses

• when a column gets stubby, \( F_y \) will limit the load
• real world has loads with eccentricity
• buckling for steel when \( F_e < 0.44F_y \)

\[
\frac{KL}{r} > 4.71 \sqrt{\frac{E}{F_y}}
\]
Columns

- **end conditions affect shape**
- **effective length factor, \( k \)**

<table>
<thead>
<tr>
<th>Buckled shape of column shown by dashed line</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical ( K ) value</td>
<td>0.5</td>
<td>0.7</td>
<td>1.0</td>
<td>1.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Recommended design values when ideal conditions are approximated</td>
<td>0.65</td>
<td>0.80</td>
<td>1.0</td>
<td>1.2</td>
<td>2.10</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Bracing

- bracing affects shape of buckle in one direction
- both directions should be checked!
Columns and Walls

- bearing & shear
Columns

- typical cross sections
Centric & Eccentric Loading

- **centric**
  - allowable stress from strength or buckling

- **eccentric**
  - combined stresses
Combined Stresses

– axial + bending

\[ f_{\text{max}} = \frac{P}{A} + \frac{Mc}{I} \]
\[ M = P \cdot e \]

– design

\[ f_{\text{max}} \leq F_{cr} = \frac{f_{cr}}{F \cdot S}. \]
Column Materials

- **wood**
- **steel**
  - *W sections can buckle about both axes*
- **concrete & masonry**
  - *compression primarily in concrete*
  - *reinforcement for bending*